

# Linear Adversarial Concept Erasure

Shauli Ravfogel, Michael Twiton, Yoav Goldberg and Ryan Cotterell



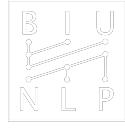
# Motivation

Neural models learn rich representations

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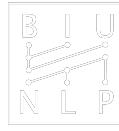
Neural models learn rich representations  
But can we control their content?

# Controlled Representation Learning



- Often, we want to make sure some concept is *not* encoded.

# Controlled Representation Learning

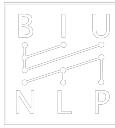


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  - Word embeddings without tense distinctions
  - Sensitivity to content, but not to style
  - **Representations that do not leak protected attributes**

# The linear concept subspace hypothesis

Useful use case: the concept lives in low-dimensional **subspace** within the representation space.

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**How can we identify the concept subspace?**

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We formulate an **adversarial game** between a projection matrix  $P \in \mathcal{P}_k$  that tries to remove the information, and a predictor  $\theta \in \Theta$  that tries to recover it.

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Predictor that tries to  
minimize it

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# Side trip: Iterative Nullspace Projection (INLP)

## Null It Out: Guarding Protected Attributes by Iterative Nullspace Projection

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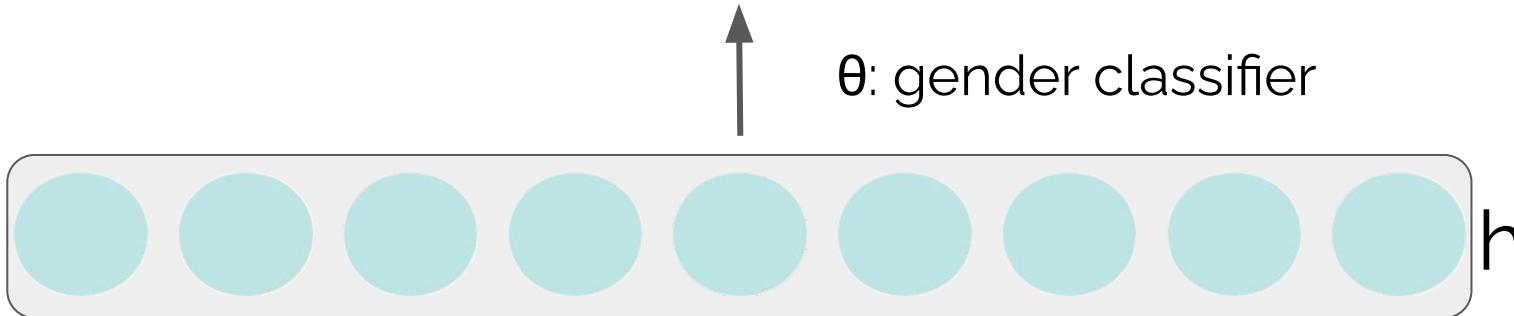
<sup>1</sup>Computer Science Department, Bar Ilan University

<sup>2</sup>Allen Institute for Artificial Intelligence

<sup>3</sup>Independent researcher

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# Nullspace projections

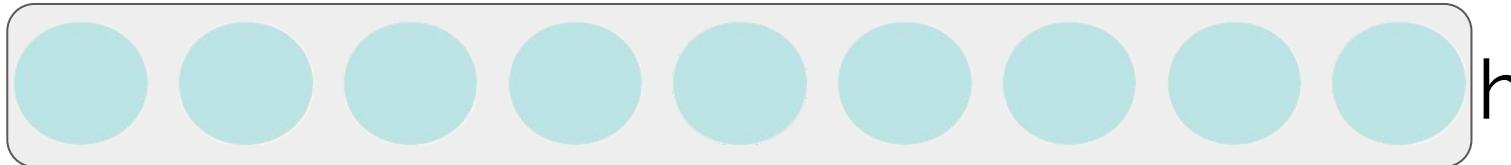


# Nullspace projections

Female!



