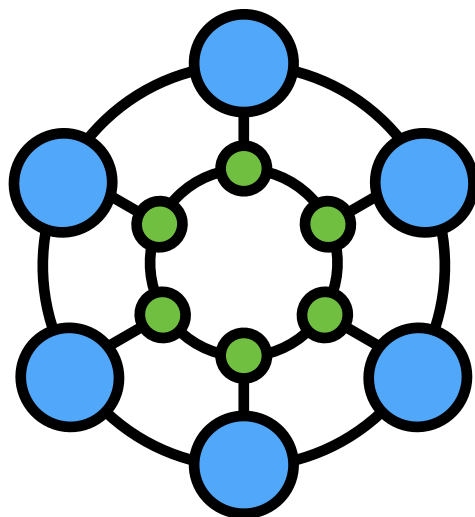
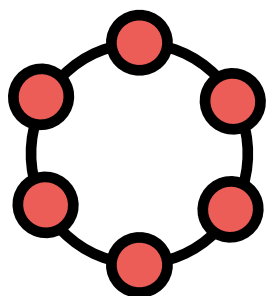


Permutation Search of Tensor Network Structures via Local Sampling

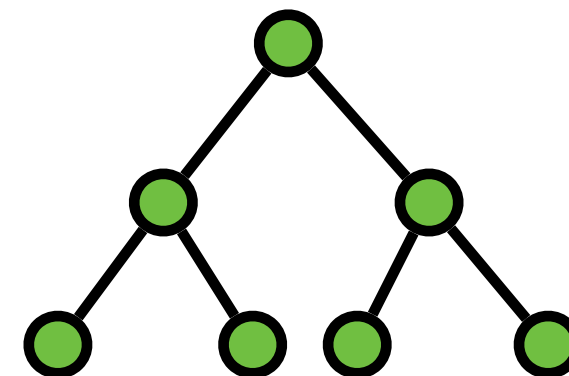
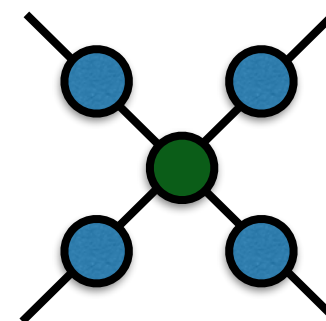


Chao Li

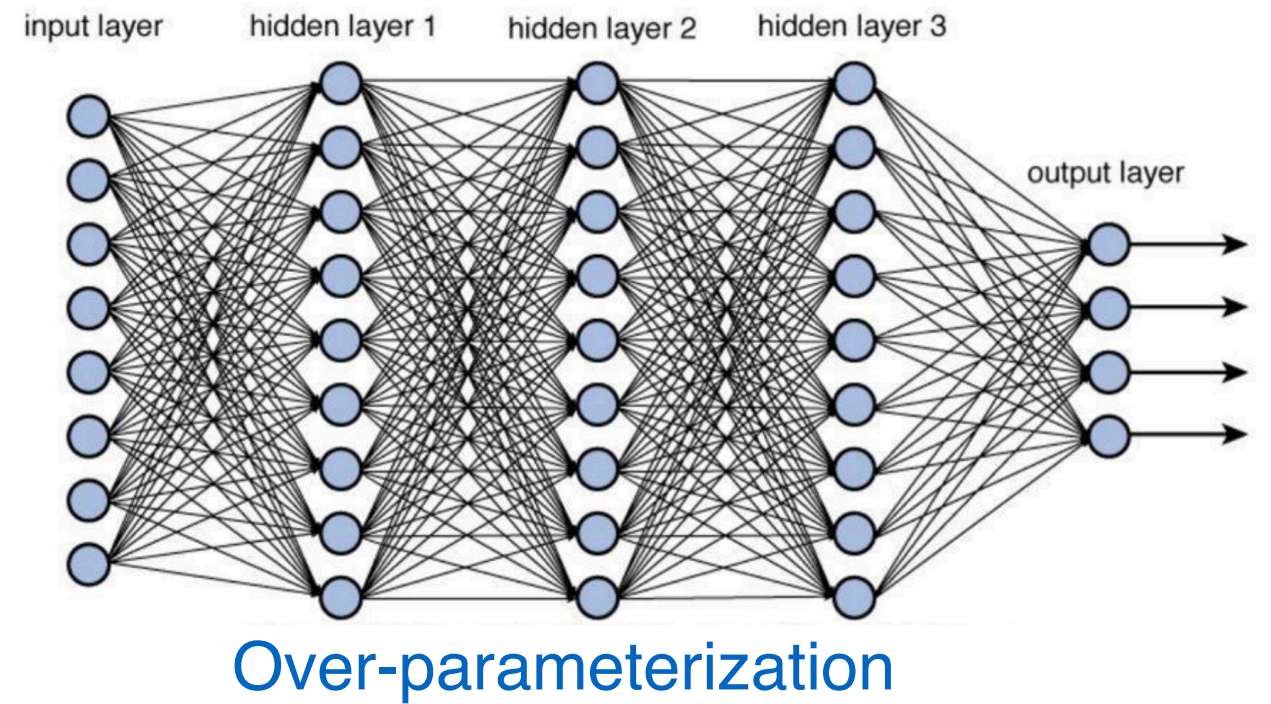
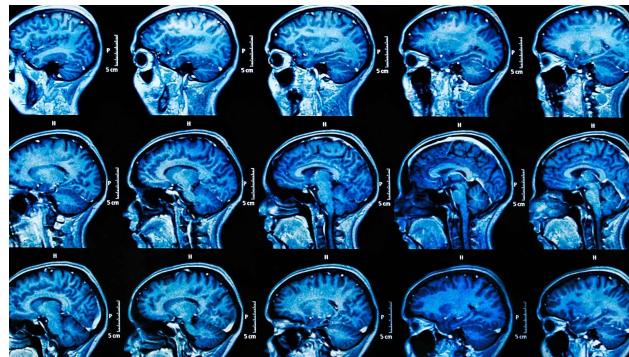
RIKEN-AIP

Paper ID: 2502

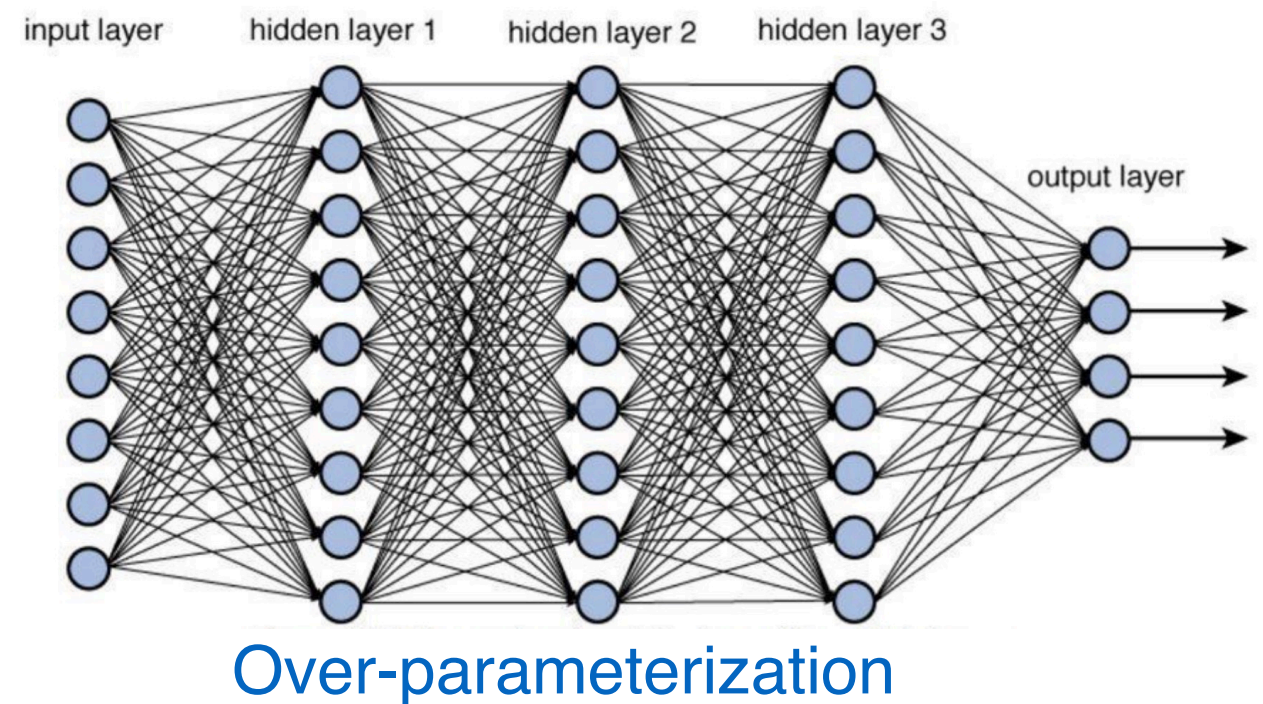
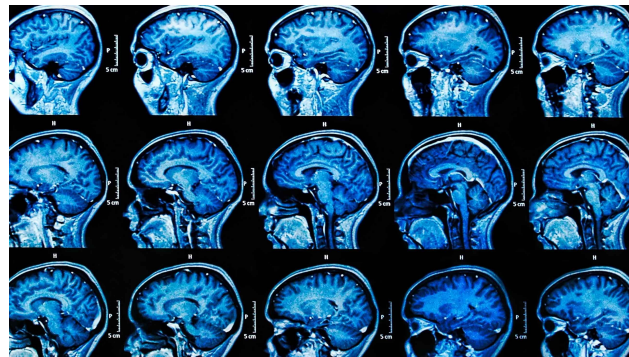
chao.li@riken.jp



Dimension Explosion in ML



Dimension Explosion in ML



- ▶ **Tensor network (TN)** is a promising framework to resolve it.
- ▶ **Statistical learning** and **probability density estimation**
- ▶ **Latent factor models** and data decomposition
- ▶ **Deep learning**

Tensor Network (TN)

Tensor Network (TN)

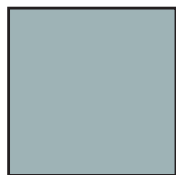
TENSOR is a multi-way number array.



