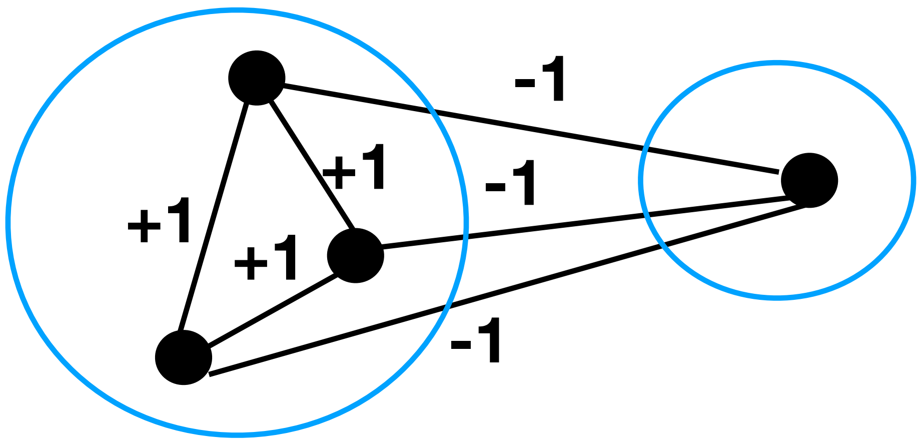
Online and Consistent Correlation Clustering

Andreas Maggiori
Vincent Cohen-Addad, Silvio Lattanzi, Nikos Parotsidis

EPFL Google

Correlation Clustering - Problem Statement

Input: a complete signed graph G = (V, E, s) where s(e) = ‘+’ or ‘-‘ for every edge e



0 disagreements 😀

Goal: output a clustering which minimises *disagreements*

‘+’ edge between two nodes assigned to different clusters

‘-‘ edge between two nodes assigned to the same cluster

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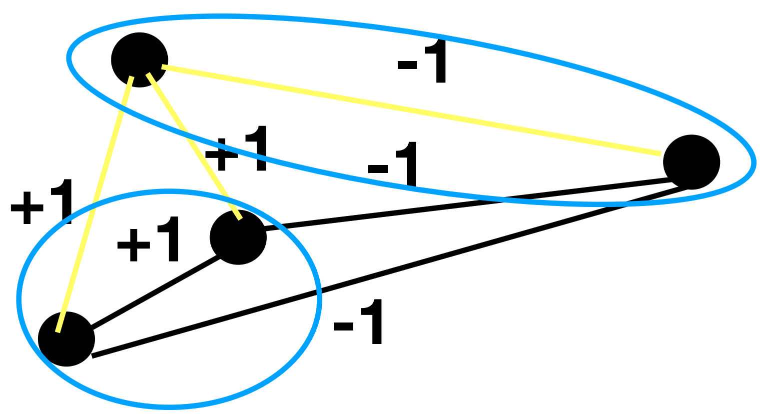

3 disagreements 🤨

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Correlation clustering is well studied.

Offline setting

- Introduced by Bansal et al. 2004. The problem is NP-HARD.
- Pivot algorithm is a 3-approximation on expectation. (Ailon et al. 2008)
- LP based 2.06-approximation algorithm (Chawla et al 2015)

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Other settings

- Distributed: Chierichietti et al 2014, Ahi et al. 2015, Pan et al. 2015, Cohen-Addad et al. 2021
- Online: Mathieu et al. 2010

Online setting

- At each time t: a node arrives, revealing all its incident edges to previously arrived nodes.
- Clustering decisions are irrevocable:
  - create a new singleton cluster with the newly arrived node; or
  - Add that node to a preexisting cluster.
- Mathieu et al. proved that any online algorithm is $\Omega(n)$- competitive. Why: difficult to distinguish if an edge is a bridge between two cliques or it is part of a clique. 😩

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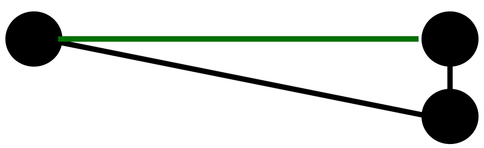

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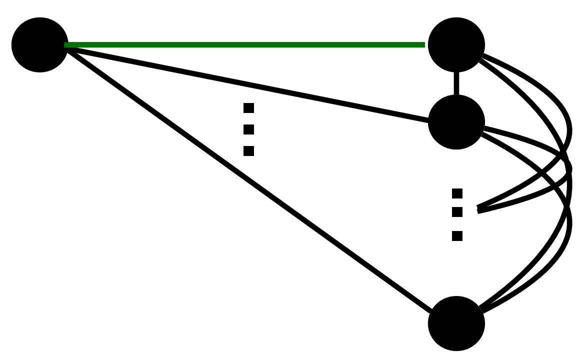

