Adaptive Gaussian Process Change Point Detection

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Change Point Detection

- Find changes in the **regime of time series**

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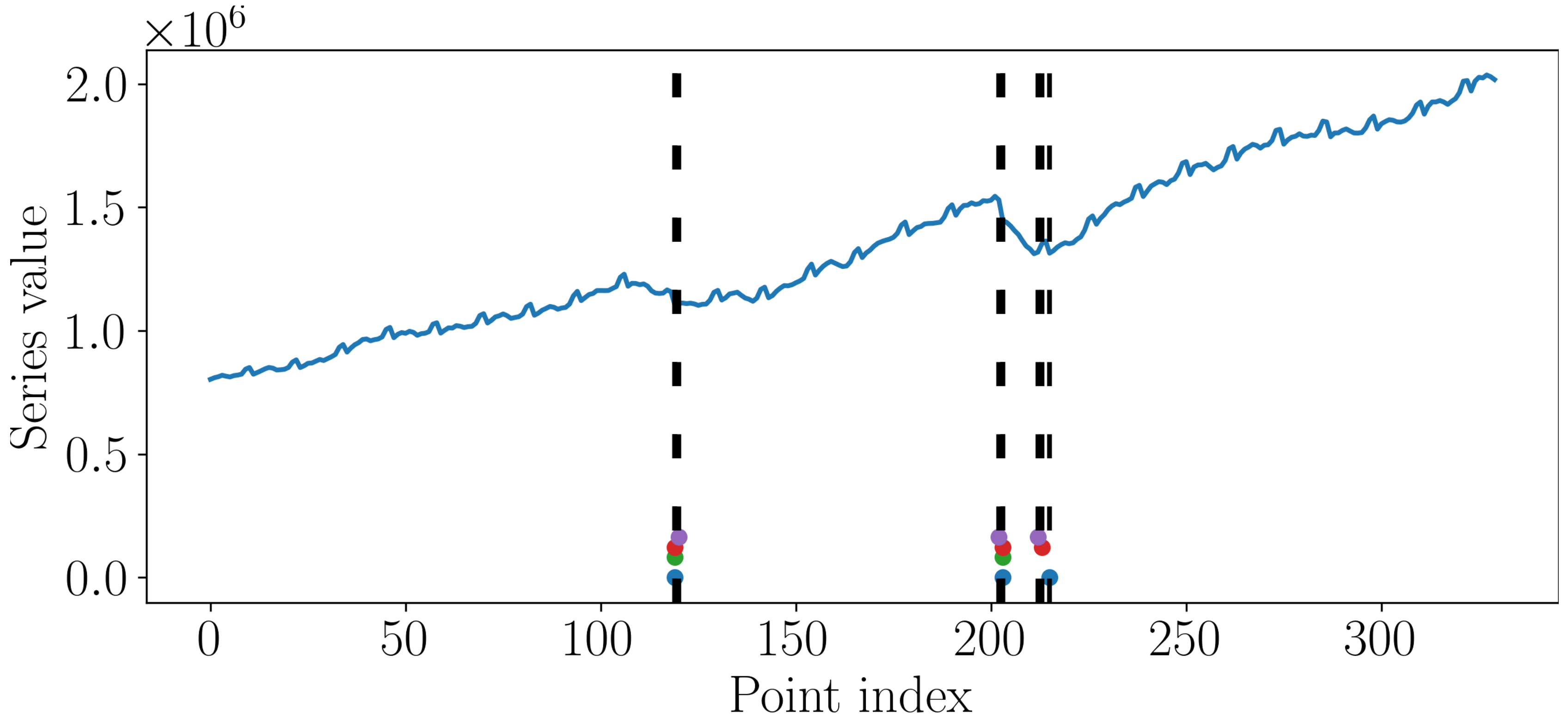
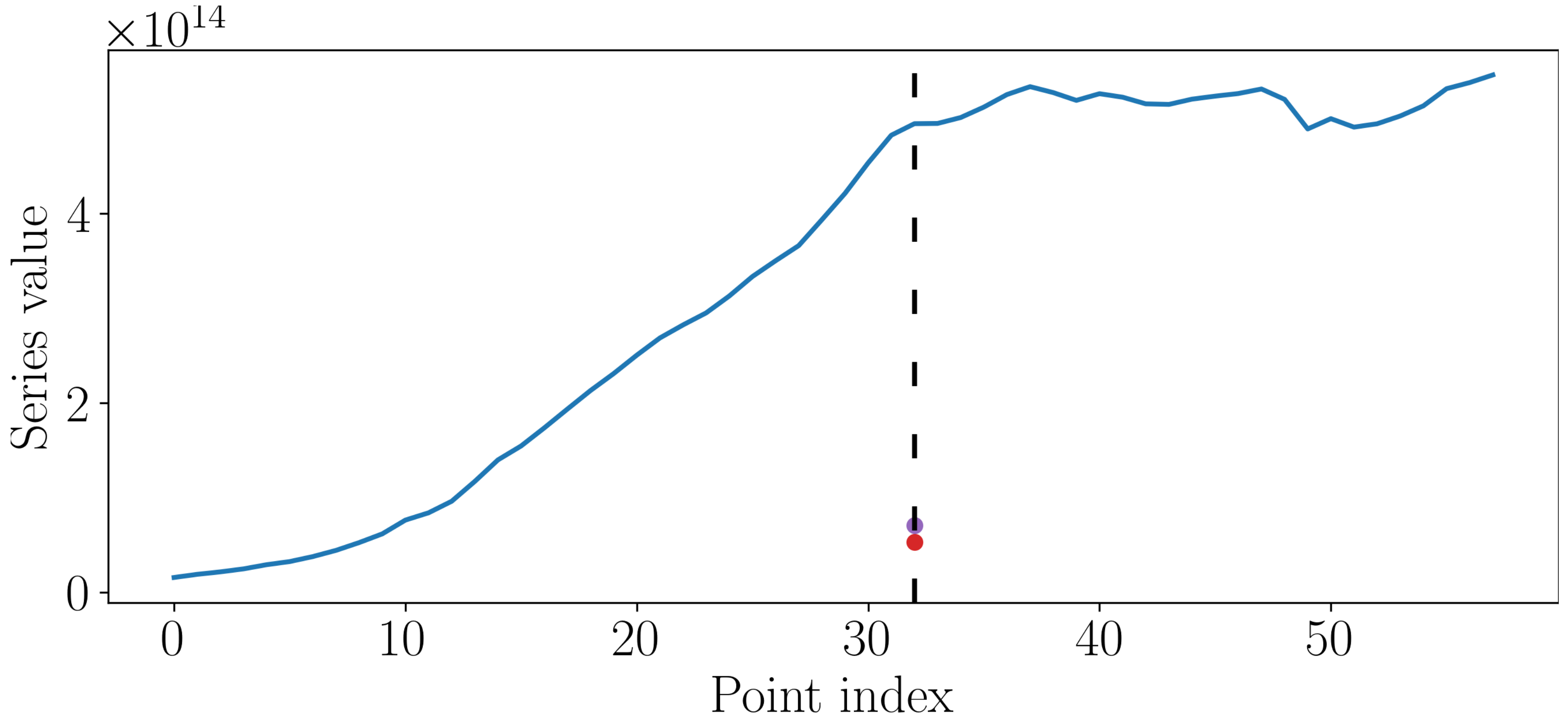
Business inventories (USA)