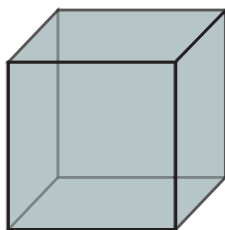
Order-0
Tensor
scalar



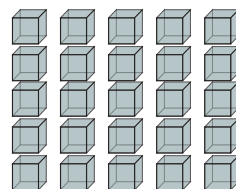
Order-1
Tensor
vector



Order-2
Tensor
matrix



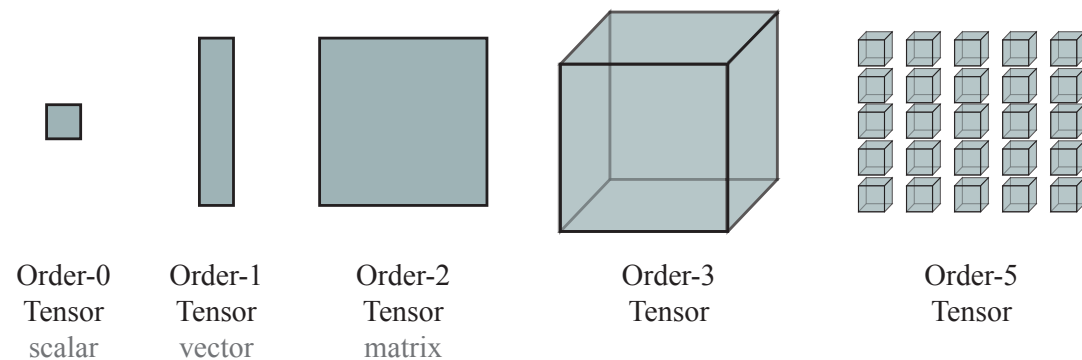
Order-3
Tensor



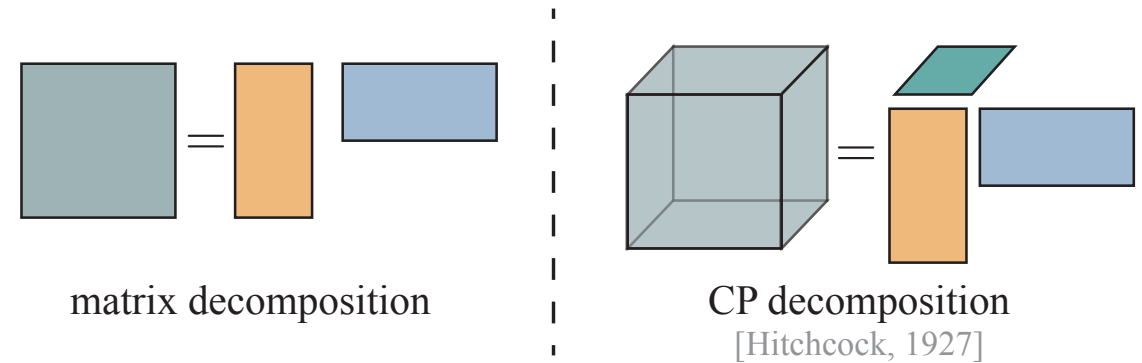
Order-5
Tensor

Tensor Network (TN)

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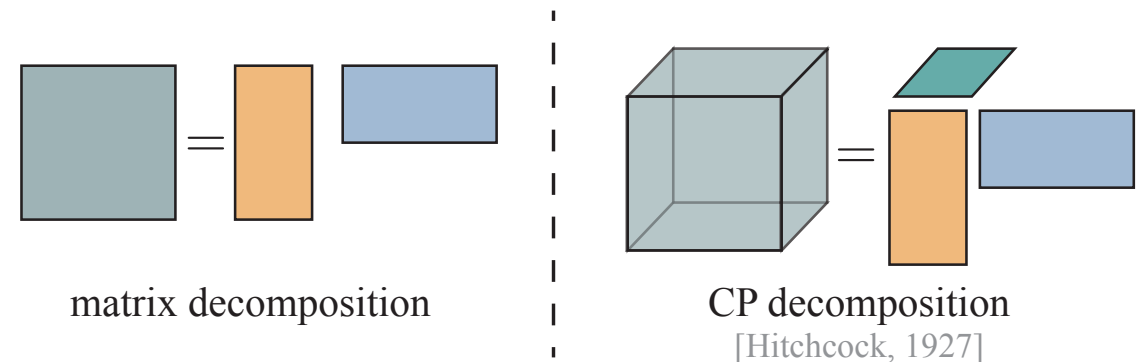
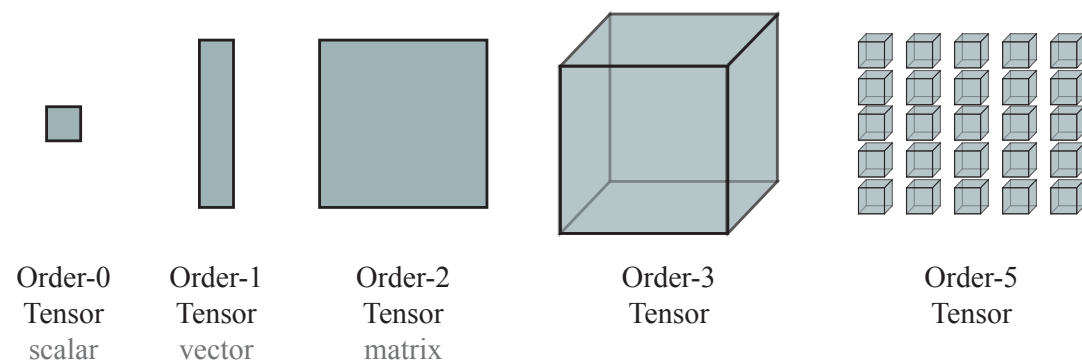
CONTRACTION: “tensor-tensor” multiplication.



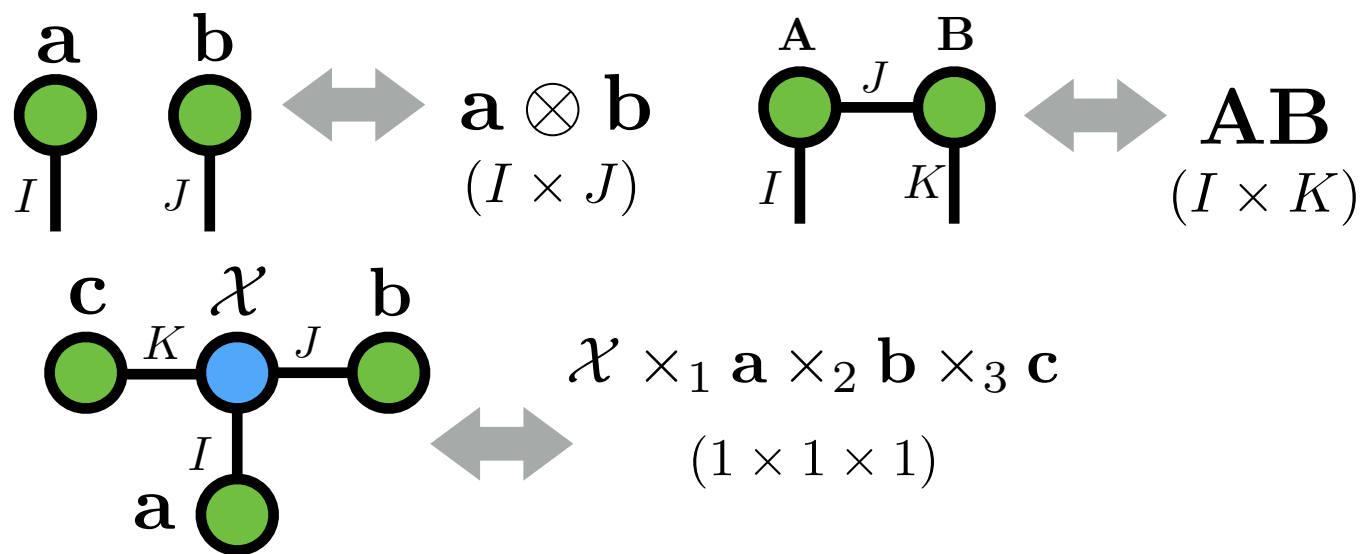
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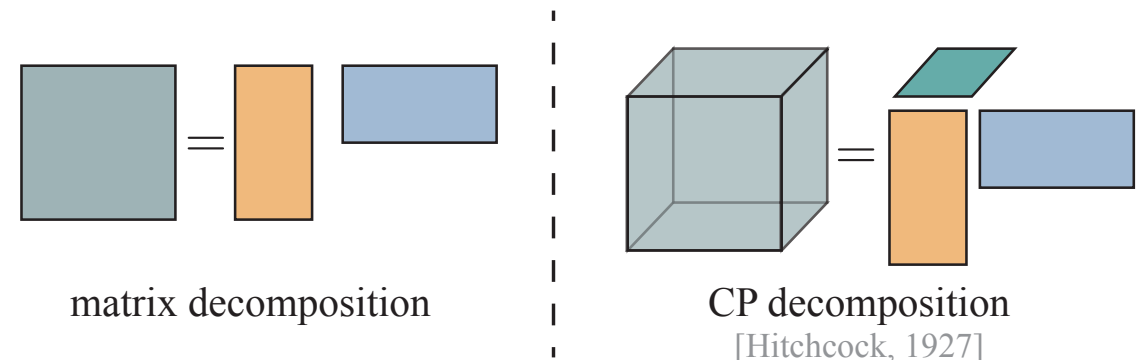
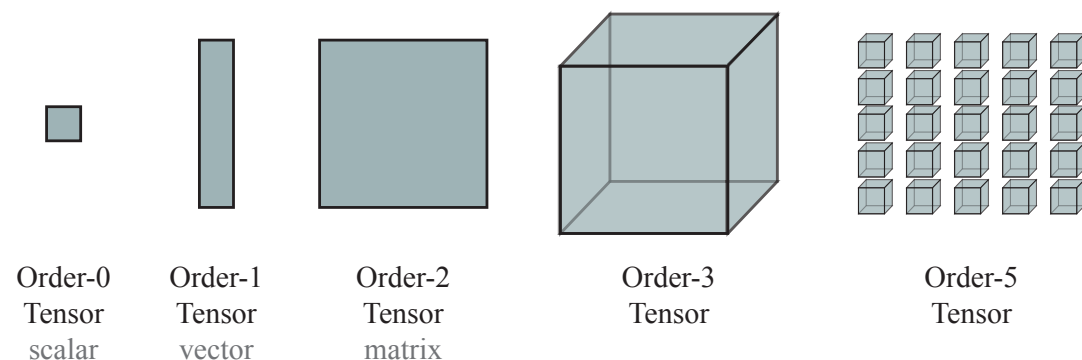
TENSOR NETWORK (TN) is an edge-labeled **graph**, which depicts a sequence of contractions among multiple tensors.



