Inductive Matrix Completion: No Bad Local Minima and a Fast Algorithm

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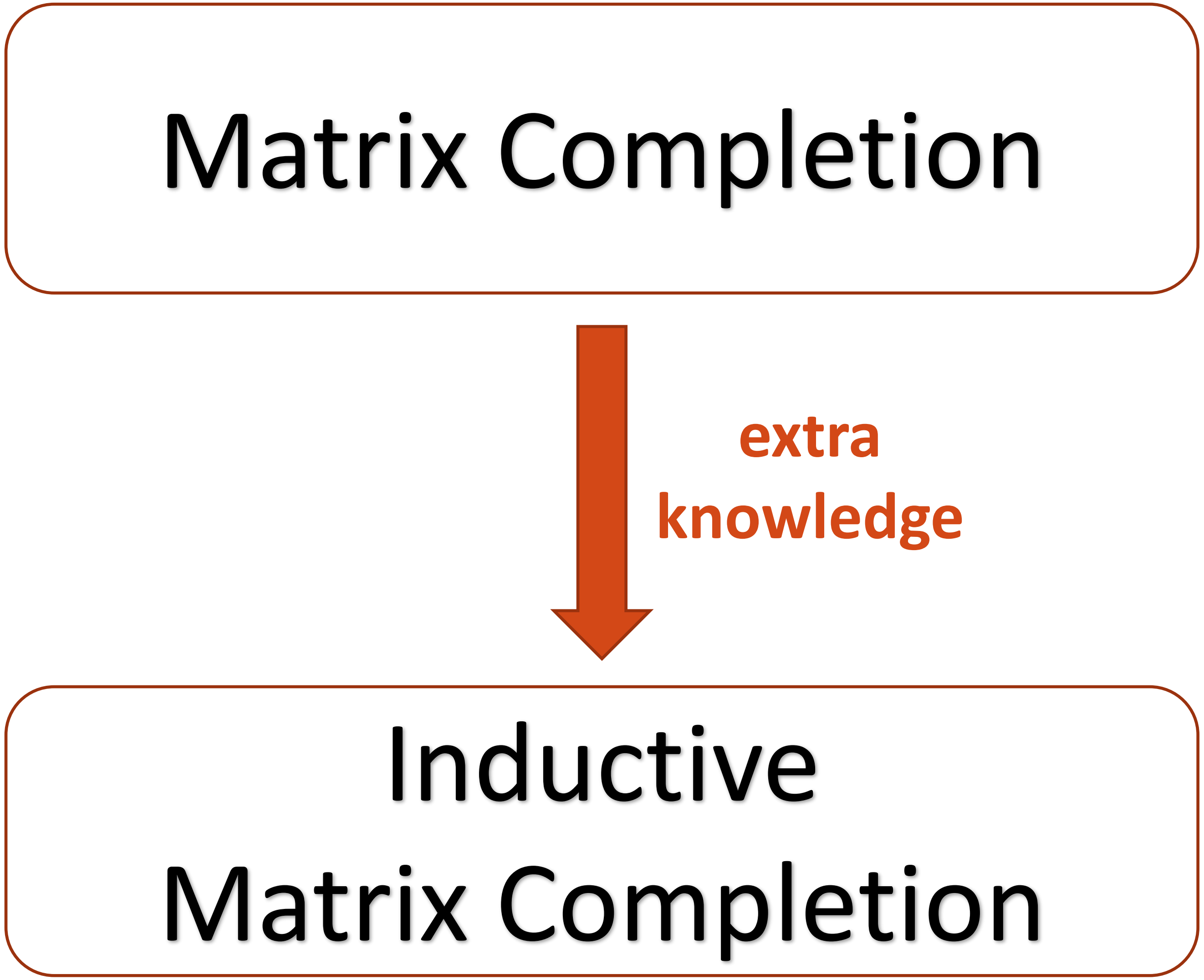
International Conference on Machine Learning 2022

Matrix Completion

$$\begin{pmatrix} ? & 4 & ? & 2 \\ ? & 2 & -3 & ? \\ -2 & ? & ? & -2 \end{pmatrix}$$