$\theta$ : gender classifier

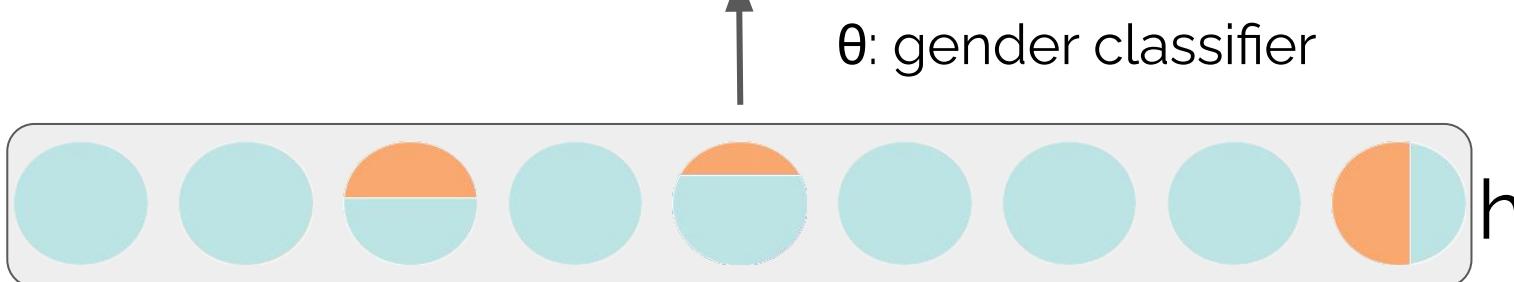


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Features that  $\theta$  finds indicative of gender

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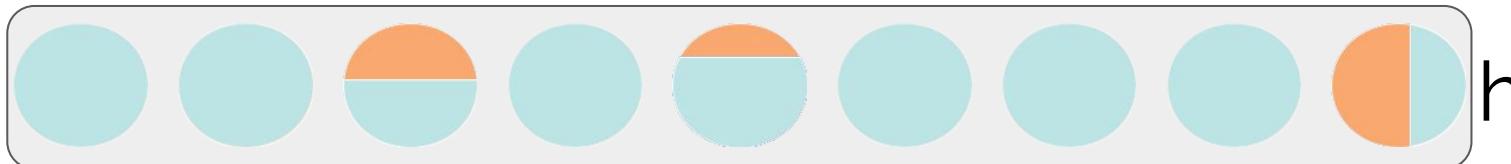


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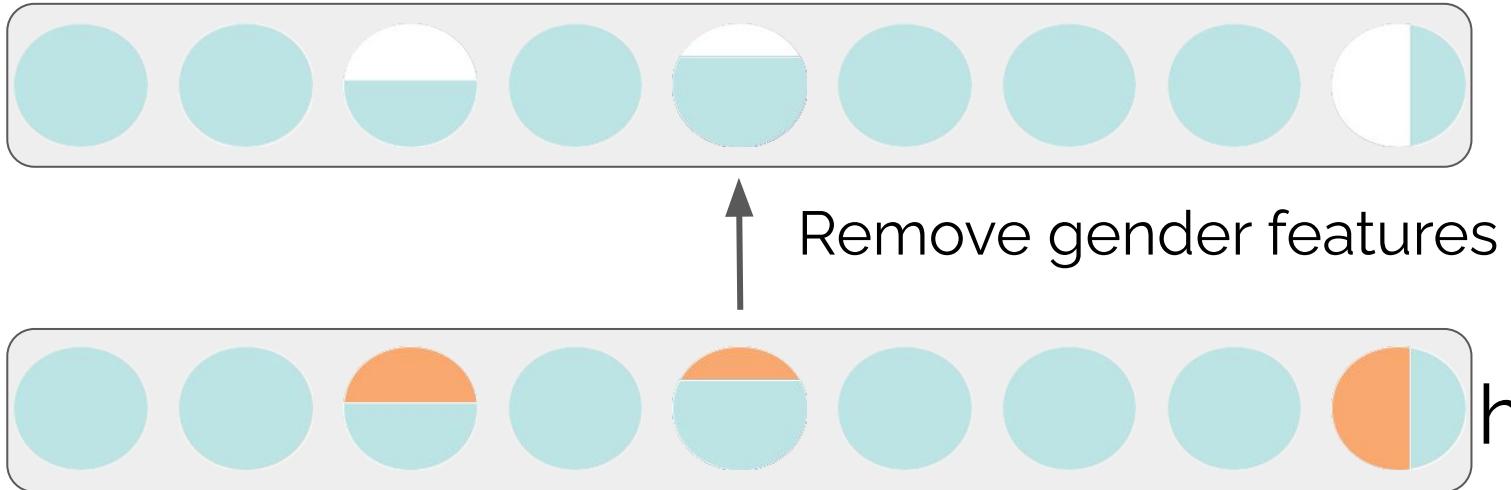