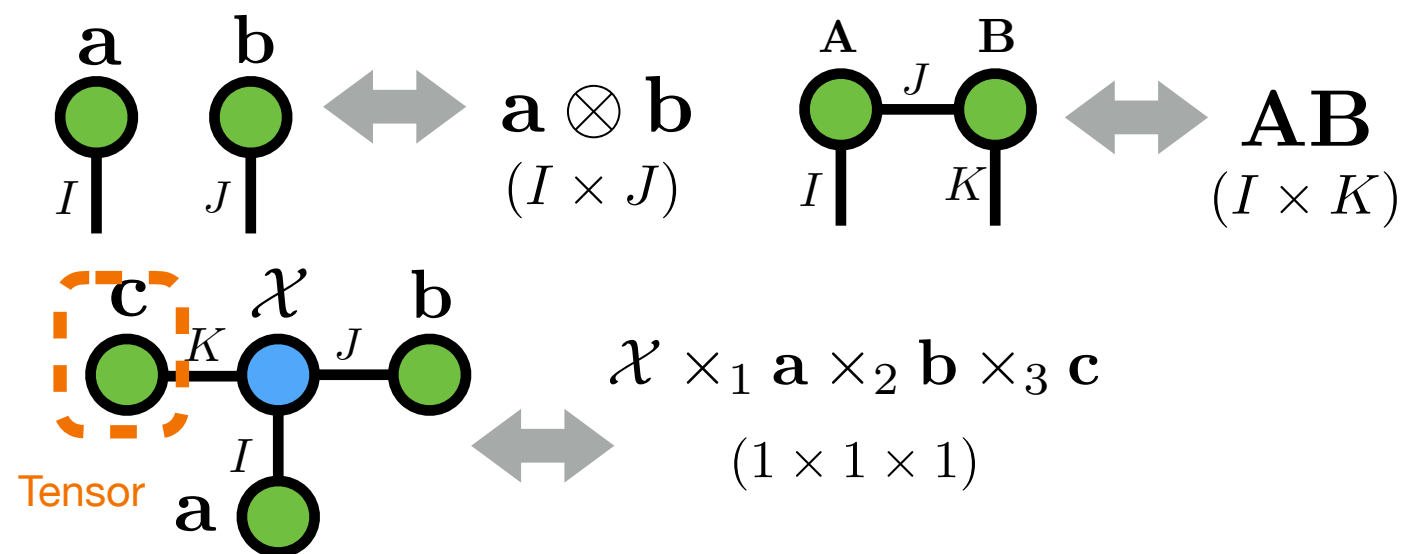
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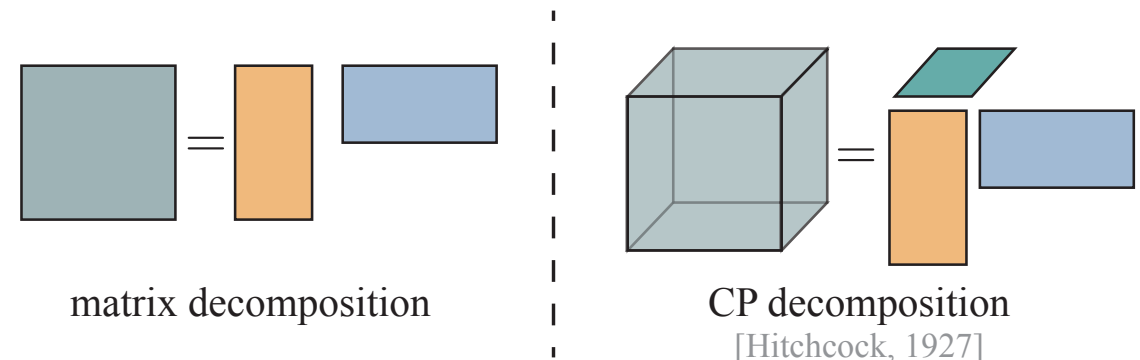
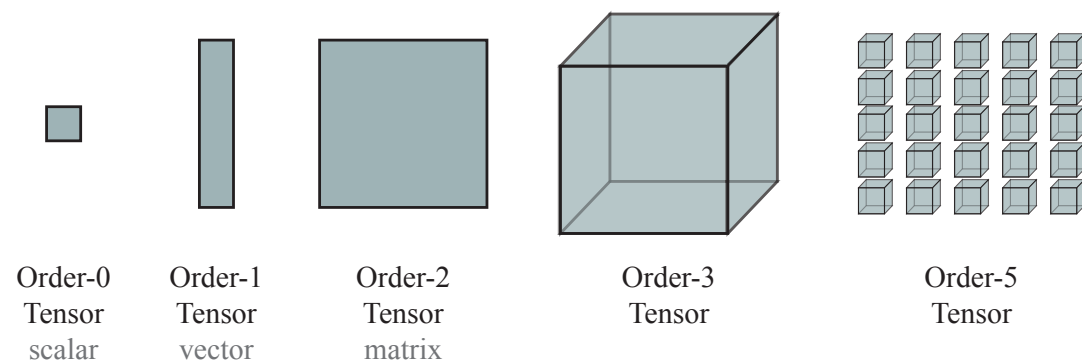
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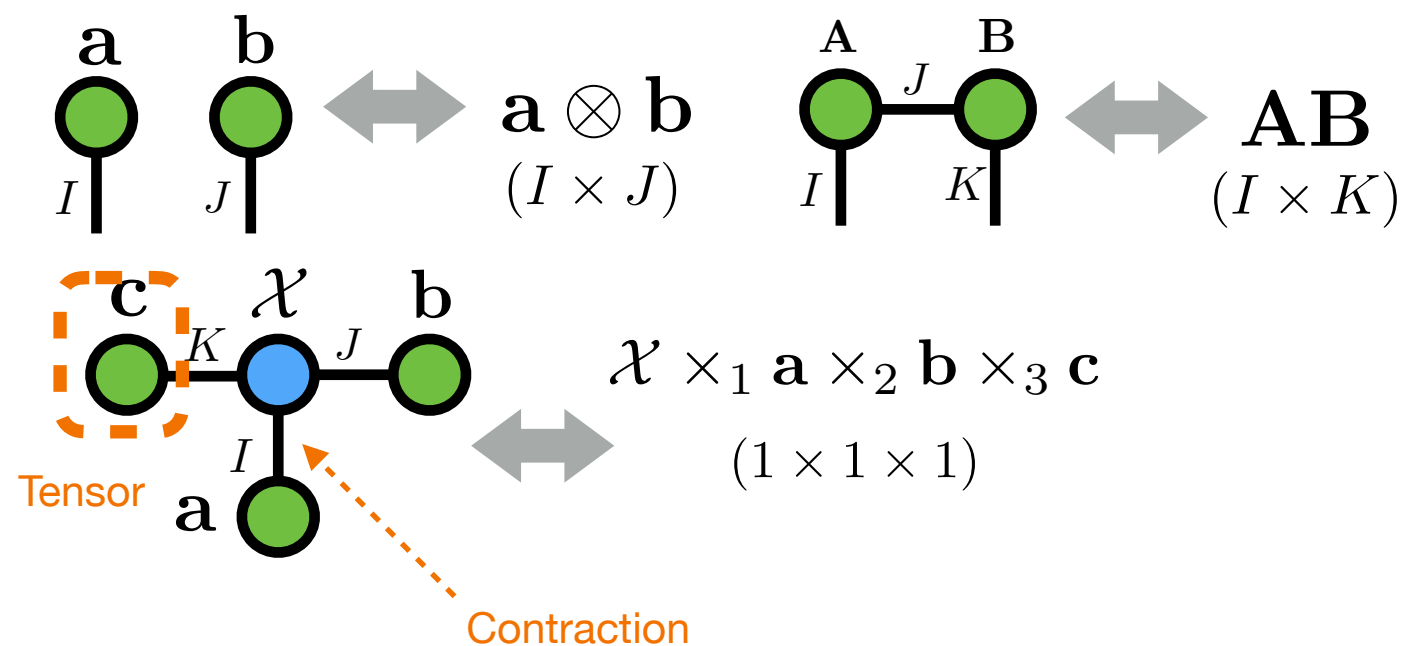
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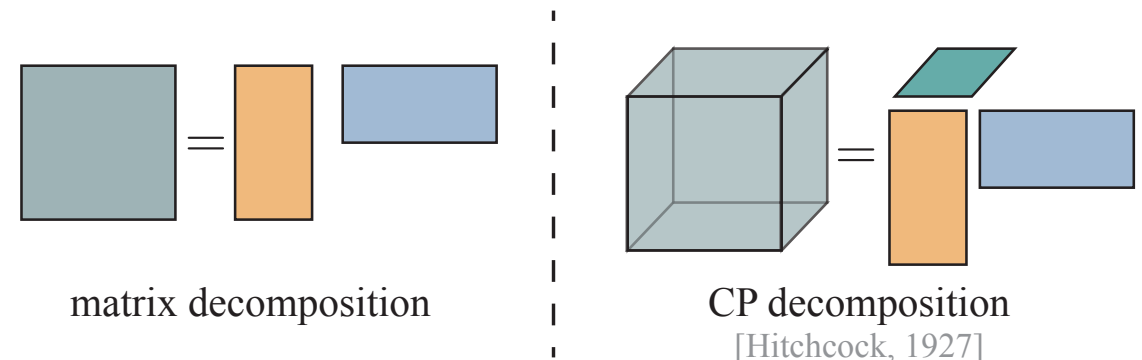
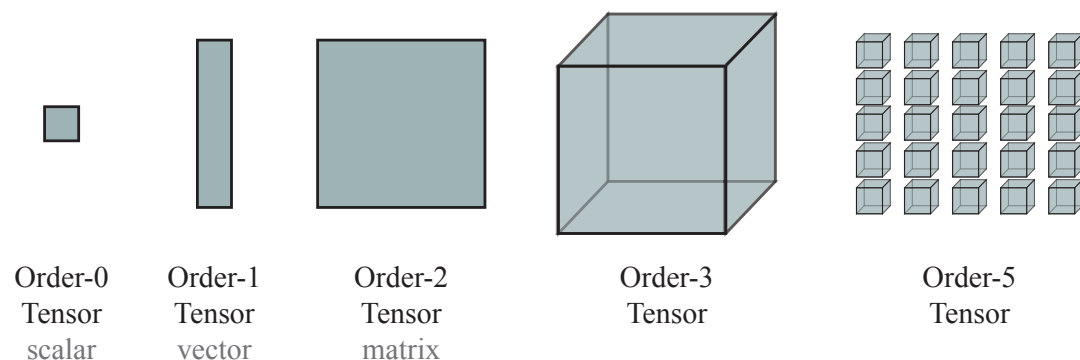
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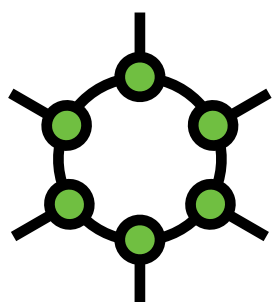
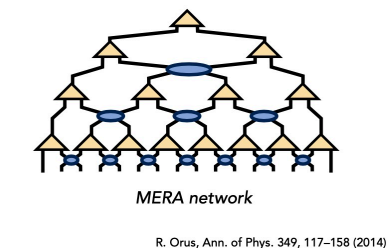
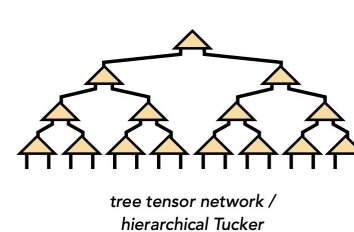
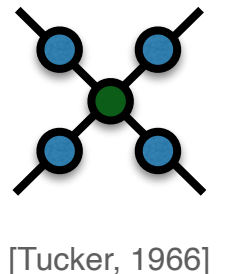
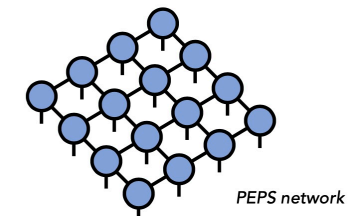
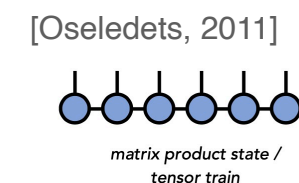
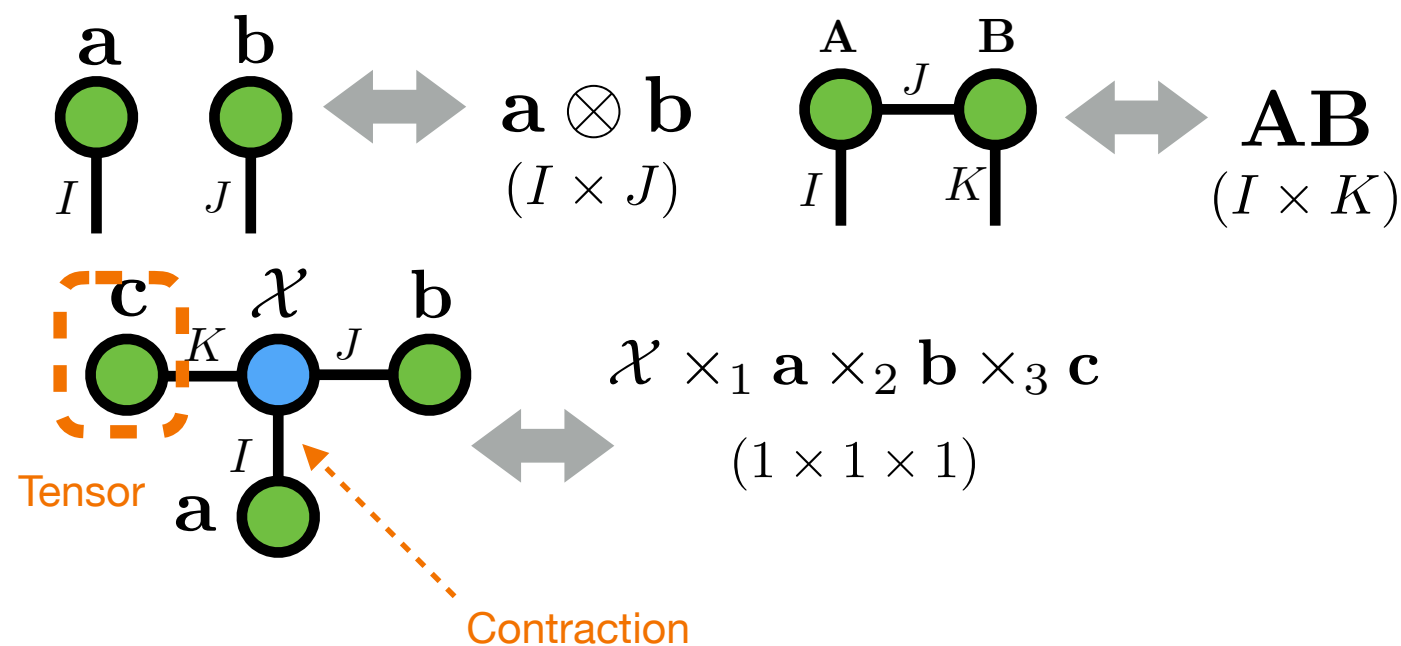
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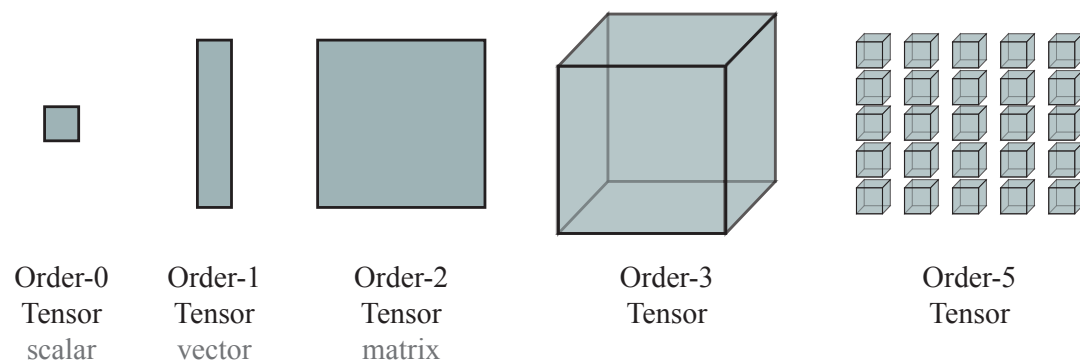