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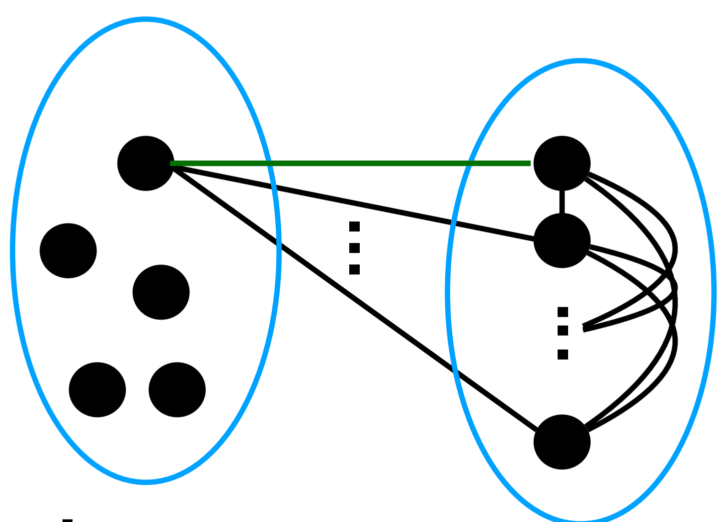

Large clique

Online setting with recourse

 # times a node changes cluster

- At each time t: a node arrives, revealing all its incident edges to previously arrived nodes.
- Clustering decisions are **NOT** irrevocable.
- Goal: having at all times a constant factor approximation while minimising the worst case recourse of a node.

Contributions:

- A constant factor approximation algorithm which achieves worst case log(n) recourse per node.
- A matching lower bound.

Online setting with recourse

Agreement Algorithm of Cohen-Addad et al 2021 as a subroutine

- Creates dense clusters with few outgoing edges
- Constant factor approximation

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Online Agreement Algorithm

1. Rerun the Agreement Algorithm

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Worst case linear recourse

😩

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Online Agreement Algorithm

1. Rerun the Agreement Algorithm
2. Stabilize the resulting clustering

- Follow (blindly) the advice of the *Agreement* Algorithm to get constant competitive ratio for free
- Change the clustering as little as possible to obtain low recourse

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Balance these two opposite objective by:

- Keep track of evolving clusters
- Use dense structure of the clustering computed by the *Agreement* Algorithm

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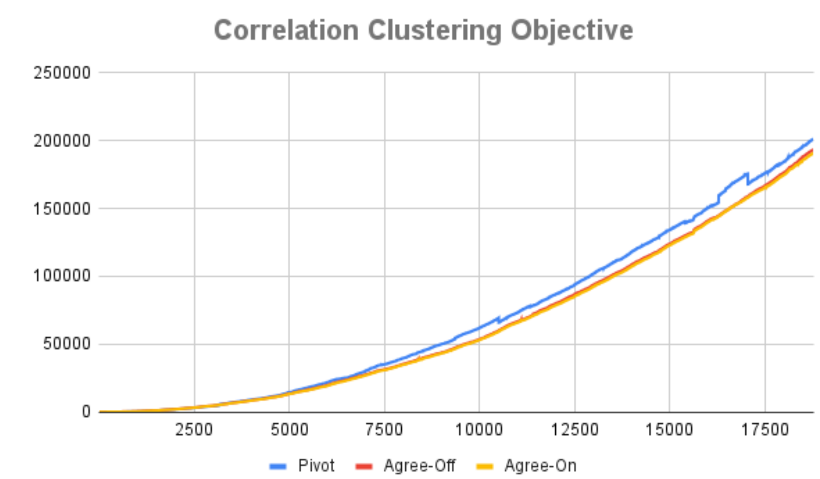

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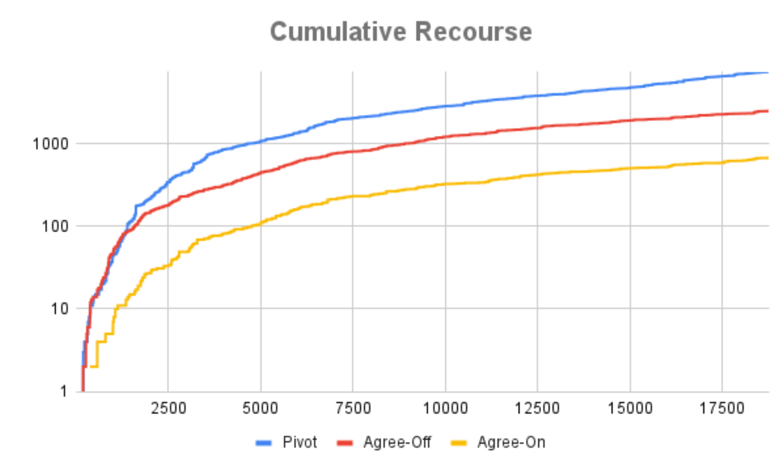

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Thank you for your attention!