Japan's GDP



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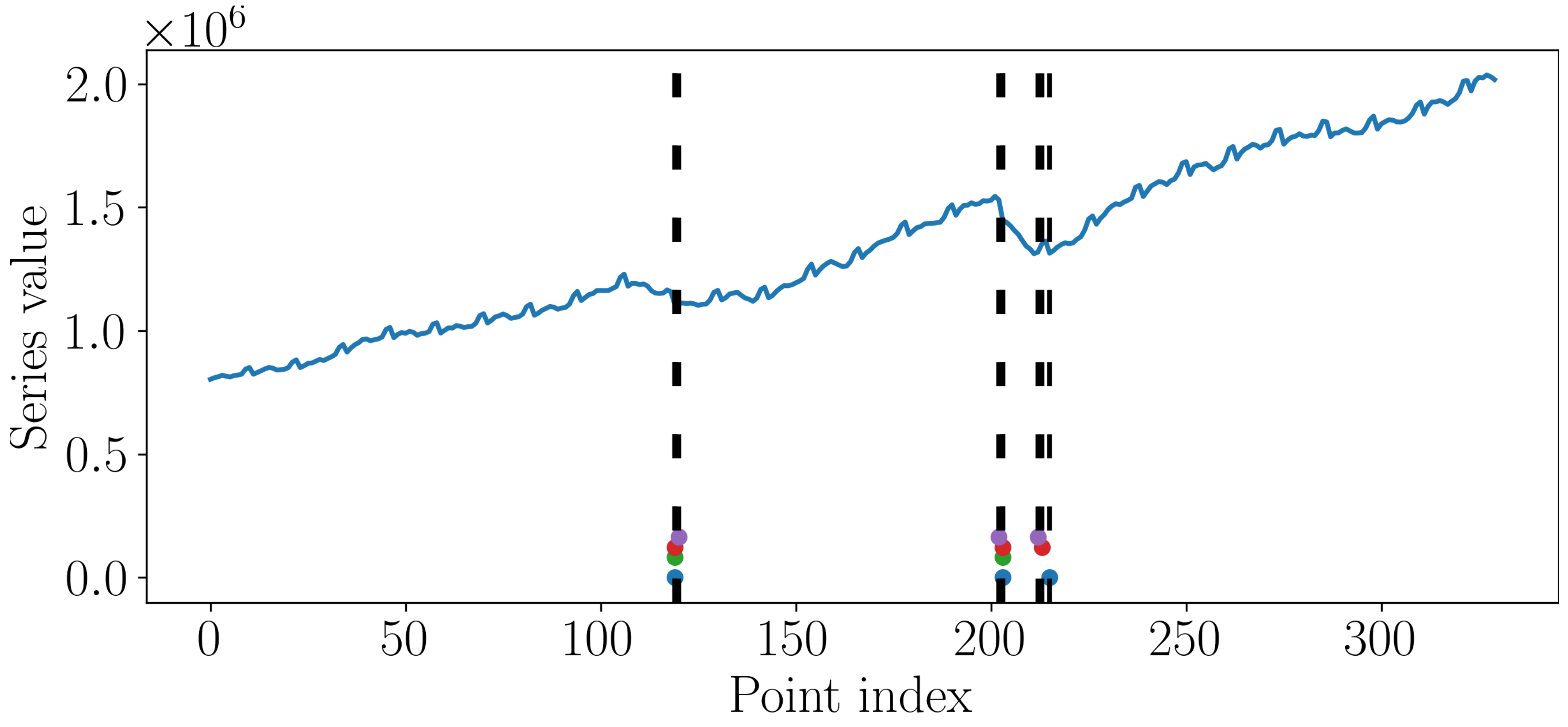

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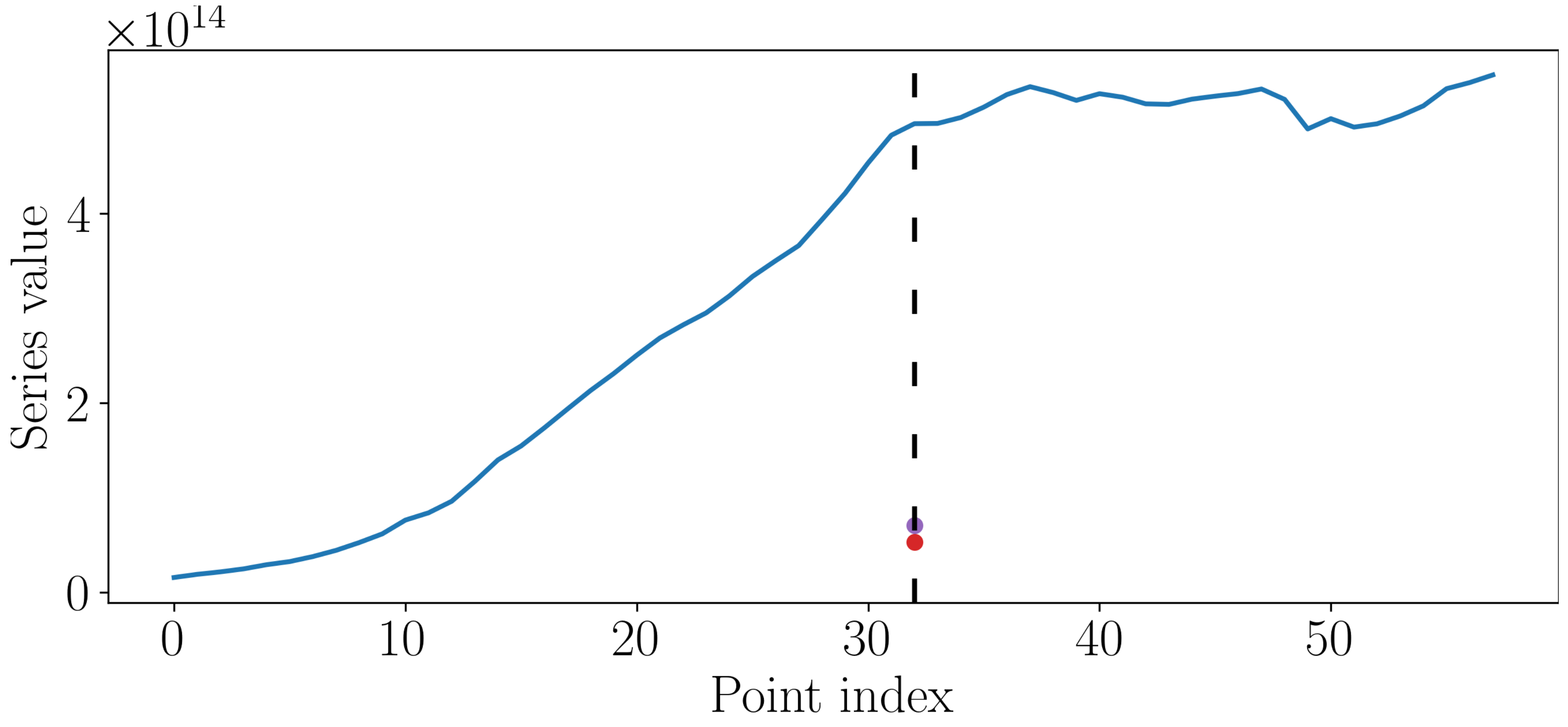