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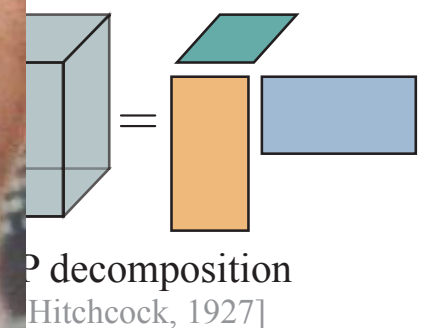


Tensor Network (TN)

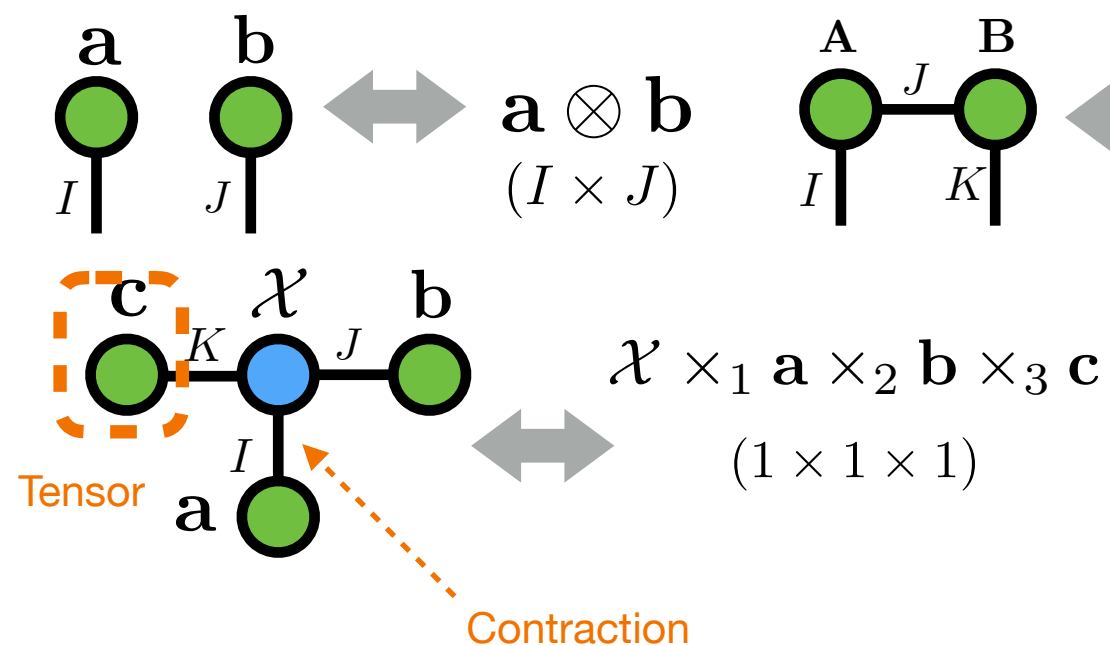
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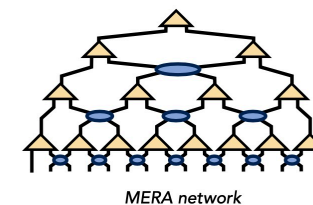
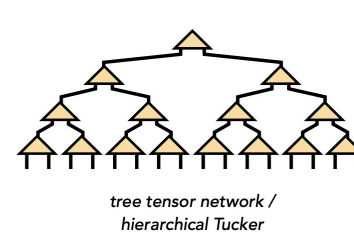


$(I \times K)$

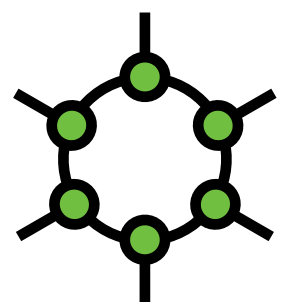
matrix product state /
tensor train

PEPS network

[Tucker, 1966]