Remove gender features



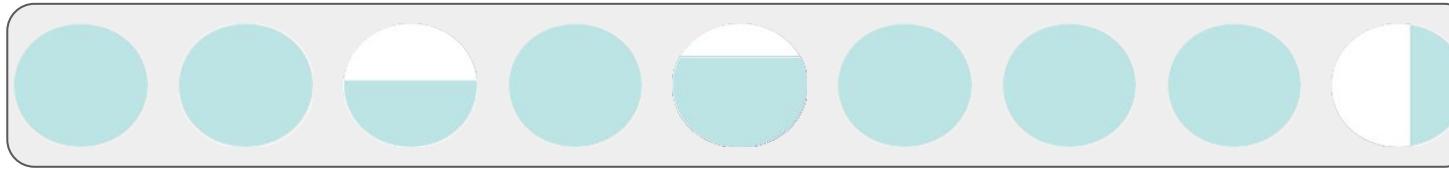
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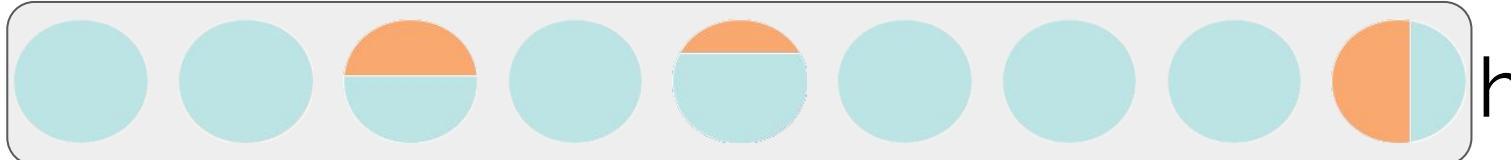


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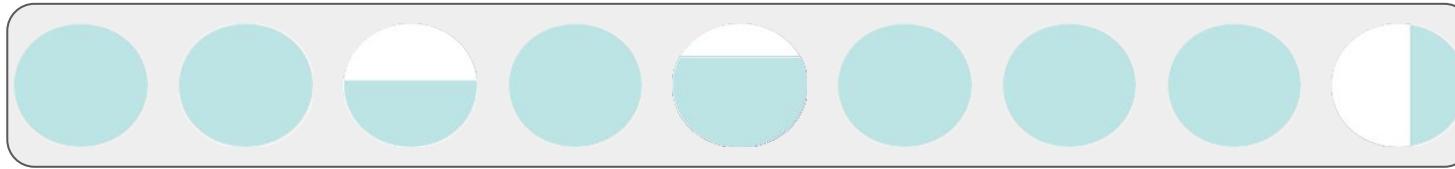
↑ Remove gender features **how?**



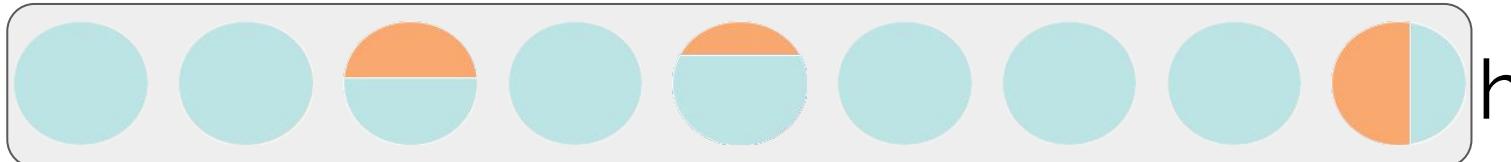
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**Project**  $h$  to the orthogonal complement of  $\theta$



