Closed-Form Diffeomorphic Transformations for Time Series Alignment

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Iñigo Martinez[1], Elisabeth Viles[2,3], Igor G. Olaizola[1]

[1] Vicomtech Foundation, Basque Research and Technology Alliance, Spain
[2] TECNUN School of Engineering, University of Navarra, Spain
[3] Institute of Data Science and Artificial Intelligence, University of Navarra, Spain

imartinez@vicomtech.org

Motivation

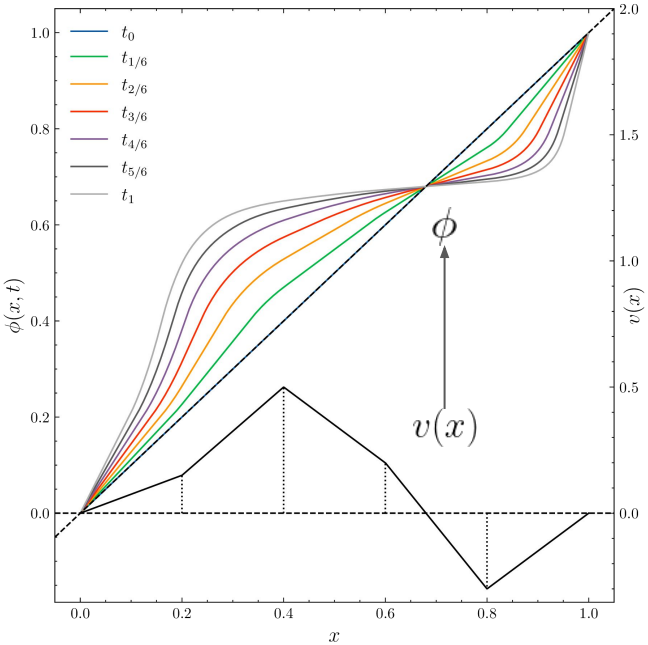
- *Temporal misalignment* impacts recognition and classification performance
- *Optimization problem:* find set of warping functions ϕ that minimize temporal variability

misaligned signals

aligned signals

Diffeomorphic Warping Functions

- Time series alignment methods call for highly expressive, differentiable and invertible warping functions, i.e. *diffeomorphisms*
- *Diffeomorphic warping functions*: generated via integration of velocity fields specified by an ODE



ODE $$\frac{d\phi^\theta(x,t)}{dt} = v^\theta(\phi^\theta(x,t))$$

Integral Eq. $$\phi^\theta(x,t) = x + \int_0^t v^\theta(\phi^\theta(x,\tau))d\tau$$

ϕ : warping function
x : temporal dimension
t : integration time
v : velocity field
θ : parameters

Optimizing Diffeomorphic Warping Functions

- Temporal Transformer Networks (TTN) can learn the optimal temporal transformations that enhance geometric invariance and minimize temporal variability

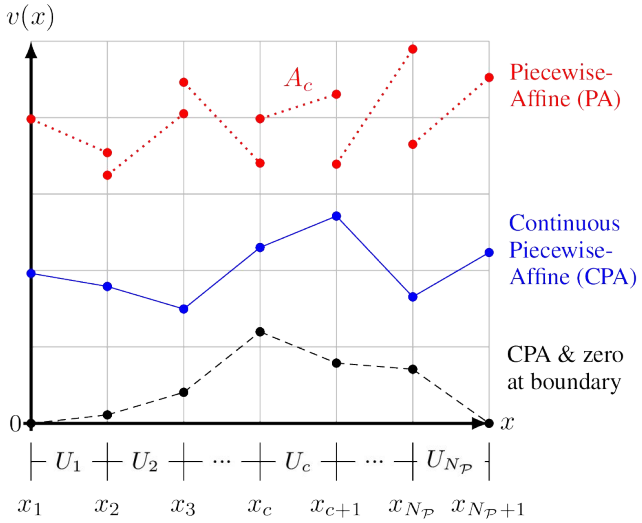
- However, neural networks that include diffeomorphic transformations require to calculate derivatives to the ODE's solution with respect to the model parameters.

$$\phi^{\theta}(x,t) = x + \int_0^t v^{\theta}(\phi^{\theta}(x,\tau))d\tau \qquad \frac{\partial \phi^{\theta}(x,t))}{\partial \theta_k} = ?$$

- Current solutions appeal to
  - Adjoint sensitivity methods
  - ResNet's Eulerian discretization
  - Ad-hoc numerical solvers and automatic differentiation

Proposed Solution

- We formulate a closed-form expression for the gradient of 1D diffeomorphic transformations under continuous piecewise-affine (CPA) velocity functions.
  - Closed-form ODE solution (forward) $\phi^{\theta}(x,t)$
  - Closed-form gradient (backward) $\dfrac{\partial \phi^{\theta}(x,t))}{\partial \theta_k}$

