Federated Minimax Optimization

Improved Convergence Analyses and Algorithms

Pranay Sharma
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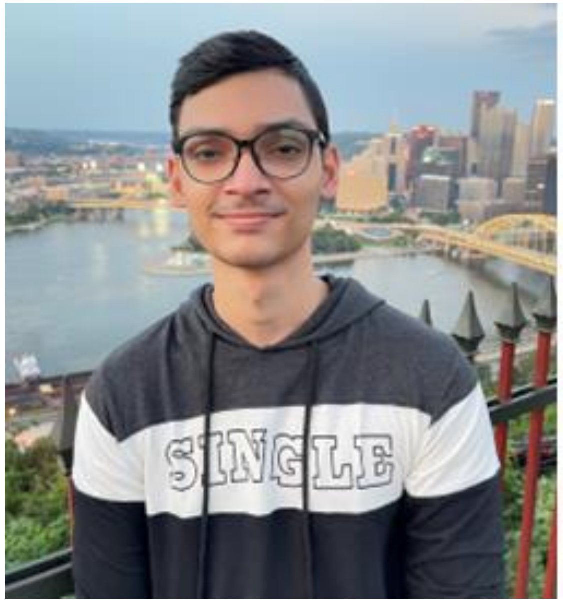

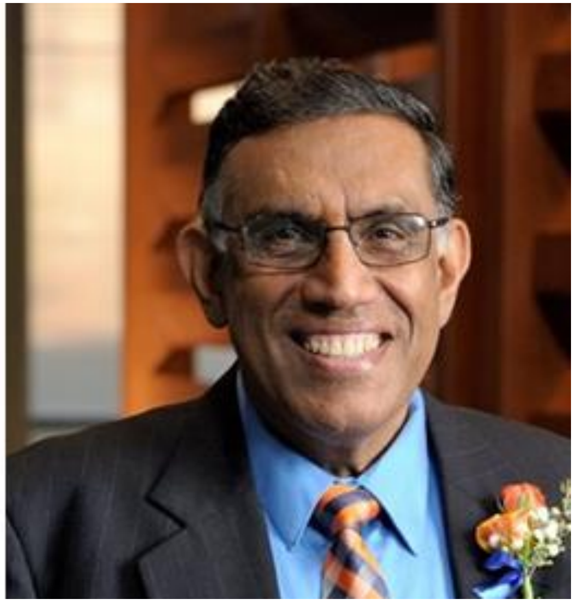

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Federated Minimax: Background

Federated Learning

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Distributed learning with

Federated Minimax: Background

Federated Learning

Distributed learning with

- **Infrequent** communication with the server

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Distributed learning with

- **Infrequent** communication with the server
- Heterogeneous data across clients

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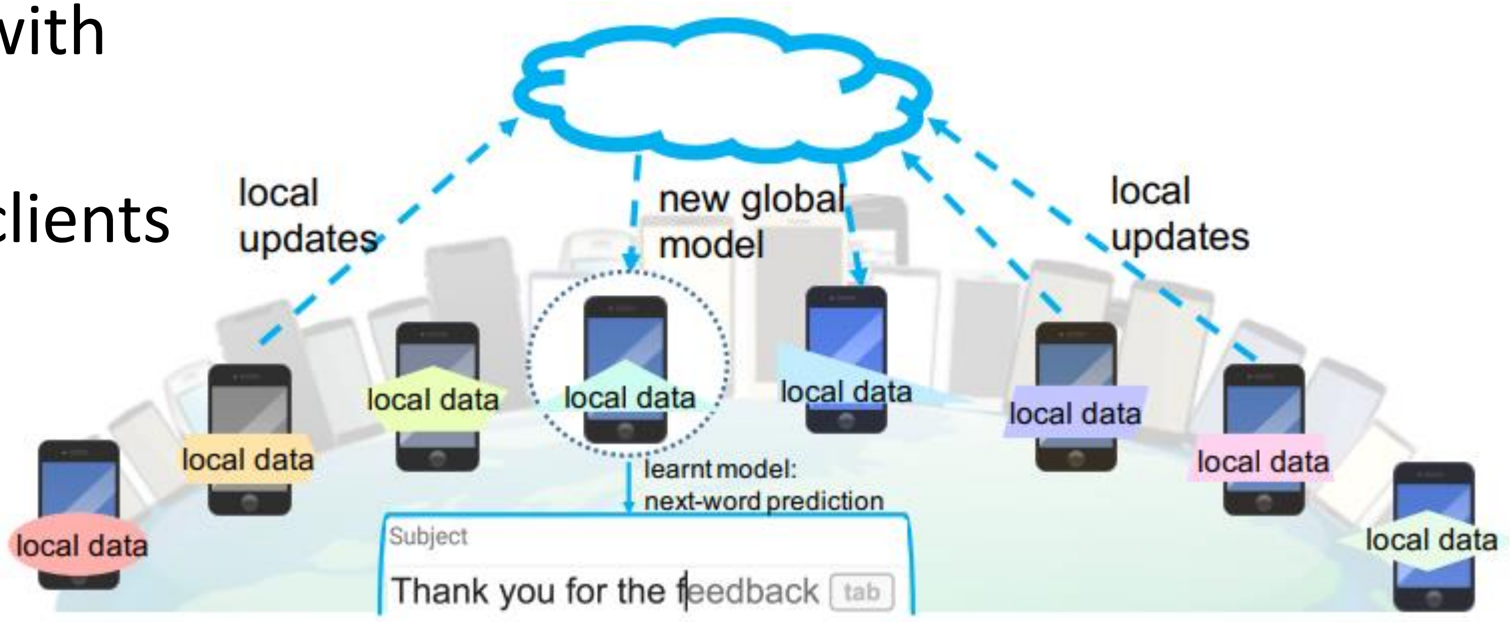