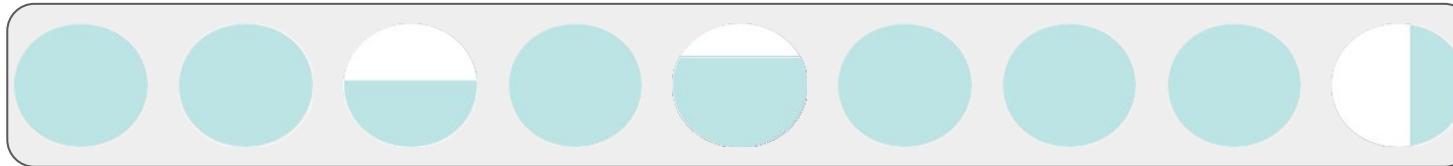
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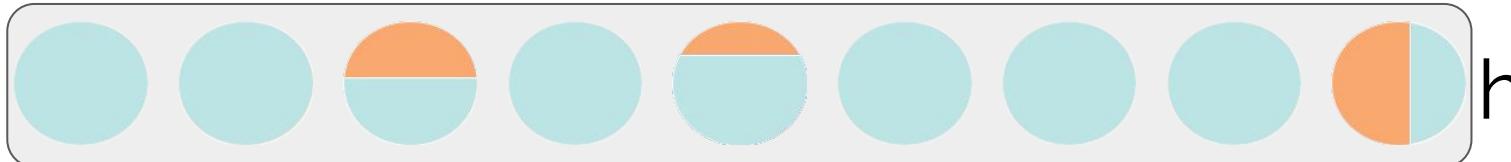
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**Iteratively**



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- 2. Rayleigh quotient losses
- 3. **Classification**

The diagram consists of three numbered items: 1. Linear regression, 2. Rayleigh quotient losses, and 3. Classification. A bracket on the right side of the list groups items 1 and 2 together, with the text 'Closed-form solution (details in the paper)' written in blue next to it. A bracket on the right side of the list groups item 3 with the text 'Gradient-based optimization' written in blue next to it.

**Closed-form solution  
(details in the paper)**

**Gradient-based optimization**

# Classification Case

The loss is an arbitrary classification loss (hinge, logistic, etc).

$$\min_{\boldsymbol{\theta} \in \mathbb{R}^D} \max_{P \in \mathcal{P}_k} \sum_{n=1}^N y_n \log \frac{\exp \boldsymbol{\theta}^\top P \mathbf{x}_n}{1 + \exp \boldsymbol{\theta}^\top P \mathbf{x}_n}$$

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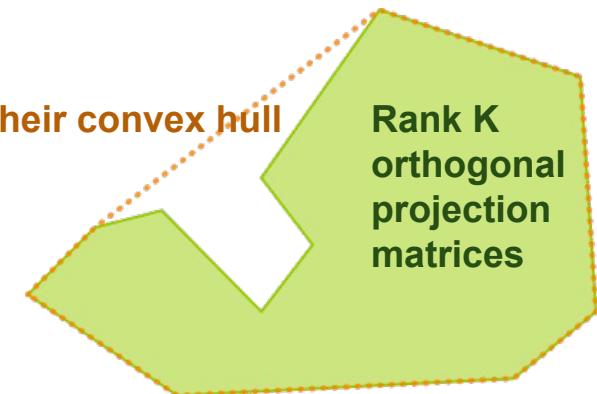
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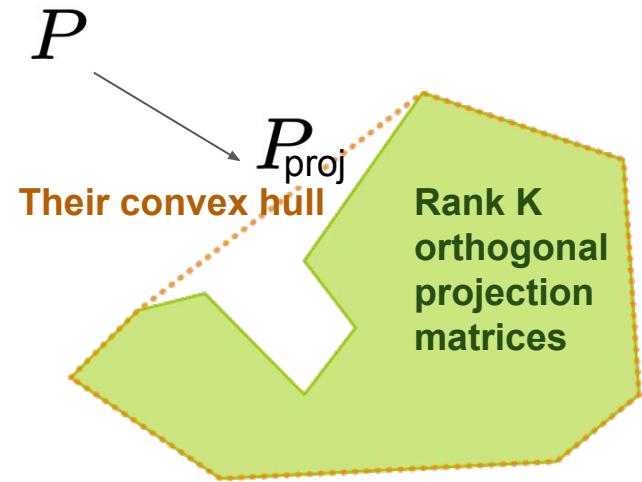
Where we define:

$$\mathcal{F}_k = \text{conv}(\mathcal{P}_k)$$



# Convex Relaxation (RLACE)

In training, we optimize over an arbitrary matrix  $P$ , and we project it to the convex hull of orthogonal projection matrices at each step.



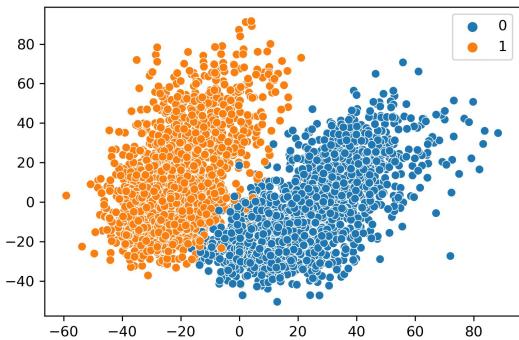
# Experimental Evaluation

We conduct experiments on GloVe embeddings and on contextualized representations of short biographies (annotated for both gender and profession).