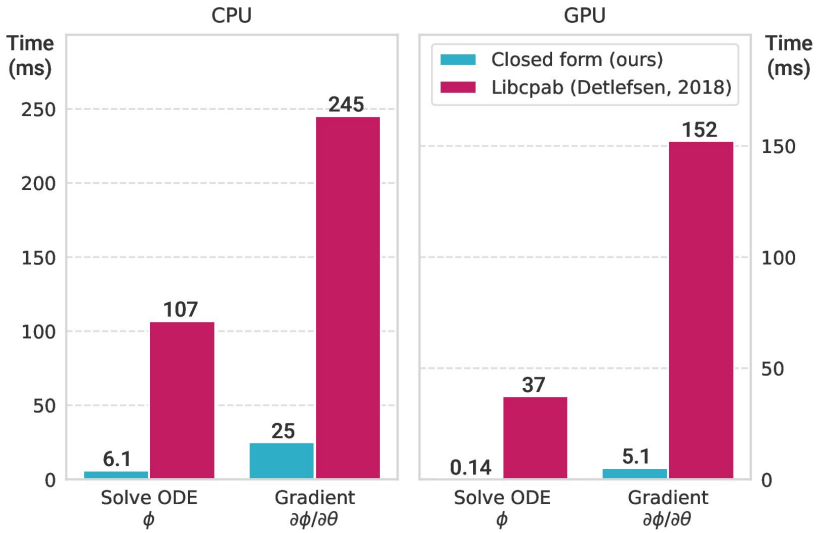


- A closed-form solution provides efficiency, speed and precision.
  - Fast computation for iterative gradient descent methods
  - Exact gradient leads to better solutions at convergence
  - Shortens chain of operations and decreases tape overhead

Results: Performance

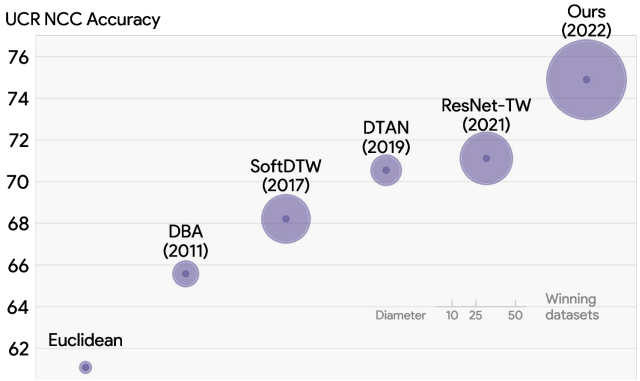
- Diffeomorphic Fast Warping DIFW library: highly optimized implementation of 1D diffeomorphic transformations on multiple backends for
  - CPU: NumPy and PyTorch with C++
  - GPU: PyTorch with CUDA

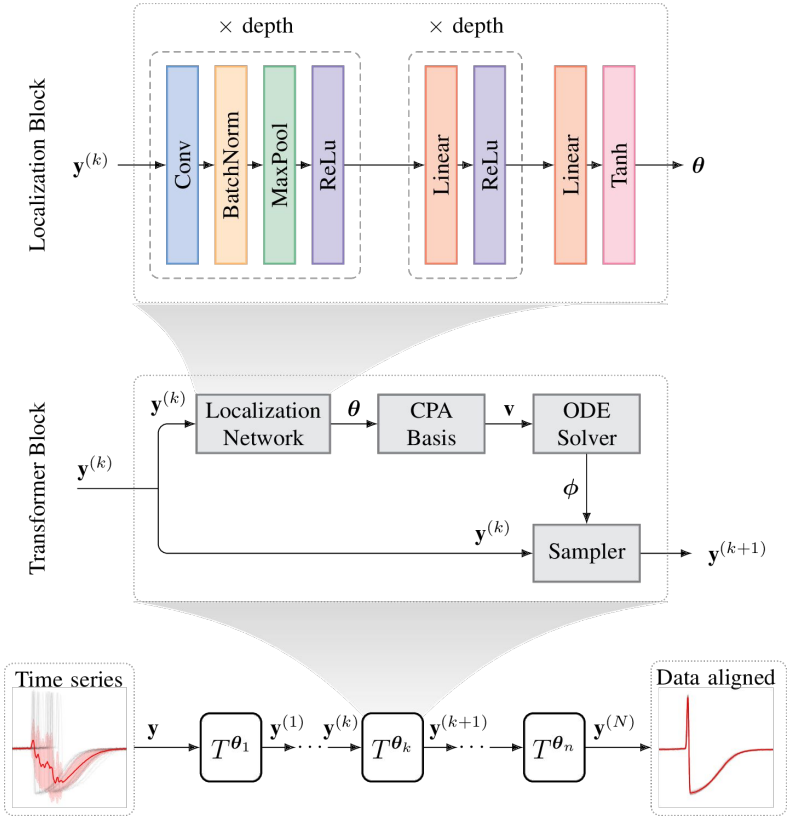
- Speed tests: **x18** / **x260** and **x10** / **x30** improvement on **CPU** / **GPU** over current solutions for forward and backward operations respectively.



