Graph-Coupled Oscillator Networks

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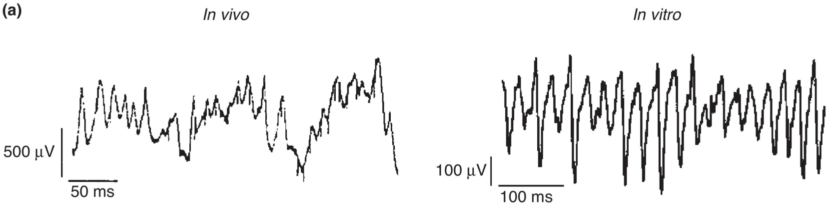
m H_ee9§n} rTn} W i i ys

Oscillators are ubiquitous in nature and engineering systems



¯ \´ n H_ee9§n} Tn} W i i l

- i 0«}n@_nenX_H9e h n§_®9§_n j Tn} j 0§ ° n}c nT n H_ee9§n}´ j 0«}n j
- N²q}0 _®_§´ nT n H_ee9§n} rSn«}_0} 0}_0 9qq}n²_h 9§_n j s
- ¯ 0ee]@0\9®0L X}9L_0 j § nT n H_ee9§n} → N²qenL_j X¤®9 j_ _j X X}9L_0 j § q}n@e0 h h_§ X9§0Ly
- K0 _}9@e0 §9@_e_§´ q}nq0}§_0 l 8 ne«§_n j Tn} §\0 n®0} h nn§_j X q}n@e0 h y

m H_ee9§n}´ _jL«H§_®O @_9 Tn} W i i

Set-up:

- ª jL_}OH§OL W}9q\ $G = (V, E \quad V \times V)$
- NLXO EI q9_} nT jnLO $\{i, j\}$ rLOjn§OL $i \quad j$s
- i nLO TO9§«}O X

W}9q\Gm i @9 OL nj X}9q\ L´j9h_H9e ´ §Oh I

$$X \quad = \sigma(F_\theta(X, t)) - \gamma X - \alpha X \qquad \begin{cases} Y \quad = \sigma(F_\theta(X, t)) - \gamma X - \alpha Y, \\ X \quad = Y \end{cases}$$

- WOjO}9e eO9}j9@eO o]jO_X\@n}\nnL Hn«qe_jX F_θ rOuXu WG i J W8¦J. . . sI

$$(F_\theta(X, t))_{ij} = F_\theta(X_i(t), X_j(t), t) \qquad i \quad j,$$

- 8H§_®9§_nj T«jH§_nj σ
- Gnj§}ne q9}9h O§O} $\gamma, \alpha > 0$

AJ1s /Bb+`2iBx iQM vB2H/b :` T?*PL,

$$Y^{n} = Y^{n \quad 1} + \quad t[\ (F\ (X^{n \quad 1}; t^{n \quad 1})) \qquad X^{n \quad 1} \qquad Y^{n \quad 1}];$$

$$X^{n} = X^{n \quad 1} + \quad tY^{n};$$

7Q` $n = 1; \dots ; N$ - M/

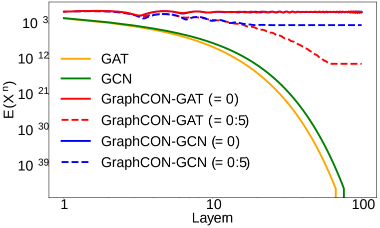
$t > 0$ iBK2@bi2T

$X^n; Y^n$?B//2M MQ/2 72 im t^n 2= b n it iBK2

p}nq0}§_0 nT W}9q\Gm i

¯0 q}n®_L0 h9§\0h9§_H9e L0»j_§_nj nT n®0} hnn§_jXI
0²qnj0j§_9e Hnj®0}X0jH0 §n ¶0}n nT e9´0}]°_ 0 K_}_H\e0§
0j0}X´

Main result:
GraphCON mitigates the oversmoothing problem



¯0 T«}§\0} \n°I

- W}9q\Gm i h_§X9§0 §\0 0²qenL_jX X}9L_0j§ q}n@e0h
- W}9q\Gm i h_§X9§0 §\0 ®9j_ _jX X}9L_0j§ q}n@e0h
- i9®0 h«e§_]e9´0} §9Hc_jX nT W i i Hn}}0 qnjL §n »²0L]qn_j§ _§0}9§_nj nT W}9q\Gm i → _X\0} 0²q}0 _®0 qn°0} nT W}9q\Gm i

¦9@e0l ¦}9j L«H§_®0 jnL0 He9 _»H9§_nj §0 § 9HH«}9H´ r g 8p _j t s nj \0§0}nq_e_H L9§9 0§ u

Homophily level	**Texas** 0.11	**Wisconsin** 0.21	**Cornell** 0.30
Wp|Wi i	78.4 ± 4.4	82.9 ± 4.2	80.3 ± 8.1
[©WG i	84.9 ± 7.2	87.7 ± 5.0	82.7 ± 5.3
WG i ^^	77.6 ± 3.8	80.4 ± 3.4	77.9 ± 3.8
W0nh]WG i	66.8 ± 2.7	64.5 ± 3.7	60.5 ± 3.7
p9_} i n}h	60.3 ± 4.3	48.4 ± 6.1	58.9 ± 3.2
W}9q\~8WN	82.4 ± 6.1	81.2 ± 5.6	76.0 ± 5.0
gdp	80.8 ± 4.8	85.3 ± 3.3	81.9 ± 6.4
W8¦	52.2 ± 6.6	49.4 ± 4.1	61.9 ± 5.1
GraphCON]W8¦	82.2 ± 4.7	85.7 ± 3.6	83.2 ± 7.0
WG i	55.1 ± 5.2	51.8 ± 3.1	60.5 ± 5.3
GraphCON]WG i	85.4 ± 4.2	87.8 ± 3.3	84.3 ± 4.8

¦9@ɐ0l ¦0 § 9HH«}9H´ _j t nj g i^~¦ ~«q0}q_²0ɐ 75u

g nL0ɐ	¦0 § 9HH«}9H´
G\0@ i 0§	75.62
g n i 0§	91.11
p i G i i	98.76
~qɐ_j0G i i	95.22
W^ i	97.23
GraphCON]W^ i	98.53
W9§0LWG i	97.95
GraphCON]W9§0LWG i	98.27
WG i	88.89
GraphCON]WG i	98.68
W8¦	k£uok
GraphCON]W8¦	98.91

¦9@ɐ0l ¦0 § h09j 9@ nɐ«§0 0}}n} nj µ^ i G **rwithout edge features, small 12k versions** }0 §}_H§0L §n h9ɐɐ j0§°n}c _¶0 nT $\sim 100k$ q9}9 h0§0} u

g nL0ɐ	¦0 § g 8N
W^ i	0.41 ± 0.008
W9§0LWG i	0.42 ± 0.006
W}9q\~8WN	0.41 ± 0.005
g n i 0§	0.41 ± 0.007
p i 8	$\mathbf{0.32 \pm 0.032}$
KW i	$\mathbf{0.22 \pm 0.010}$
WG i	0.47 ± 0.002
GraphCON]WG i	$\mathbf{0.22 \pm 0.004}$
W8¦	0.46 ± 0.002
GraphCON]W8¦	$\mathbf{0.23 \pm 0.004}$