# Results

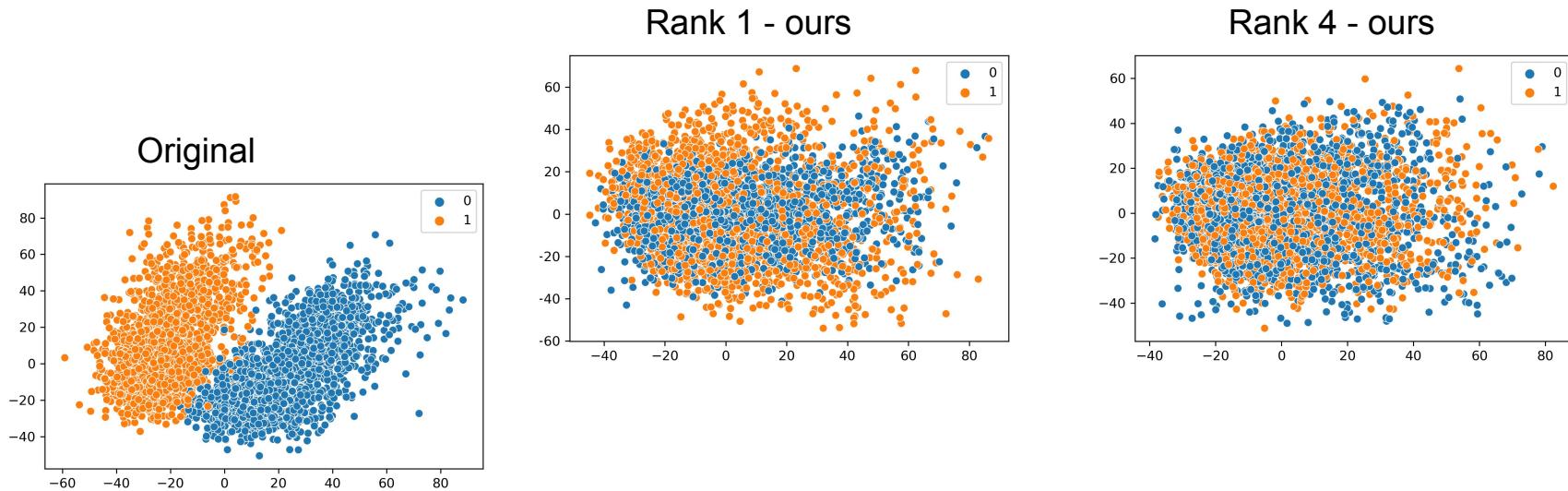
PCA: less clustering of representations by gender