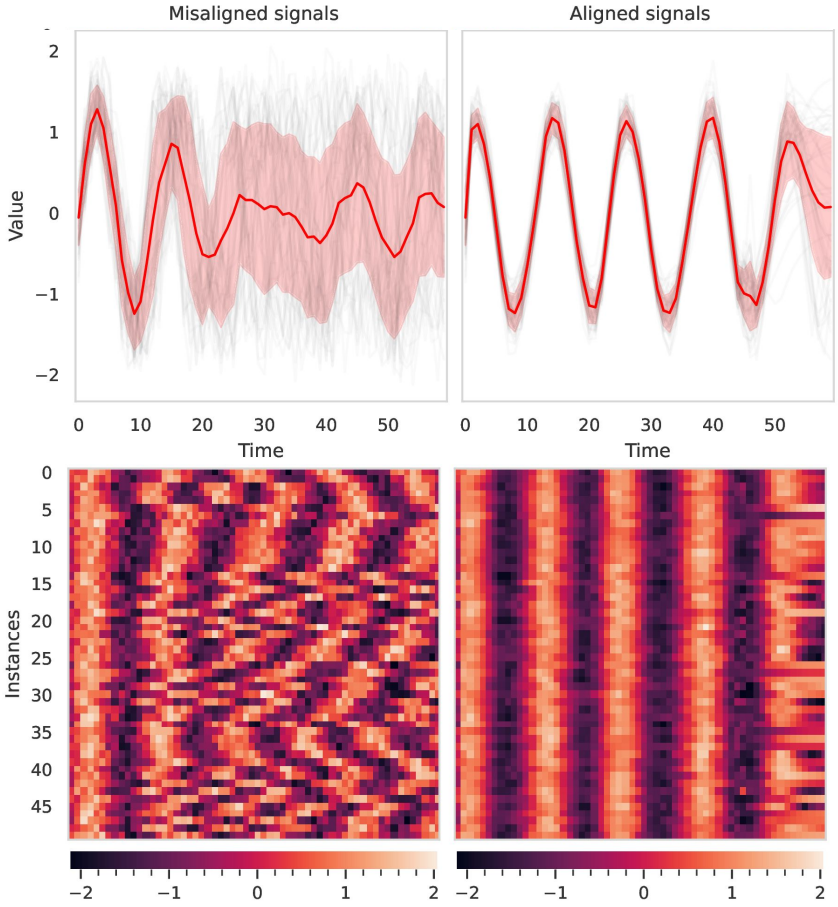
Results: Time Series Classification

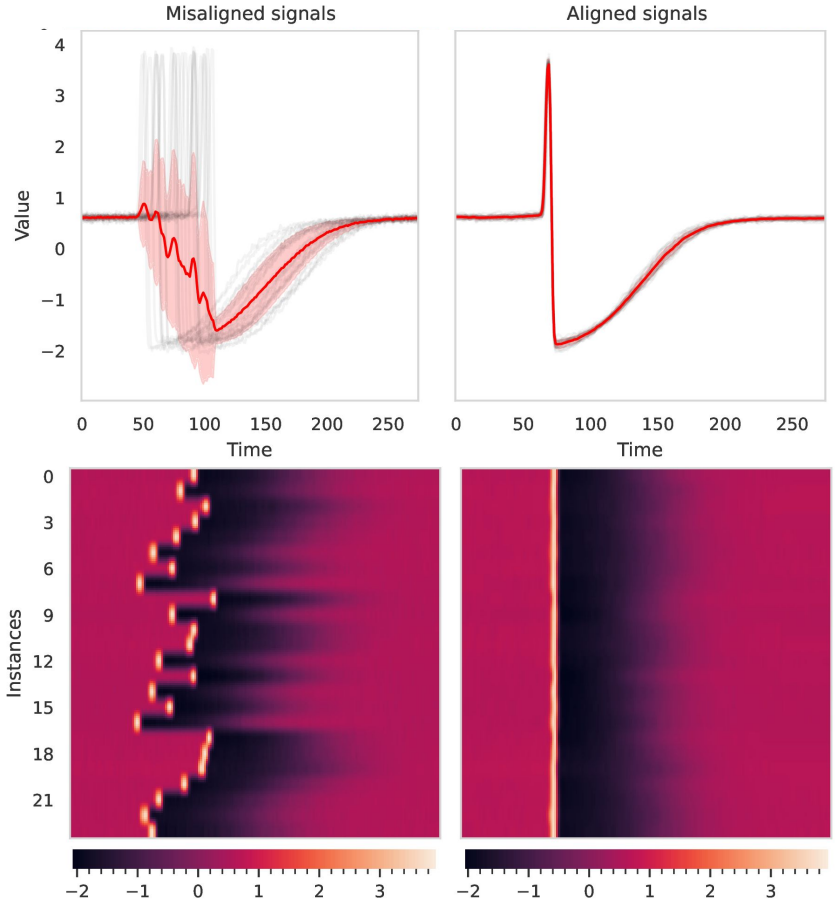
- We integrate closed-form diffeomorphic transformations into a TTN for time series alignment and classification
- Results show significant improvement in terms of efficiency and accuracy





Results: Time Series Alignment





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