Li et al., IEEE SPMag 2021.

Federated Minimax: Background

Federated Learning

Distributed learning with

- **Infrequent** communication with the server
- Heterogeneous data across clients
- Ensures client data privacy

Minimax Optimization

$$\min_{\mathbf{x}} \max_{\mathbf{y}} f(\mathbf{x}, \mathbf{y})$$

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- GANs, adversarial training of neural networks, reinforcement learning

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Minimax Optimization

$$\min_{\mathbf{x}} \max_{\mathbf{y}} f(\mathbf{x}, \mathbf{y})$$

- GANs, adversarial training of neural networks, reinforcement learning
- f is often nonconvex in $\mathbf{x}$, nonconcave in $\mathbf{y}$

Problem and Algorithm

$$\min_{\mathbf{x}} \max_{\mathbf{y}} \left\{ f(\mathbf{x}, \mathbf{y}) \triangleq \frac{1}{n} \sum_{i=1}^{n} f_i(\mathbf{x}, \mathbf{y}) \right\}$$

Problem and Algorithm

n - No. of clients

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n - No. of clients

$f_i(\mathbf{x}, \mathbf{y}) \triangleq \mathbb{E}_{\xi_i \sim \mathcal{D}_i}[L(\mathbf{x}, \mathbf{y}; \xi_i)]$

- $\mathcal{D}_i$ - Local data distribution

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- $\mathbf{y}^i \leftarrow \mathbf{y}^i + \eta_y \widetilde{\nabla}_{\mathbf{y}} f_i(\mathbf{x}^i, \mathbf{y}^i)$

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Infrequent averaging

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Infrequent averaging

- Every τ iterations
- Restart with the average

Theoretical Results I

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$f(\mathbf{x}, \mathbf{y})$ is nonconvex in $\mathbf{x}$, **strongly concave** in $\mathbf{y}$

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- Optimal in ϵ

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- Communication rounds: $\mathcal{O}\left(\frac{1}{\epsilon^3}\right)$

Comparison with Existing Work

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	Sample Complexity	Communication Rounds
Lin et al., 2020 ($n = 1$)	$\mathcal{O}(\epsilon^{-4})$	-

- Needs $\mathcal{O}(\epsilon^{-2})$ batch-size

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- $\mathcal{O}(1)$ batch-size
- Linear speedup in n

Summary

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$f(\mathbf{x}, \mathbf{y})$ is nonconvex in $\mathbf{x}$

1. strongly concave in $\mathbf{y}$
2. PL in $\mathbf{y}$
3. Concave in $\mathbf{y}$
4. 1-Point-Concave in $\mathbf{y}$

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$f(\mathbf{x}, \mathbf{y})$ is smooth and

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- Local update algorithms
- $\mathcal{O}(1)$ batch-size
- Optimal/SOTA complexity

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