Efficient Low Rank Convex Bounds for Pairwise Discrete Graphical Models

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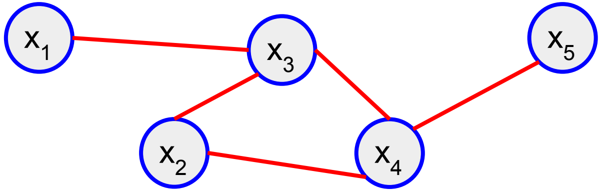
Graphical Models (GMs)

Idea: represent a multivariate function as the sum of many *simple* functions

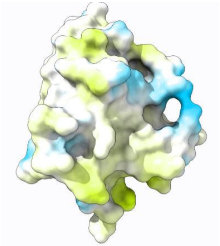


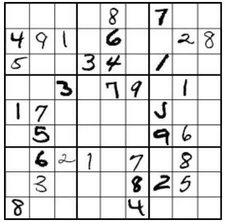
- $X = \{x_1, \dots, x_n\}$
- $x_i \in \{1, \dots, d_i\},\ i = 1, \dots, n$

$$\Theta_M = \underbrace{\sum_{(x_i,x_j)\in E} \theta_{ij}}_{\text{Pairwise potentials}} + \underbrace{\sum_{x_i\in X} \theta_i}_{\text{Unary potentials}} + \underbrace{\theta_\varnothing}_{\text{Constant term}}$$

Maximum a posteriori (MAP) assignment problem: find an assignment of all variables that maximizes the posterior probability

Applications:





Linear and Semidefinite Programming Approaches

- Solving MAP is decision NP-complete
- Solving to optimality is not always practical

Goal: efficient tight *upper* and *lower* bounds

Linear Programming (LP)	Semidefinite Programming (SDP)
Convergent message passing (TRW-S)	Interior point methods
Scales well ✅	Quickly limited in size ❌
Loose bounds on hard instances ❌	Tight bounds ✅

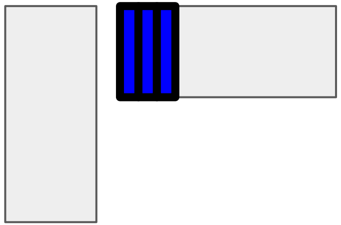
Efficient Semidefinite Programming Through Factorization

