# IDYNO: Learning Nonparametric DAGs from Interventional Dynamic Data

Tian Gao, Debarun Bhattacharjya, Elliot Nelson, Miao Liu, Yue Yu

IBM Research, Lehigh University

tgao@us.ibm.com

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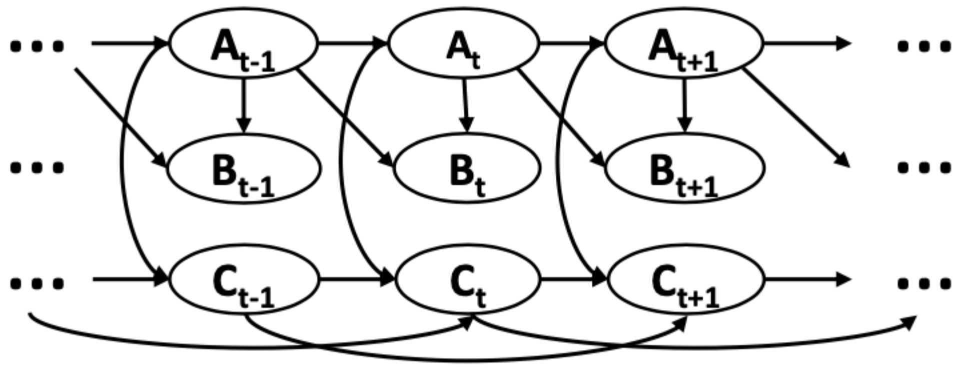
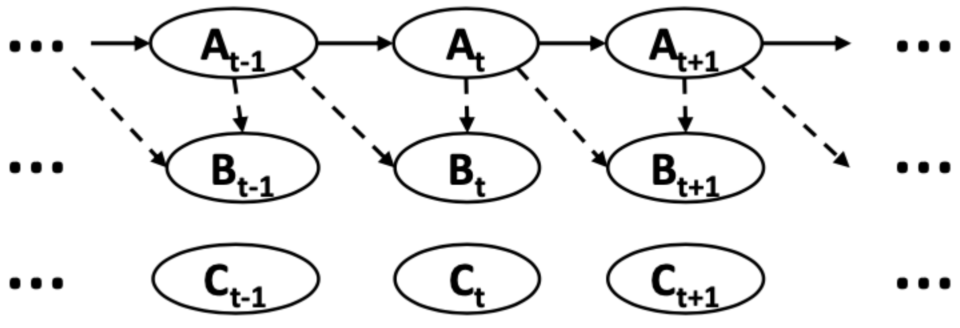
# Overview

- Directed acyclic graph (DAG) structure learning for dynamic Bayesian networks (DBN)
- IDYNO: capable to handle both observational and *(hard/soft)* *interventional* time series data
  - Optimization with potentially nonlinear objectives and continuous DAG constraints




(a)




(b)

# Related Works: DAG Learning

**I.I.D. Data (BN)**

- Score-based and constraint-based approaches [Spirtes et al, ’01; Chickering, ’02; Tsamardinos et al, ‘06]

- NOTEARS: Continuous algebraic characterization of the DAG [Zheng et al, ’18]

$$\min_{\theta, \mathbf{W}} L_\theta(\mathbf{X}; \mathbf{W}) - \lambda\Omega(\theta, \mathbf{W}) \quad s.t. \quad h(\mathbf{W}) = 0,$$

$$h(\mathbf{W}) = \mathrm{Tr}(e^{\mathbf{W}}) - d$$

**Time Series Data (DBN)**

- (Structured) vector auto-regressive (SVAR) [Swanson & Granger, ‘97; Reale & Wilson, ‘01]

- Constraint-based [Malinsky, ’19]

- DYNOTEARS: Continuous constraint [Pamfil, ‘20]

# Interventional Data

**Observation**

- SVAR Model with Structure Equation Models (SEM)

$$\min_{\theta,\mathbf{W},\mathbf{A}} L_\theta(\mathbf{X};\mathbf{W},\mathbf{A}) - \lambda\Omega(\theta,\mathbf{W}) \;\; s.t. \;\; h(\mathbf{W}) = 0,$$

$$X_t = X_t\mathbf{W} + X_{t-1}\mathbf{A_1} + ... + X_{t-p}\mathbf{A_p} + Z_t,$$

**Intervention**

- DCDI: Interventional DAG Learning [Brouillard et al, '20]

- For each interventional family $k$:

$$p^{(k)}(X) := \prod_{j\notin I_k} p_j^{(1)}(x_j|x_{\pi_j^G}) \prod_{j\in I_k} p_j^{(k)}(x_j|x_{\pi_j^G})$$

# IDYNO: Formulation

- Linear SEM

$$\min_{\mathbf{W},\mathbf{A}} \frac{1}{n} \sum_{k=1}^{K} \sum_{j=1}^{d} \Big( || \left(\mathbf{X} - \mathbf{X}\mathbf{W}_{(\mathbf{1})} - \mathbf{Y}\mathbf{A}_{(\mathbf{1})}\right)_j ||_2^{2(1-r^{\mathcal{I}}_{kj})}$$
$$+ || \left(\mathbf{X} - \mathbf{X}\mathbf{W}_{(\mathbf{k})} - \mathbf{Y}\mathbf{A}_{(\mathbf{k})}\right)_j ||_2^{2r^{\mathcal{I}}_{kj}} \Big) + \lambda\Omega(\theta)$$
$$s.t. \quad Tr(e^{\mathbf{W}}) - d = 0 \qquad \textbf{(8)}$$