Standard assumption: underlying matrix has low rank

$$\begin{pmatrix} 2 & 4 & -6 & 2 \\ 1 & 2 & -3 & 1 \\ -2 & -4 & 6 & -2 \end{pmatrix}$$



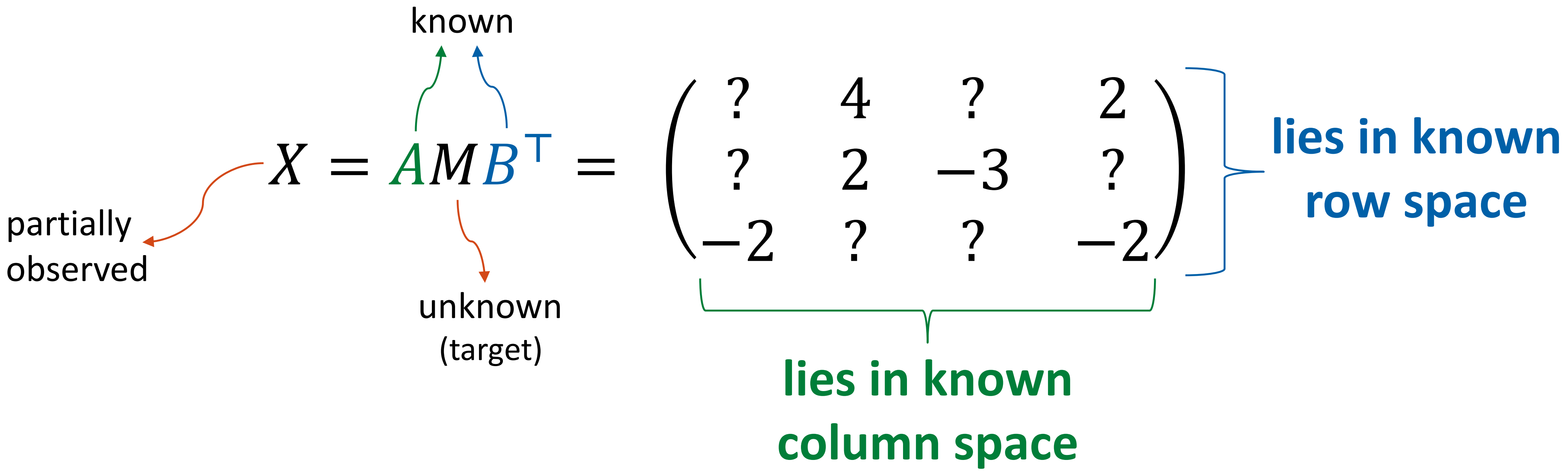
Matrix Completion

extra knowledge

Inductive Matrix Completion

[Jain & Dhillon ’13]
[Xu, Jin & Zhou ’13]

Inductive Matrix Completion



known

$$X = AMB^\top = \begin{pmatrix} ? & 4 & ? & 2 \\ ? & 2 & -3 & ? \\ -2 & ? & ? & -2 \end{pmatrix}$$

partially observed

unknown (target)

lies in known row space

lies in known column space

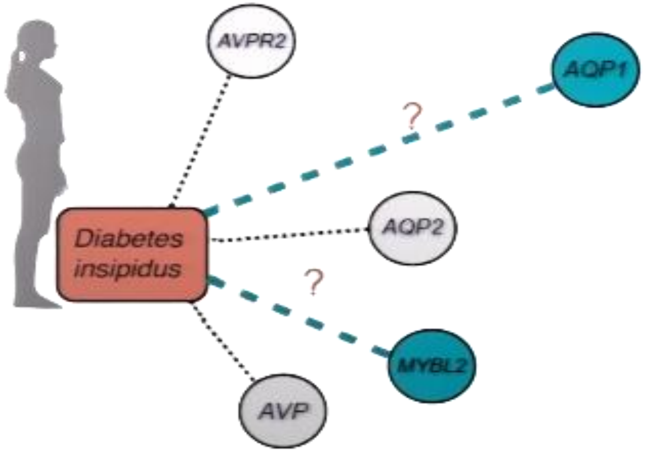
→ Matrix Completion in reduced dimensions (dim M < dim X)

