Fair and Fast k-Center Clustering for Data Summarization

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[1]CoW Protocol

[2]ETH Zürich

SUMMARIZATION PROBLEMS

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We serve some of the most demanding end markets, from aerospace & defense, bill validation and counterfeit detection, and oil & gas, to chemical, pharmaceutical ...

https://en.wikipedia.org › wiki › Crane_(bird)

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Family: Gruidae; Vigors, 1825
Kingdom: Animalia
Class: Aves
Order: Gruiformes

Common crane · Whooping crane · Sarus crane · Red-crowned crane

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Leading the global overhead **crane** industry with the largest service network and a complete range of lifting equipment.

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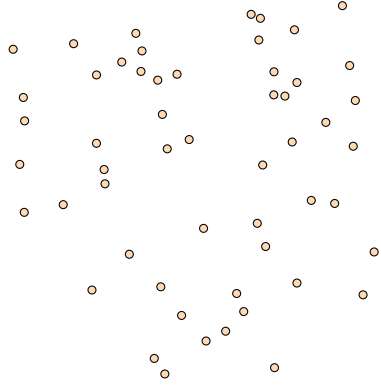

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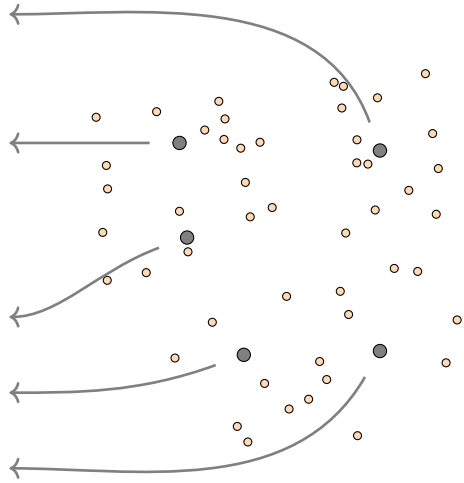


k-CENTER PROBLEM

k-Center problem

Input:

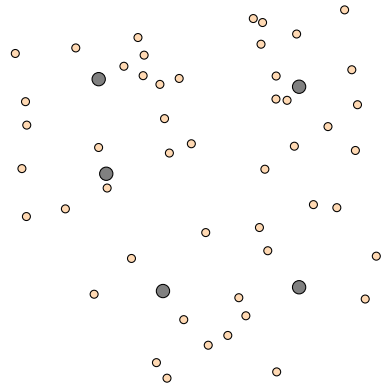
- metric space (X, d)
- integer k

Output: set of centers $C \subseteq X$ s.t

- $|C| \leqslant k$

Goal:

- minimize the distance: $r := \max_{x \in X} d(x, C)$



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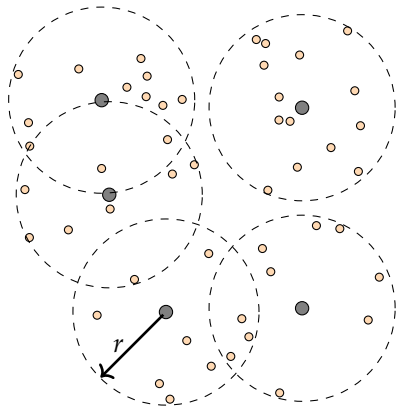

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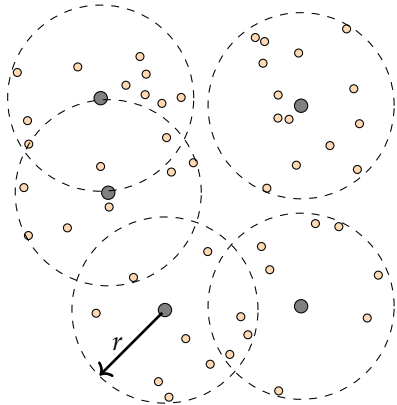
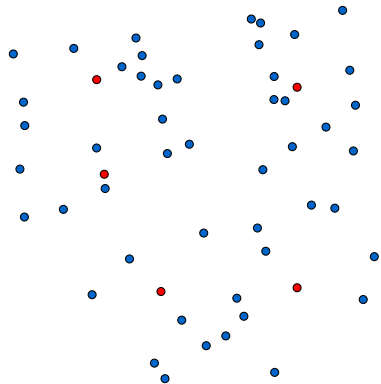
Known results:

- $(2 - \epsilon)$-hard (APX-hard)
- 2-approx. (Gonzalez algorithm)



REPRESENTATIVE k-CENTER PROBLEM



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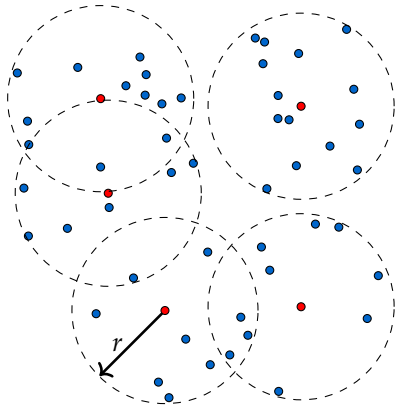

REPRESENTATIVE k-CENTER PROBLEM

Representative k-Center problem

Input:

- metric space (X, d)
- integer $k \in \mathbb{Z}_{\geqslant 0}$
- partition $\{X_1, \ldots, X_\gamma\}$
- integers $a_i \leqslant b_i$ for $i \in [\gamma]$