Original



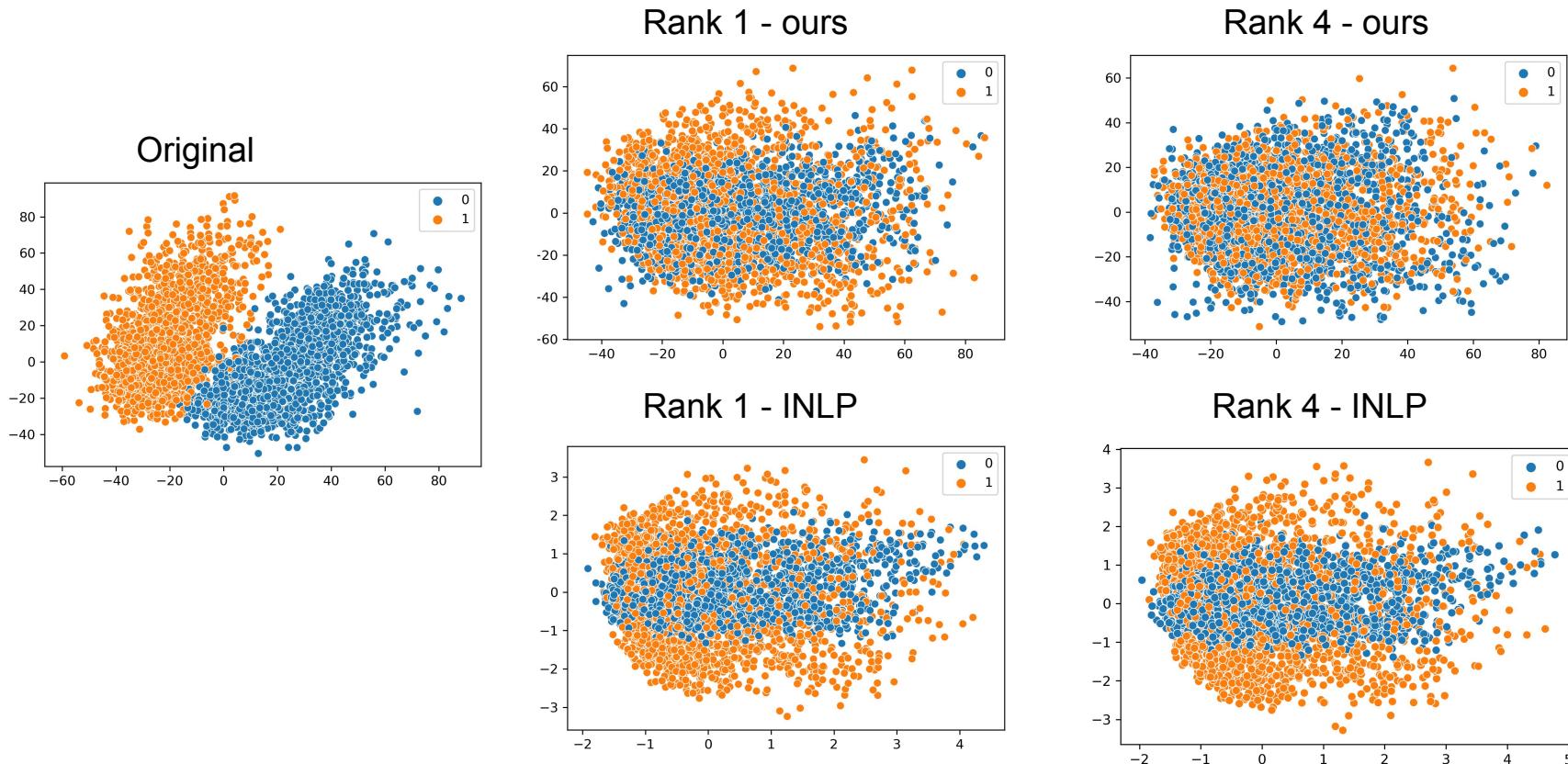
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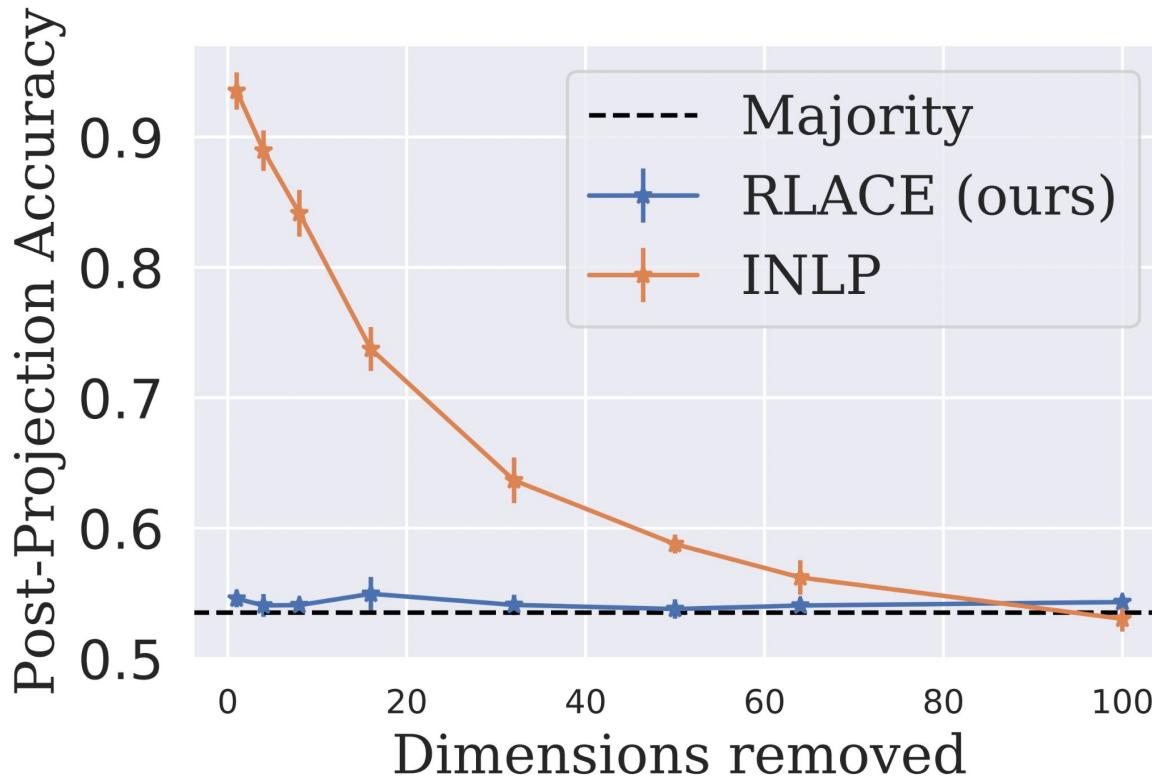