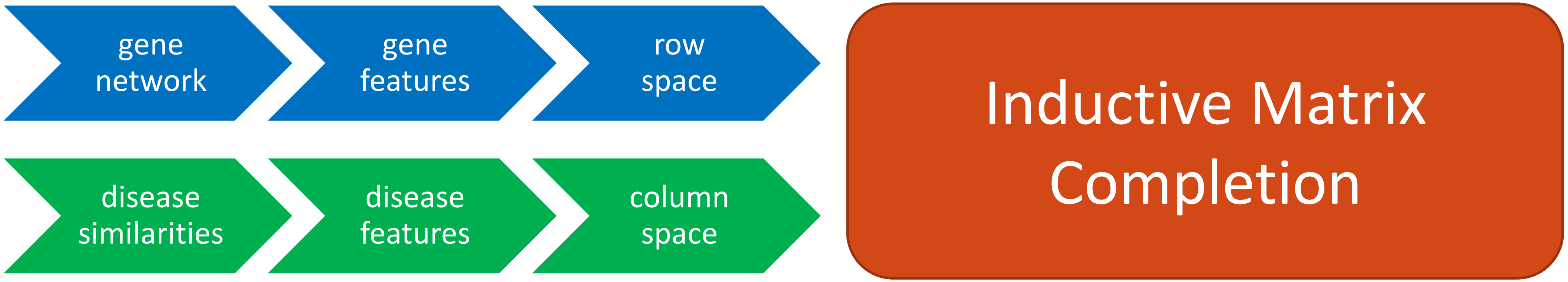
[Natarajan & Dhillon ’14]

Example: Gene-disease prediction

$$\begin{array}{cc} & \text{genes} \\ \text{diseases} & \begin{pmatrix} ? & 1 & ? \\ ? & ? & 0 \\ 0.5 & ? & ? \end{pmatrix} \end{array}$$

Goal: complete unknown associations





gene network

gene features

row space

disease similarities

disease features

column space

Inductive Matrix Completion

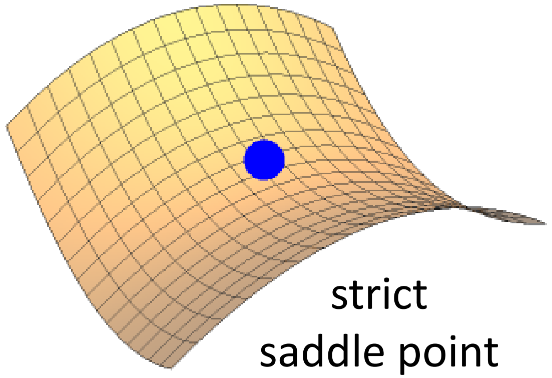
Our contributions

- No bad local minima guarantee
- Provable rank estimation scheme
- GNIMC: fast and provable Gauss-Newton based algorithm

No Bad Local Minima Guarantee

In general, Inductive Matrix Completion is **nonconvex**

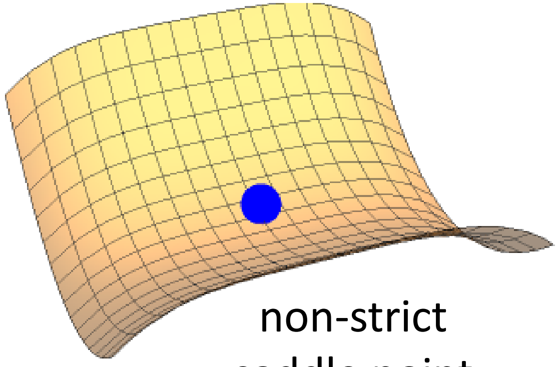
Theorem: (informal) under certain conditions, all critical points are either global minima or strict saddle points



strict
saddle point

Novelty:

- holds with few observations
- no regularization required



non-strict
saddle point

Underlying technique: creating a bridge to Matrix Sensing

by proving Restricted Isometry Property (RIP)