- Aim: efficient optimization of GMs with arbitrary potentials and # of states

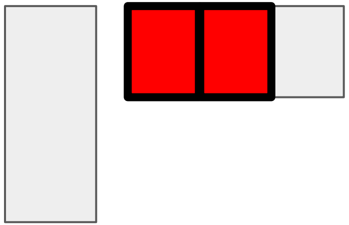
Burer-Monteiro scheme

Exactly one state constraint

X = 

X = 

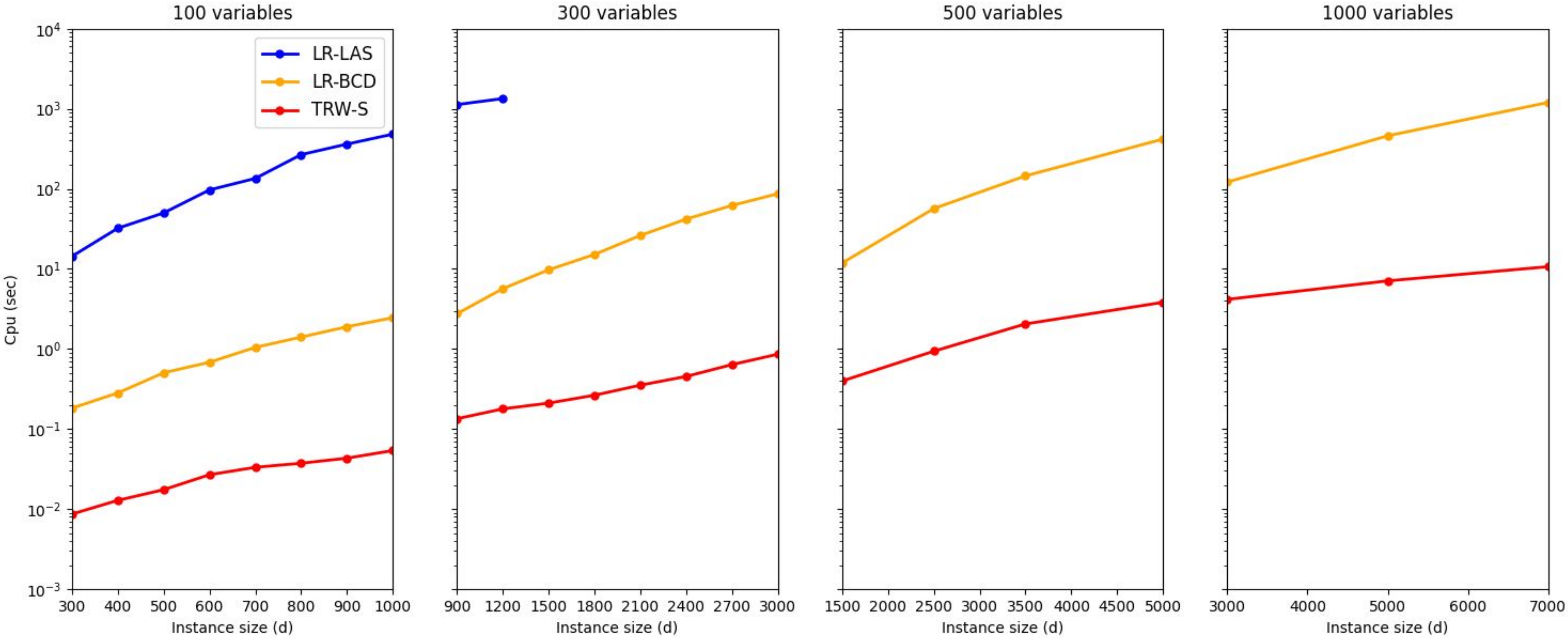
Penalization
LR-LAS

X = 

Block-coordinate descent
LR-BCD

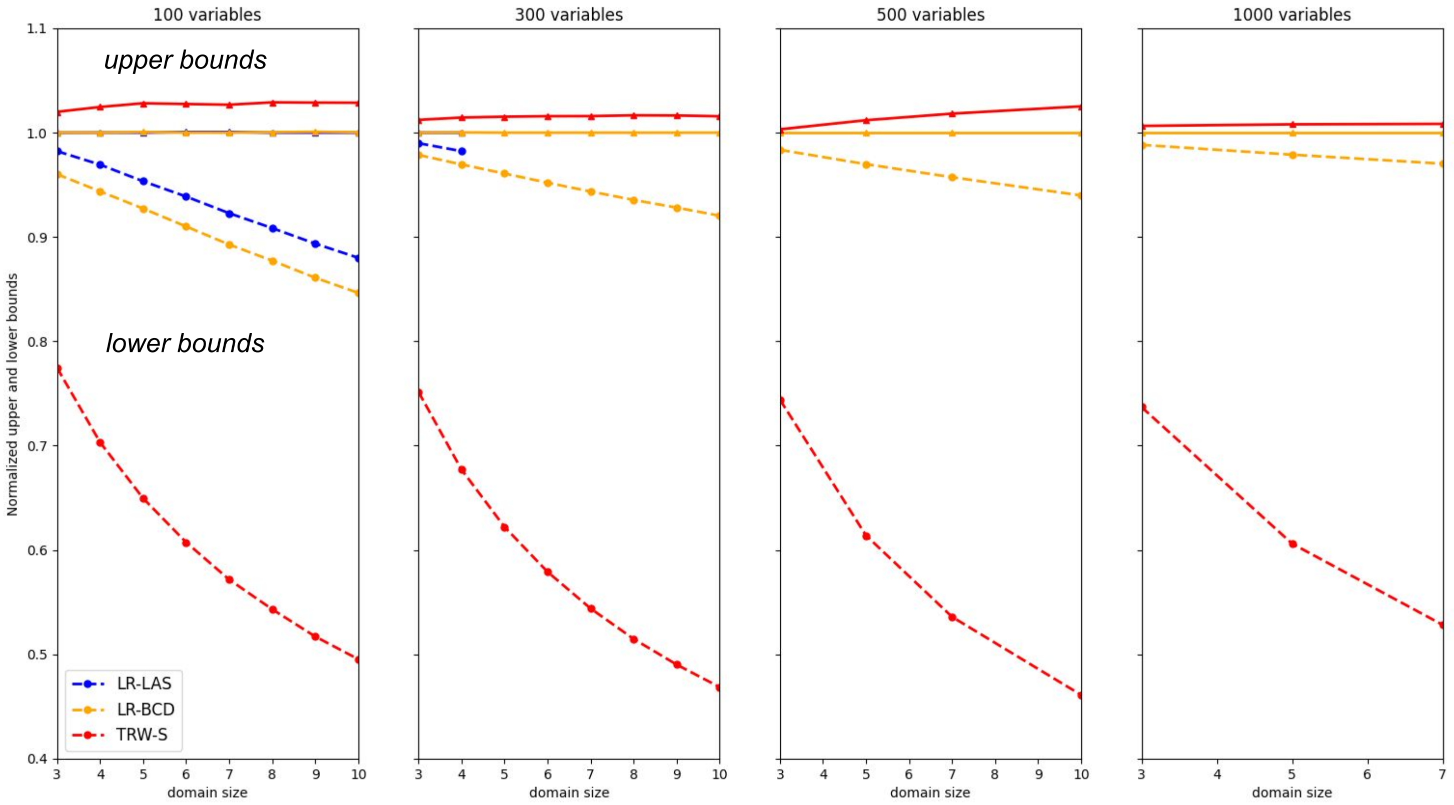
Non-increasing algorithms

Empirical Time Complexity



Upper and Lower Bounds

Random problems



Real instance: 34 296 vars

LP gap = [0, 128 878]

SDP gap = [102 900, 122 694]

Conclusion

Efficient MAP solving on arbitrary pairwise GMs
(# of states, potentials)

Far tighter bounds than LP / convergent message passing on dense hard instances

Scales to sizes not reachable by interior point solvers

- **Link to GitHub:** https://github.com/ValDurante/LR-BCD
- **Link to arXiv paper:** https://arxiv.org/abs/2111.12491

Thank you for your attention !