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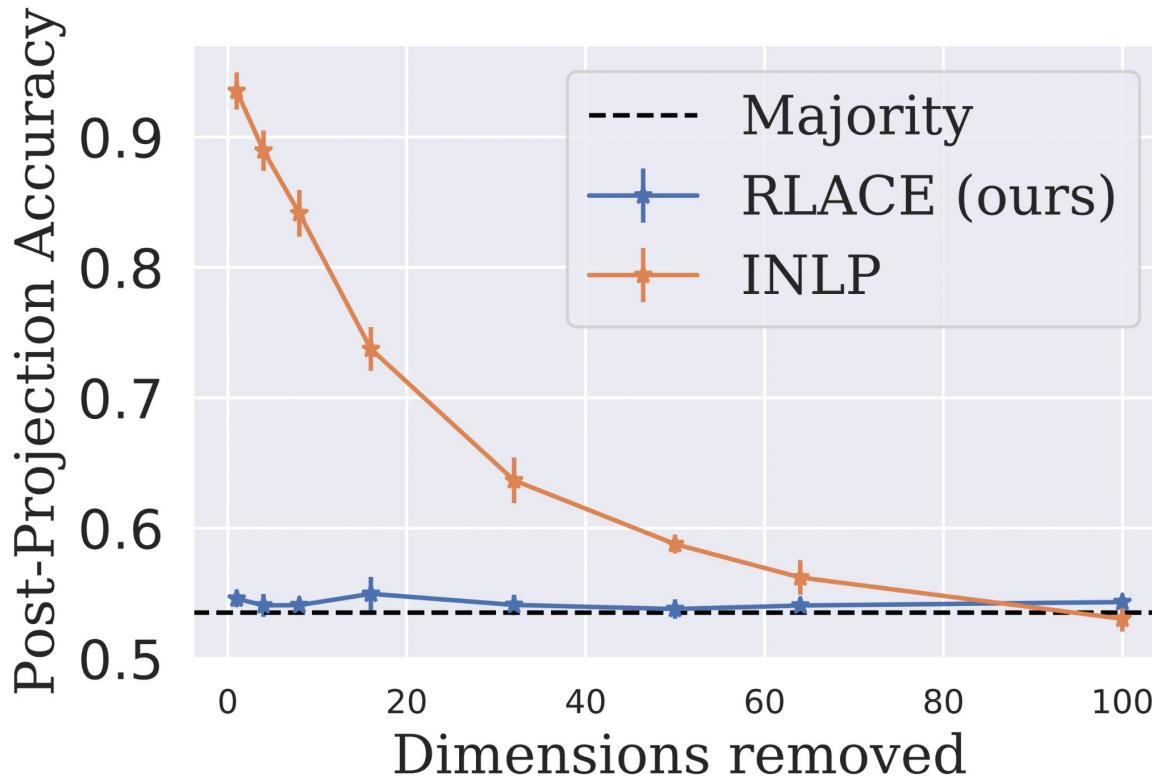
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# Comparison with INLP - BERT (bios data)



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Additional results in the paper.

# Application on Images



Original

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Original

Smile

# Application on Images



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- We present analytical solutions in some cases, and provide a relaxation which works well for others.
- High level conclusion: it's sometimes valuable to constrain our model (in contrast to the “more parameters is better” trend)

# Thanks!