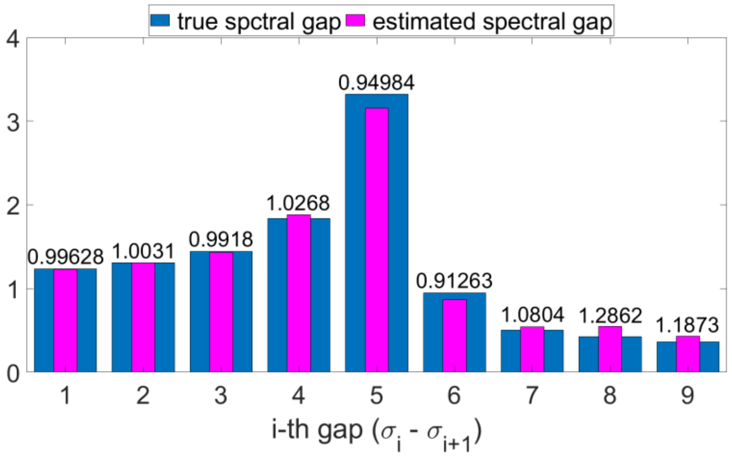
Rank estimation scheme

To complete the matrix, many algorithms (e.g. gradient descent) require its rank as input

Our rank estimator is based on the spectral gaps between pairs of consecutive singular values

Theorem: (informal) Let X be an $n \times m$ matrix, whose row and column subspaces are known. Then our estimator recovers rank(X) given $\mathcal{O}(\log(n) + \log(m))$ randomly observed entries

- First rank estimation guarantee
- The rank of X may be either exact or approximate
- The observed entries may be corrupted by noise



GNIMC algorithm

GNIMC: Gauss Newton Inductive Matrix Completion

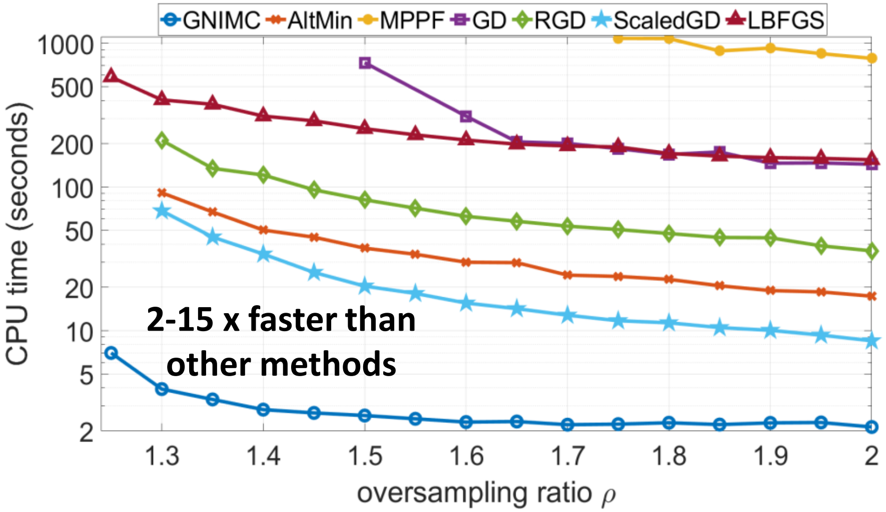
Based on [Zilber & Nadler, GNMR, SIMODS (2022), to appear]

Theorem: (informal) Let X be an $n \times m$ matrix, whose row and column subspaces are known. Suppose we randomly observe $\mathcal{O}(\log(n) + \log(m))$ of its entries.

Then starting from a sufficiently accurate initialization, GNIMC successfully recovers X. Moreover, in the absence of noise, GNIMC recovers X at a *quadratic* rate.

Bonus: no hyperparameter tuning required!

20000×1000 matrix with $0.005\% - 0.007\%$ observed entries



Python + Matlab implementations of GNIMC

as well as other IMC algorithms are available at

github.com/pizilber/IMC

Thank you