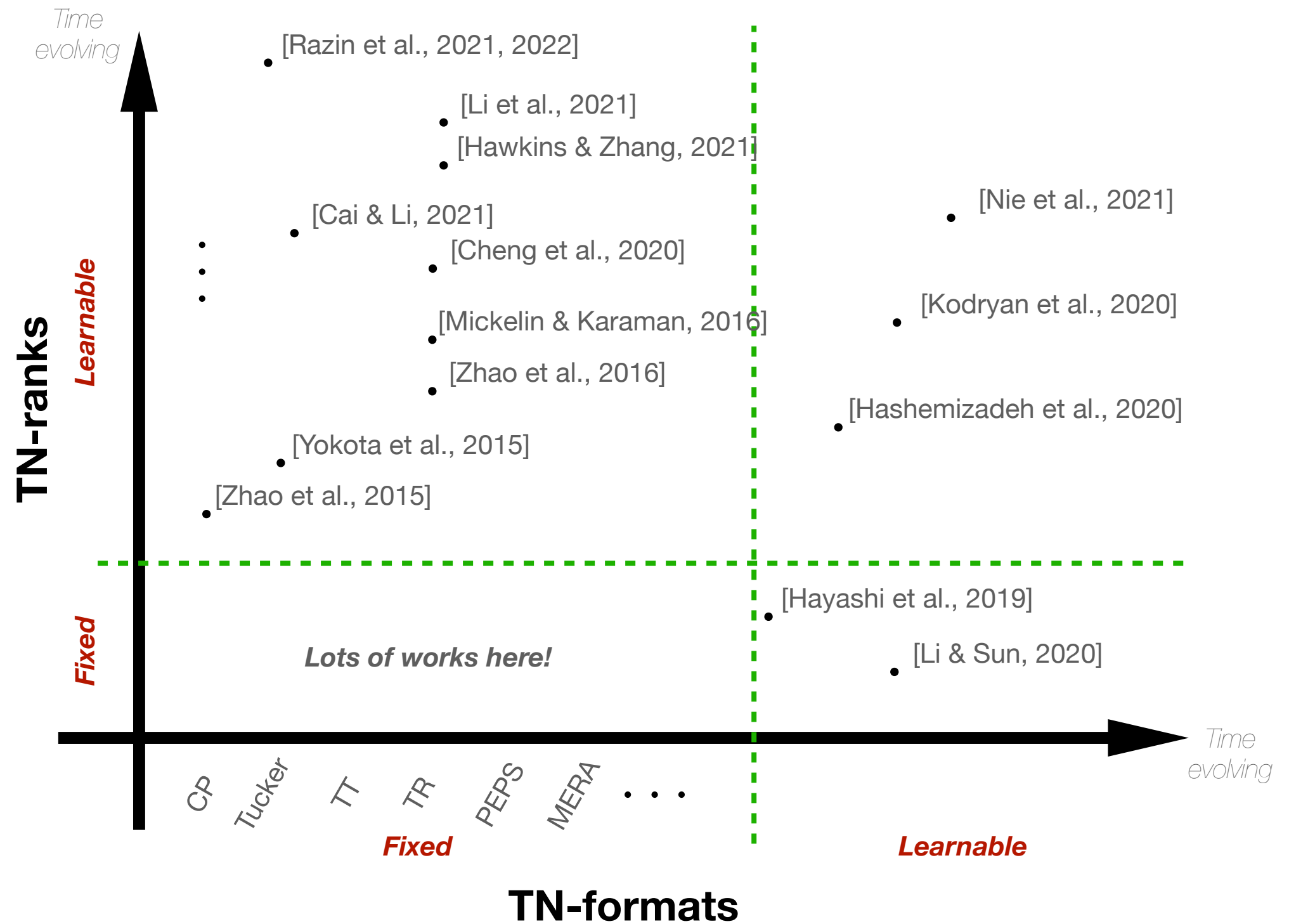


R. Orus, Ann. of Phys. 349, 117–158 (2014)



[Zhao et al, 2016]

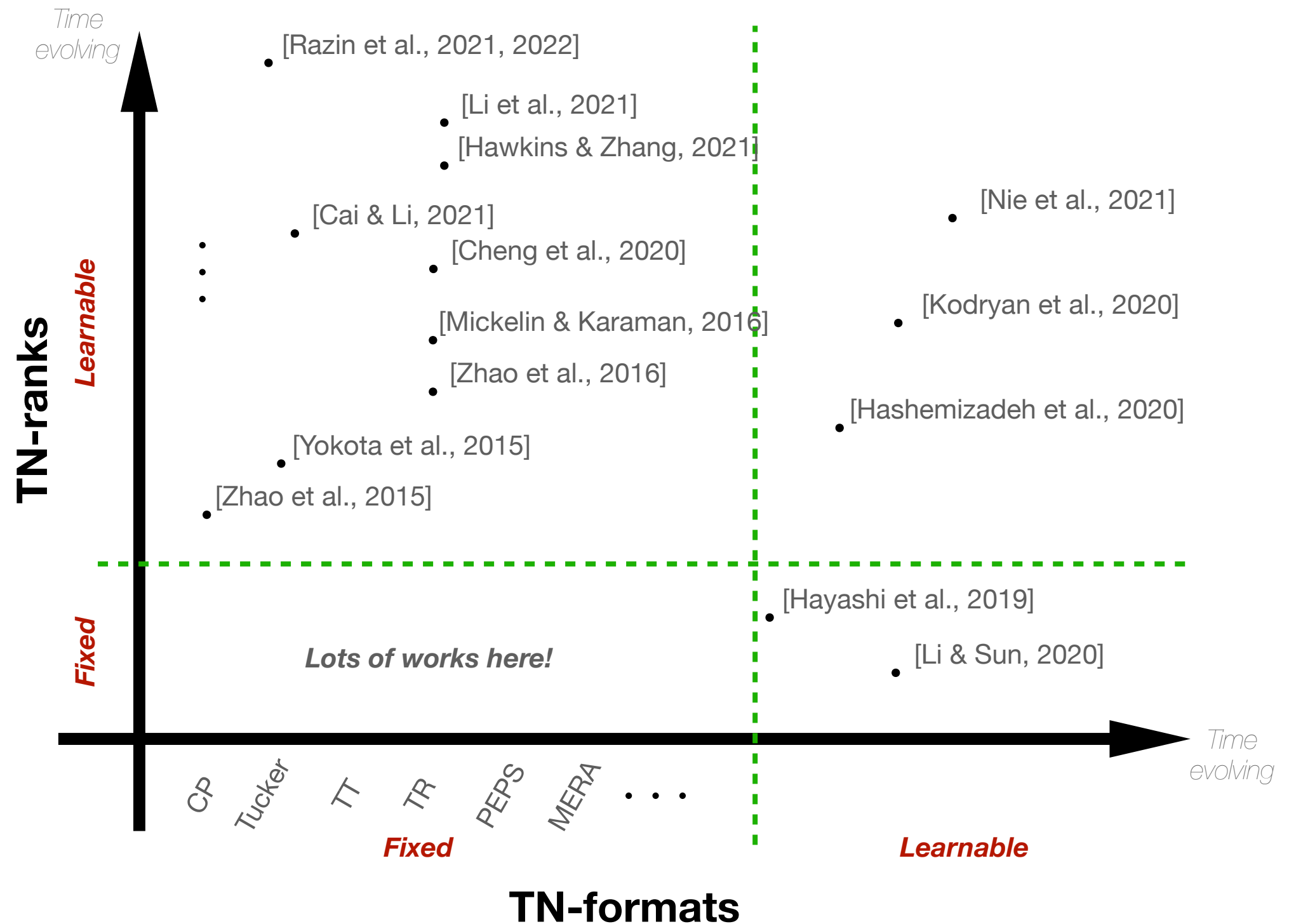
Tensor Network Structure Search (TN-SS)



Tensor Network Structure Search (TN-SS)

Involved Algorithm

Spectral Decomp.
Bayesian Inf.
Reinforcement L.
Implicit regul.
Greedy search
Random sampling



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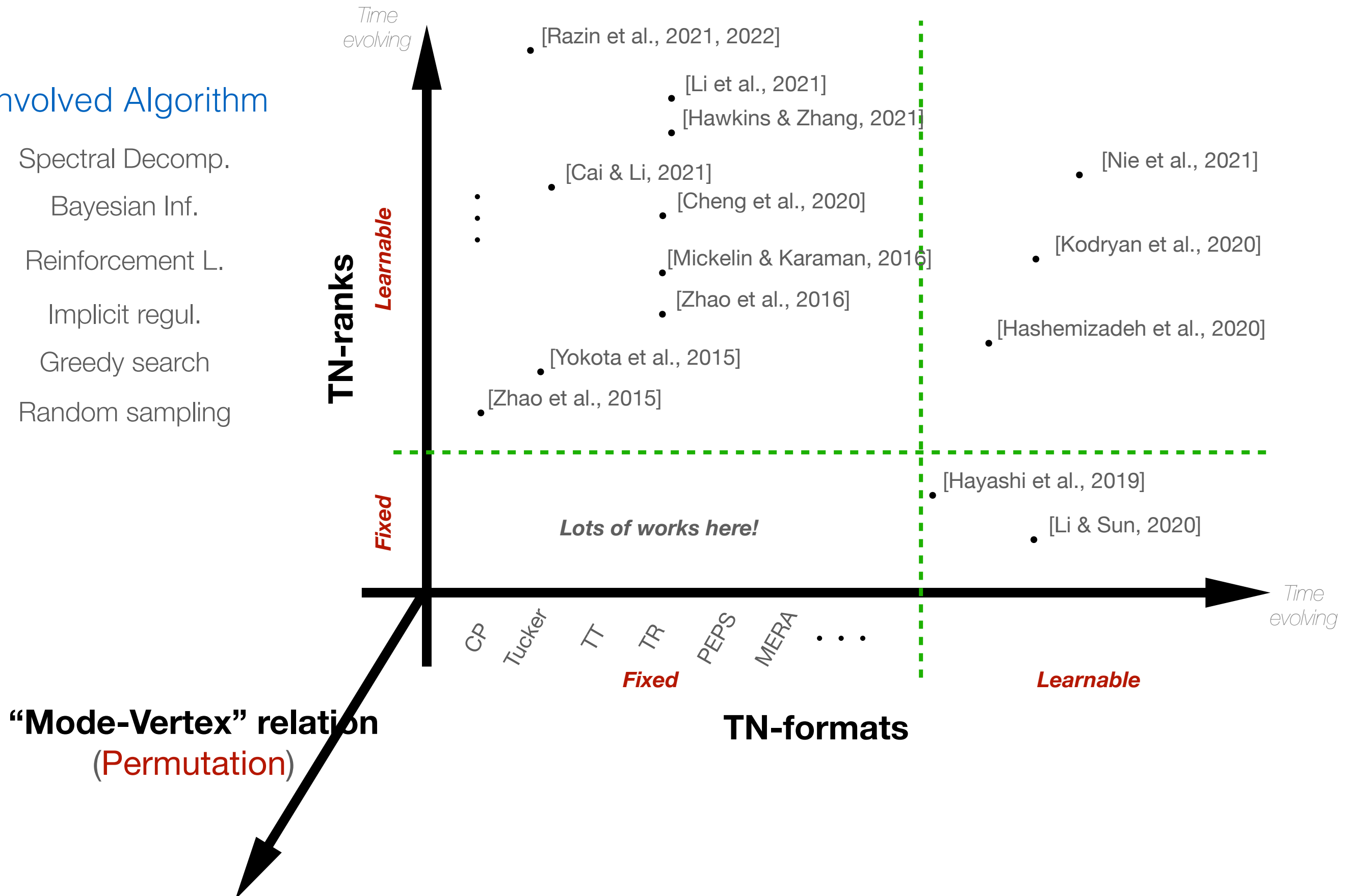
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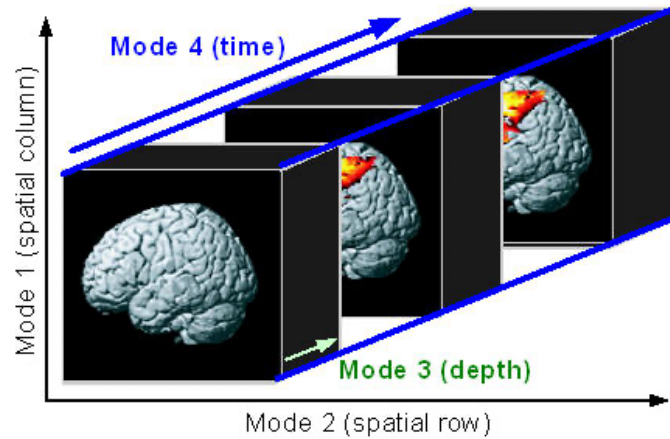
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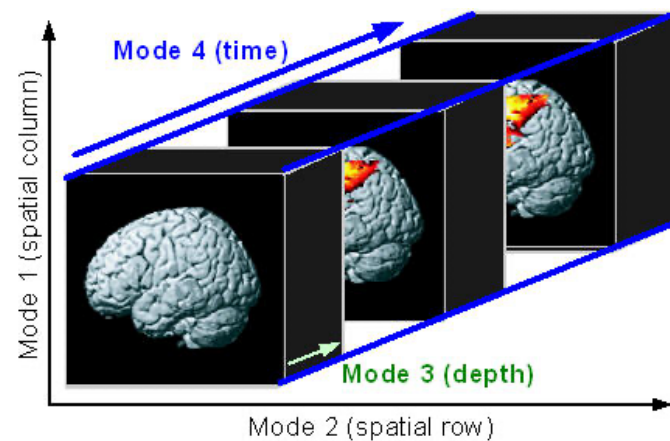