Where $\left[r^I_{kj}\right] \in \{0,1\}^{k\times d} = 1$ when $x_j$ is intervened on with interventional family $k$

# IDYNO: Formulation

- Nonlinear Time Series Data

$$\min_{W,A,\theta} \frac{1}{n} \sum_{k=1}^{K} \sum_{j=1}^{d} L_j(\mathbf{X}, MLP(\mathbf{X}; \theta_j, \mathbf{A}_{(\mathbf{1})}, \mathbf{W}_{(\mathbf{1})}))^{1-r^{\mathcal{I}}_{kj}}$$

$$L_j(\mathbf{X}, MLP(\mathbf{X}; \theta_j, \mathbf{A}_{(\mathbf{k})}, \mathbf{W}_{(\mathbf{k})}))^{r^{\mathcal{I}}_{kj}}$$

$$+ \lambda_a ||A_j^{(1)}||_{1,1} + \lambda_w ||W_j^{(1)}||_{1,1} \tag{12}$$

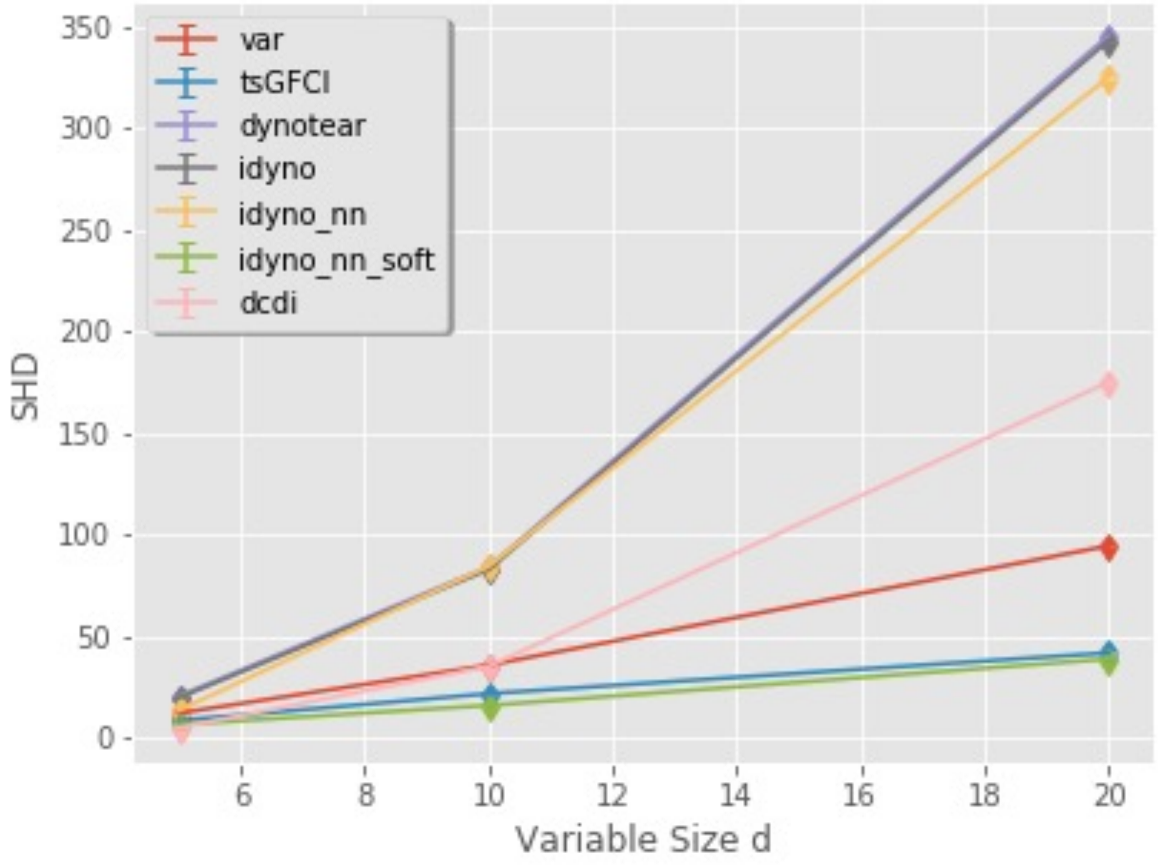
$$s.t. \quad Tr(e^{\mathbf{W}}) - d = 0 \tag{13}$$

- **Identifiability**: Under some assumptions, the learned graph is $(I, D_s)$-Markov equivalent to the ground true graph $G^*$
  - $D_s$: DAGs that correspond to stationary dynamics with constant-in-time inter-slice and intra-slice conditional distributions
  - $(I, D_s)$-Markov equivalent: $D_s \subset DAG$ which are $I$-Markov equivalent to $G^*$

# Empirical Evaluation

*Table 1.* SHD Results for Synthetic Linear Datasets

| Dataset | DYNOTEARS | IDYNO |
|---|---|---|
| Observational | 2.0± 0.0 | 2.0± 0.0 |
| Interventional | 32± 0.4 | **19± 0.3** |

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# Questions?

tgao@us.ibm.com