Output: set of centers $C \subseteq X$ s.t

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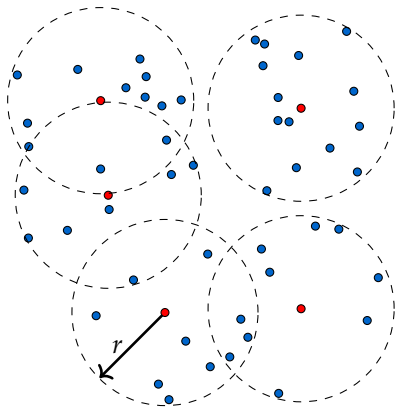

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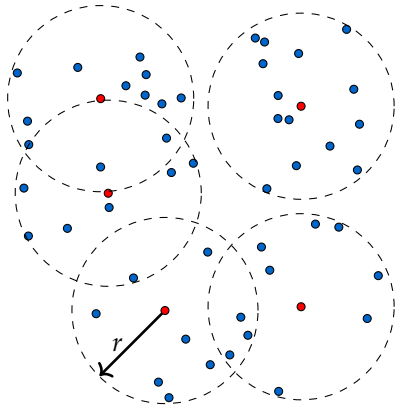
- Chen et.al, Kleindessner et al., Jones et.al $\rightarrow$ 3 **approx. in** $O(nk)$ **time**



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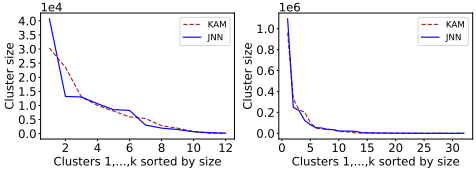

(a) Diabetes: $k = 12$ (b) Electric: $k = 32$

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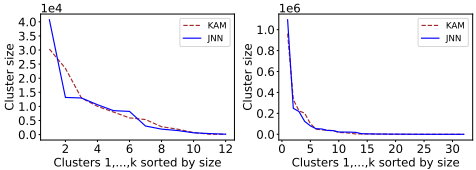

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Priv-Rep k-Center problem

Input:

- metric space (X, d)
- integer $k, L \in \mathbb{Z}_{\geqslant 0}$
- partition $\{X_1, \ldots, X_\gamma\}$
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Output: set of centers $C \subseteq X$, assignment $\phi : X \to C$ s.t

- $|C| \leqslant k$
- $a_i \leqslant |C \cap X_i| \leqslant b_i$ for $i \in [\gamma]$
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Goal:

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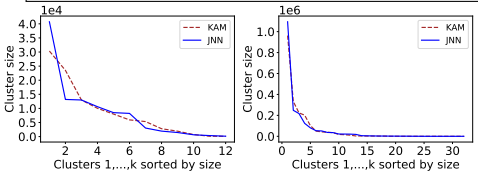

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NAIVE IDEA: Expand Rep-k-Center solution

Open: 2 blue, 1 red. **What if $L = 2$?**

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Main theorem:

There is a 15 approximation algorithm for PRIV-REP-KC that runs in time $O(nk^2 + k^5)$.

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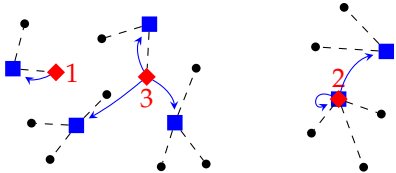

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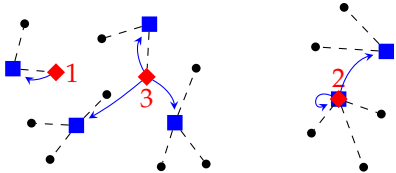

Algorithm sketch

1. Create $q \leqslant k^2$ backbones $(\Pi_1, \eta_1), \ldots, (\Pi_q, \eta_q)$.
2. **for** $i = 1, \ldots, q$ **do**:
   - Solve MIN-REP-REALIZATION for backbone (Π_i, η_i).
   - Compute private clustering with centers C_i of smallest possible radius r_i.
3. **return** the clustering with smallest r_i.

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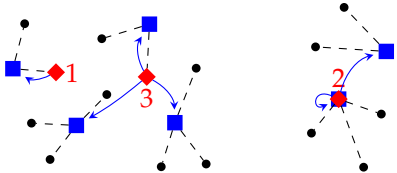

REMARKS:

- Beating nk^2 time algorithm is hard, even for computing private assignment for given set of centers.
- Achieving overall $O(nk^2 + k^5)$ running time is highly nontrivial.
- Backbones can be extended to combine privacy with other constraints.

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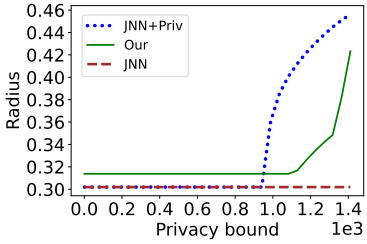
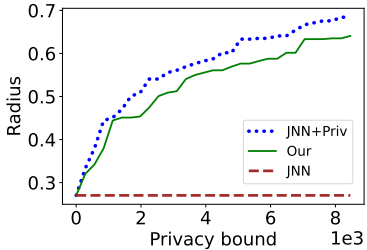
EXPERIMENTS



(a) Query: $k = 7$



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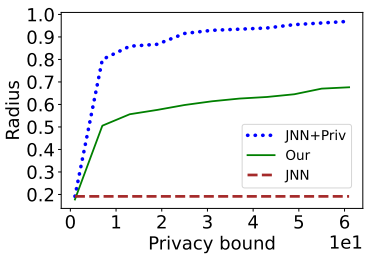

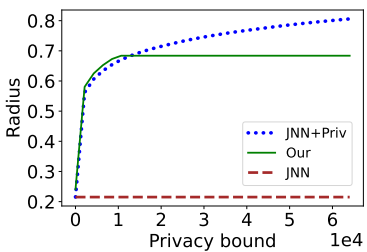

Thank you