Tensor Network Permutation Search (TN-PS)



4th-order tensor
X,Y,Z-axis and T (time)

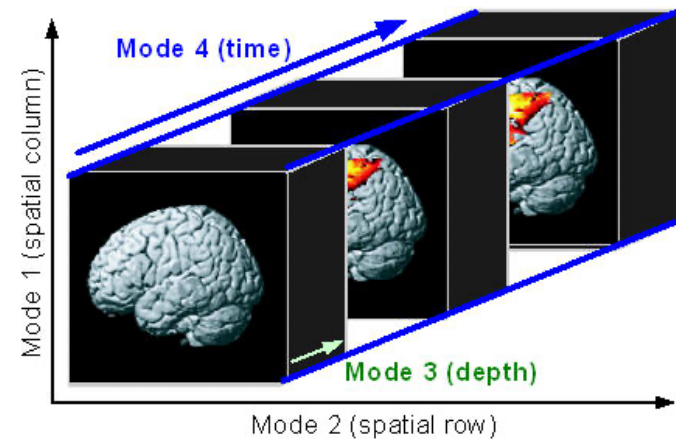
Tensor Network Permutation Search (TN-PS)



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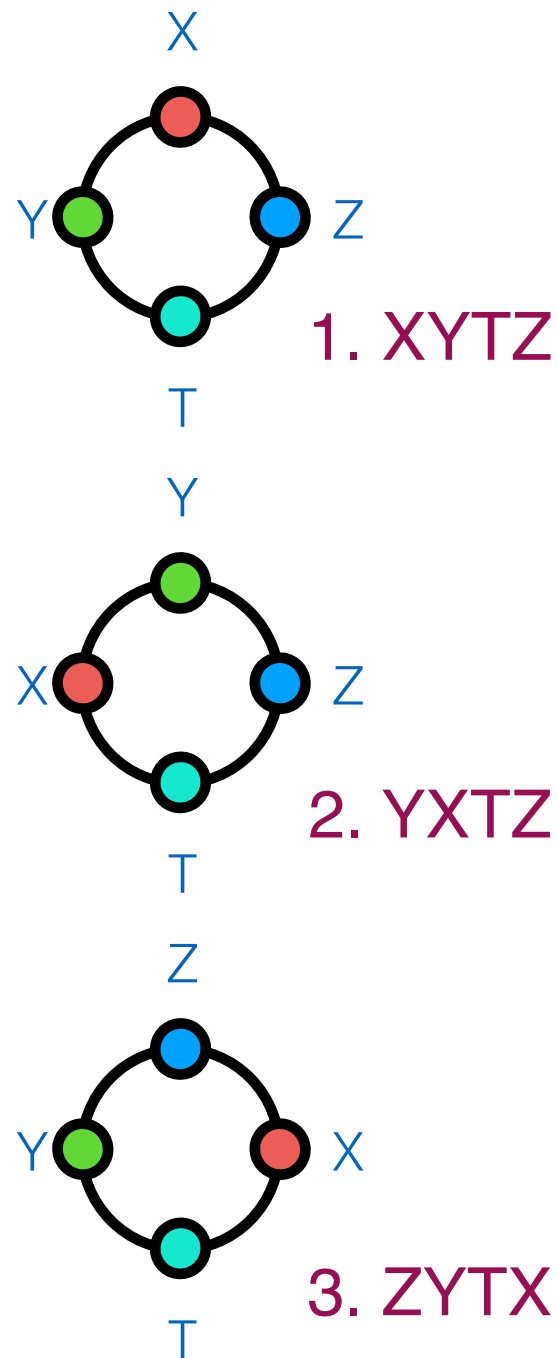
TN approx.
→
cycle

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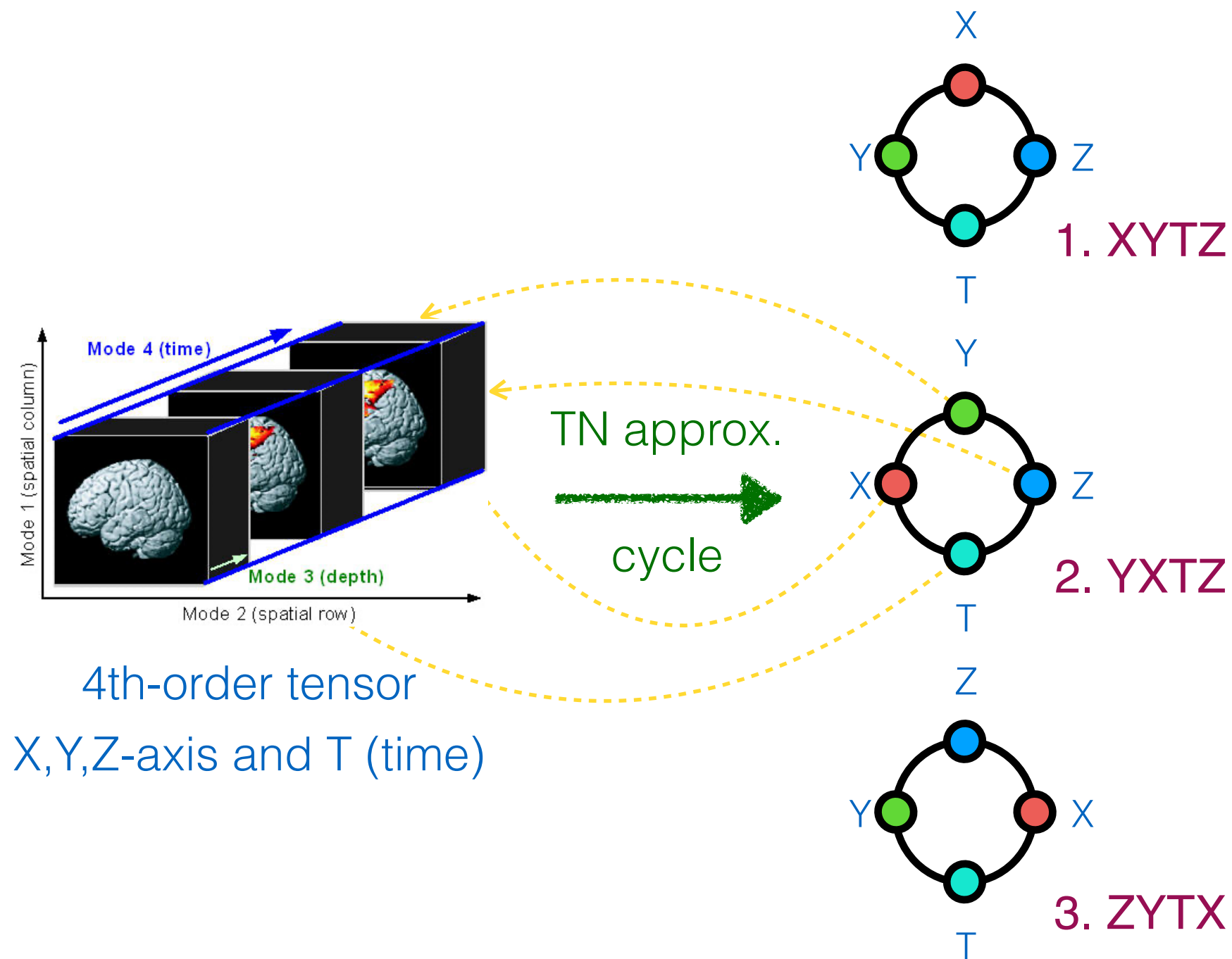


4th-order tensor
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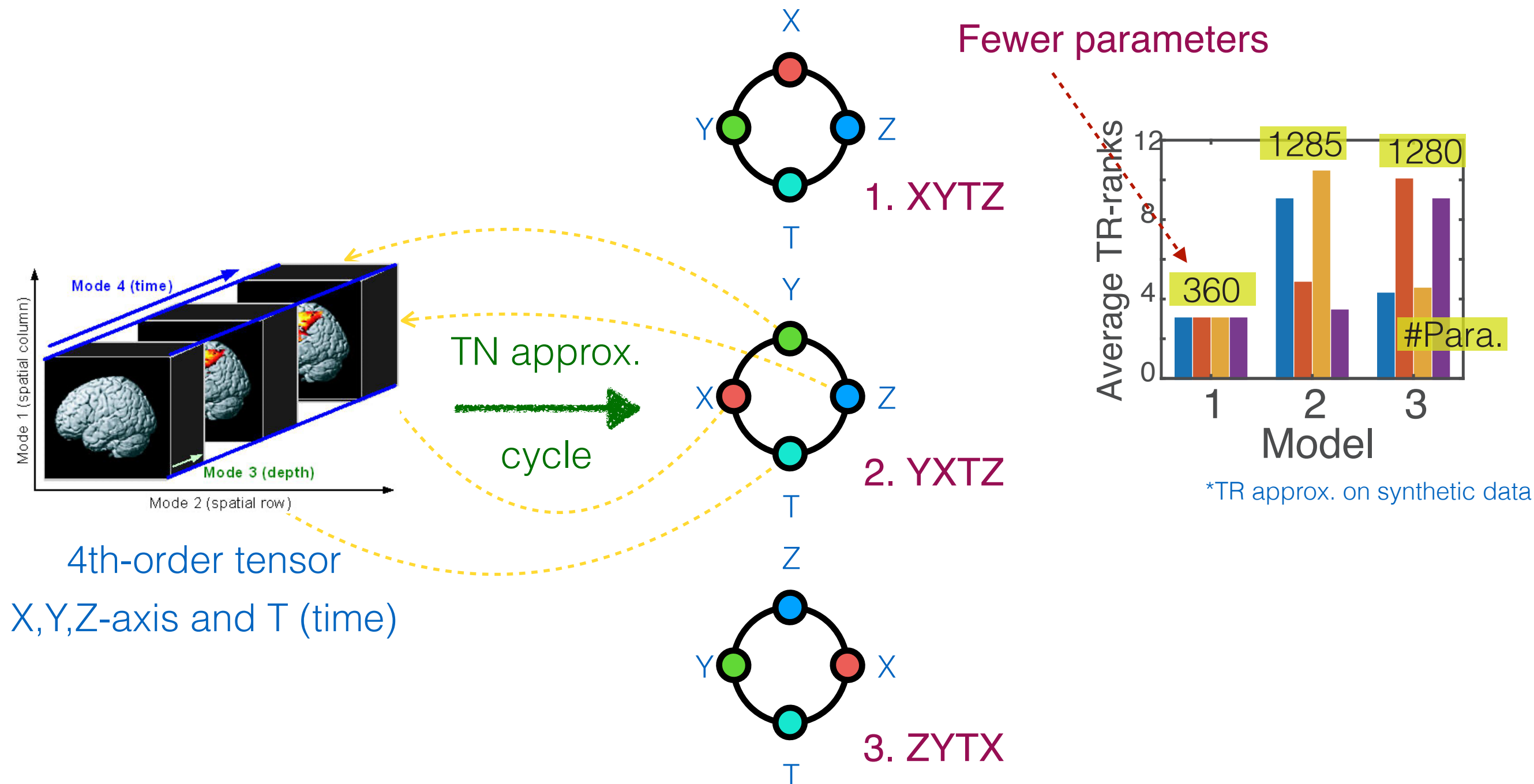
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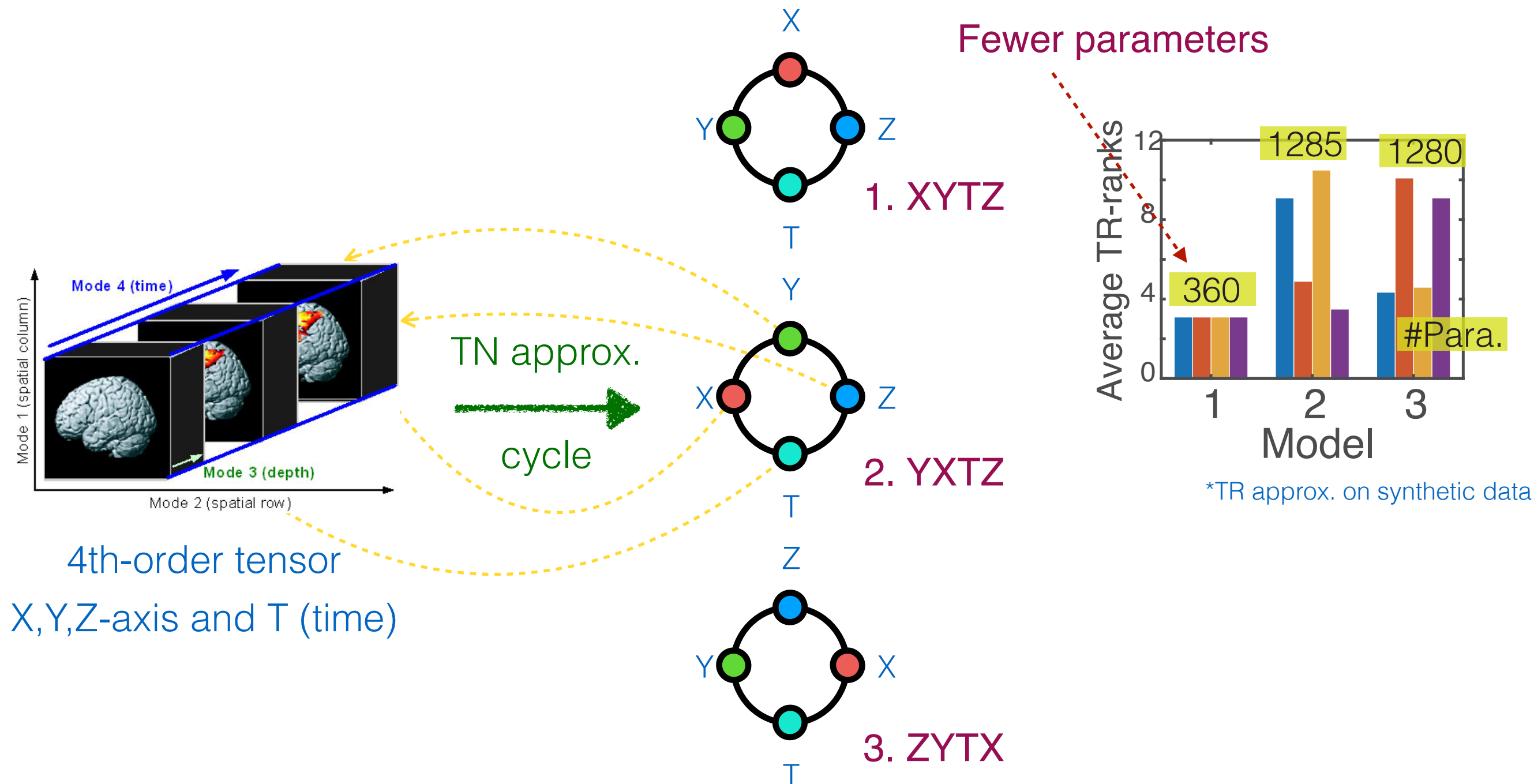
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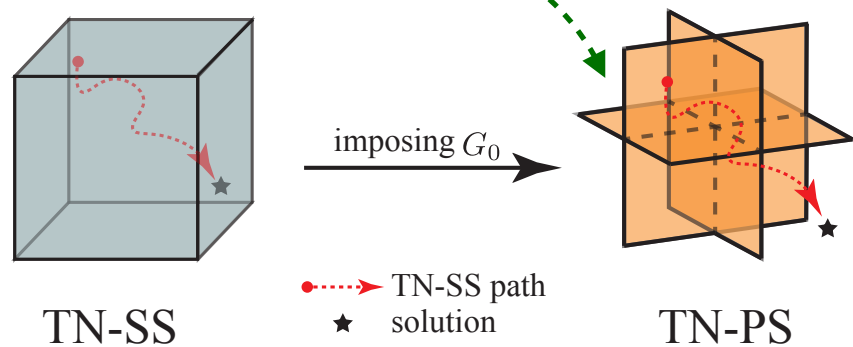
The goal of TN-PS is to search for good “mode-vertex” mappings and ranks.

Main Contributions

Main Contributions

1. Optimization model for TN-PS

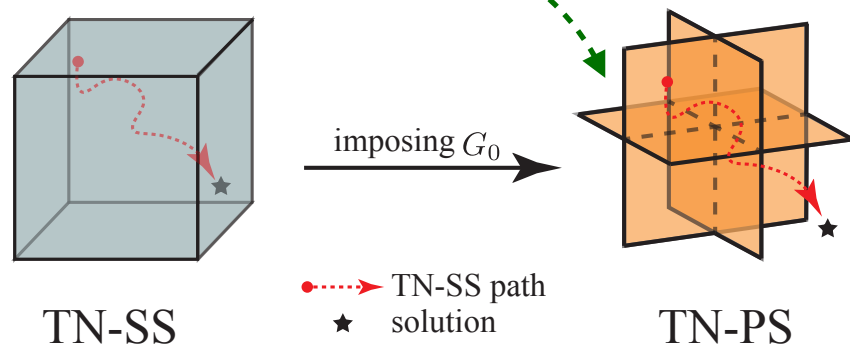
$$\min_{(G,r) \in \mathbb{G}_0 \times \mathbb{F}_{G_0}} \phi(G,r), \quad s.t. \mathcal{X} \in TNS(G,r).$$



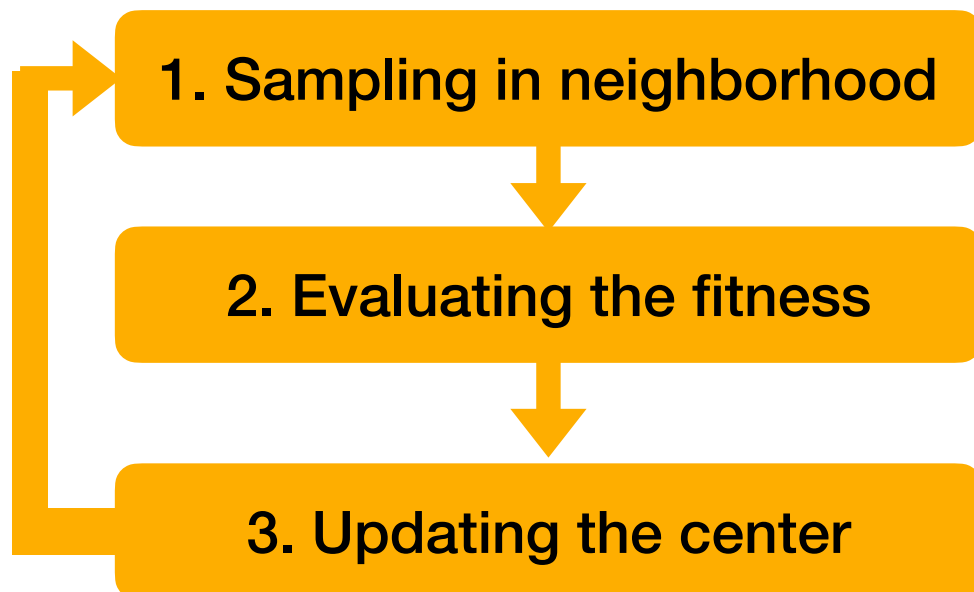
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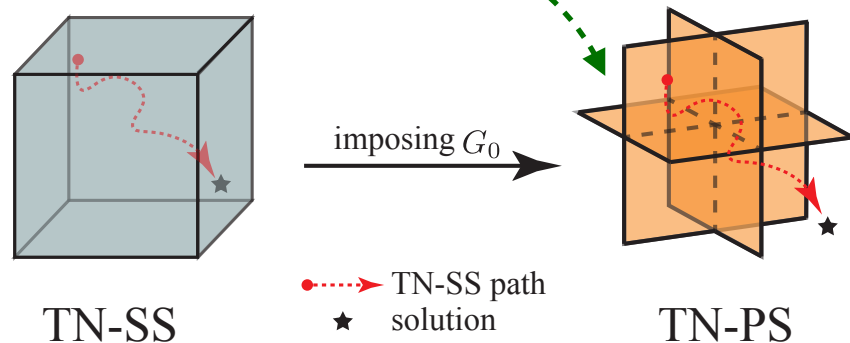
2. TNLS: Algo. based on local-sampling



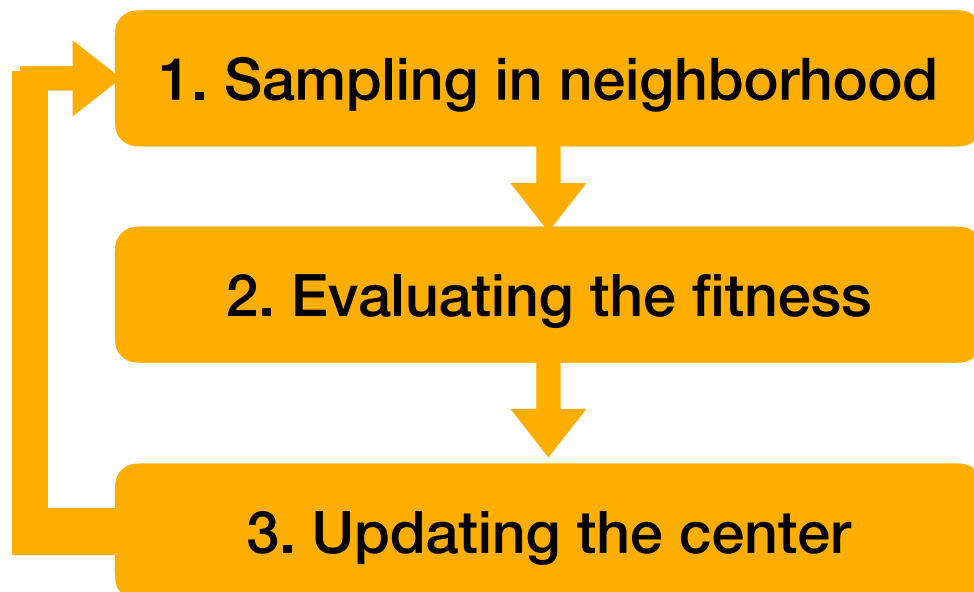
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2. TNLS: Algo. based on local-sampling



3. Theoretical results

1. Solving TN-PS is non-trivial.

The number of mapping candidates grows exponentially fast with the tensor order for most practical TN formats.

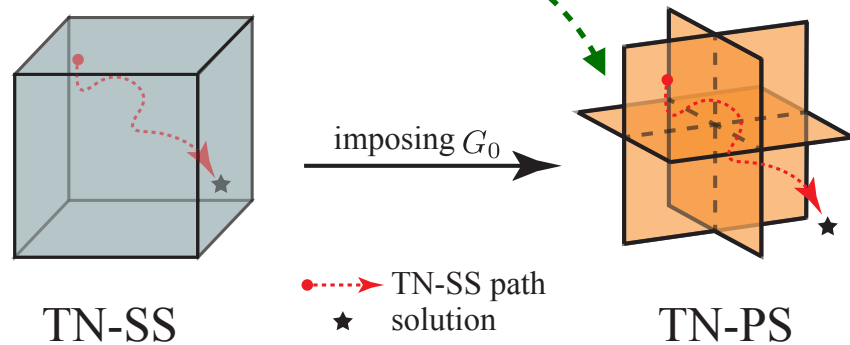
2. Basic geometry for new search space

The New (semi)-metric and its induced neighborhood is rigorously proved.

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2. TNLS: Algo. based on local-sampling

1. Sampling in neighborhood

2. Evaluating the fitness

3. Updating the center

3. Theoretical results

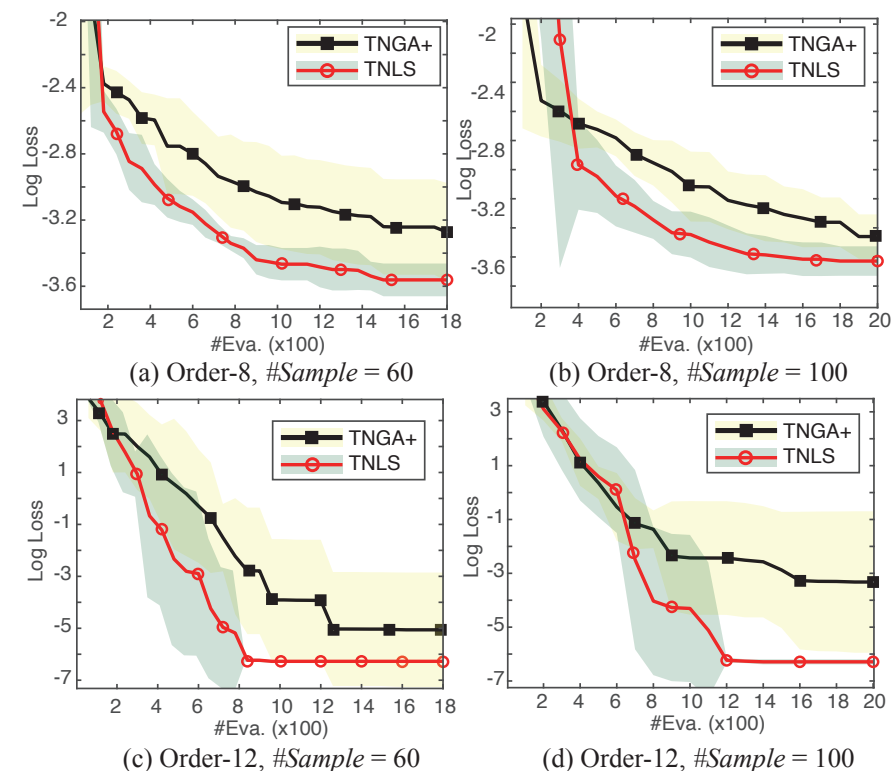
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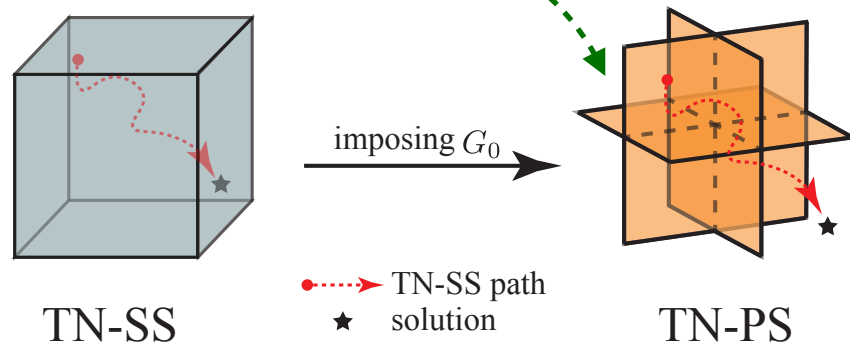
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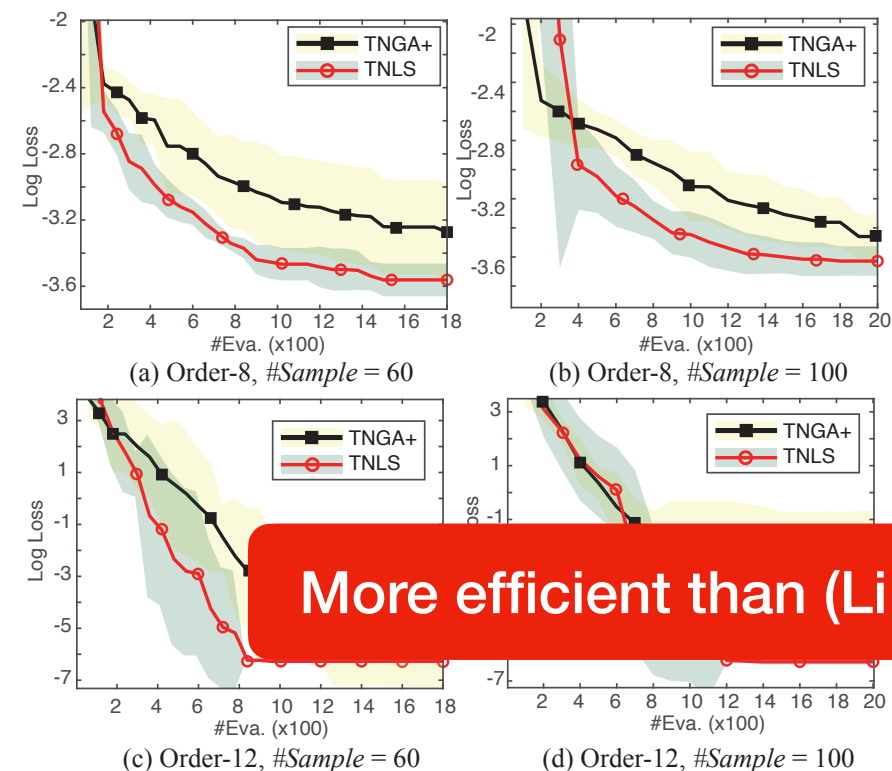
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More efficient than (Li and Sun,2020)

Contributors

Corresponding to:

Qibin Zhao <qibin.zhao@riken.jp>

Chao Li <chao.li@riken.jp>



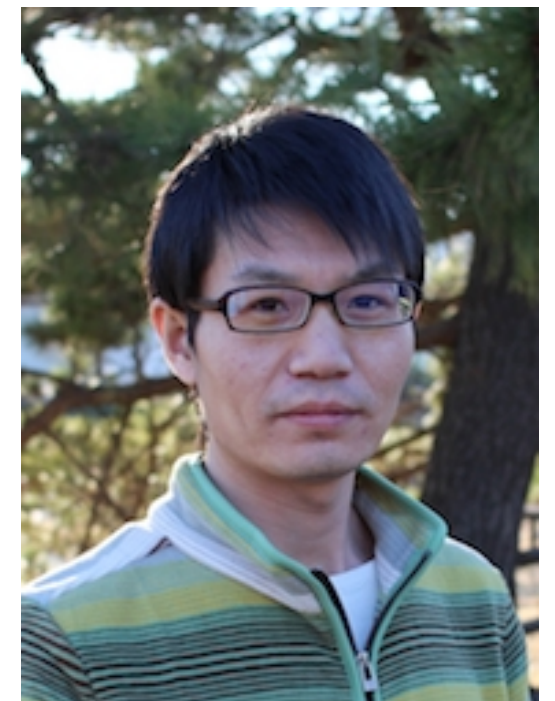
Chao Li



Junhua Zeng



Zerui Tao



Qibin Zhao