- Changes in the **kernel** and **mean** function of a **Gaussian process**

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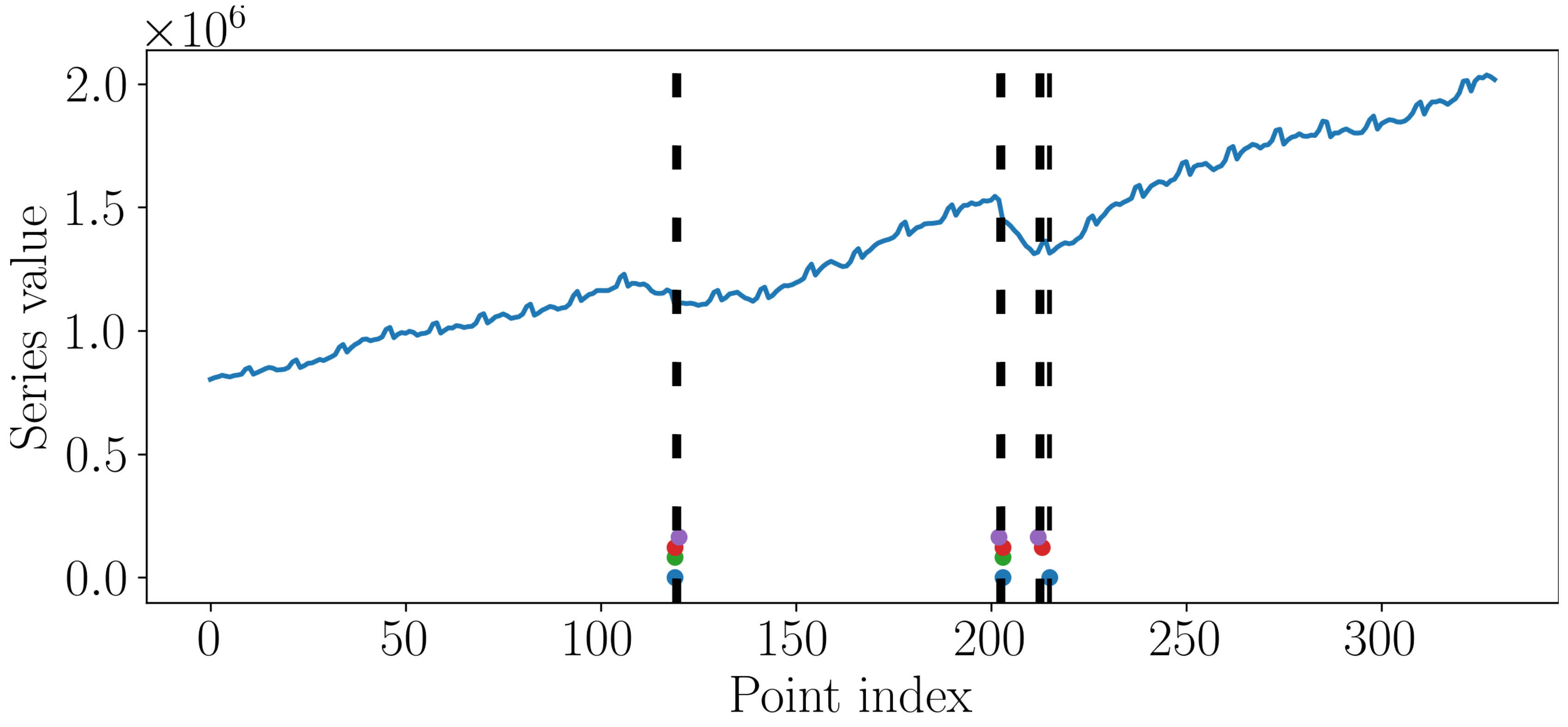

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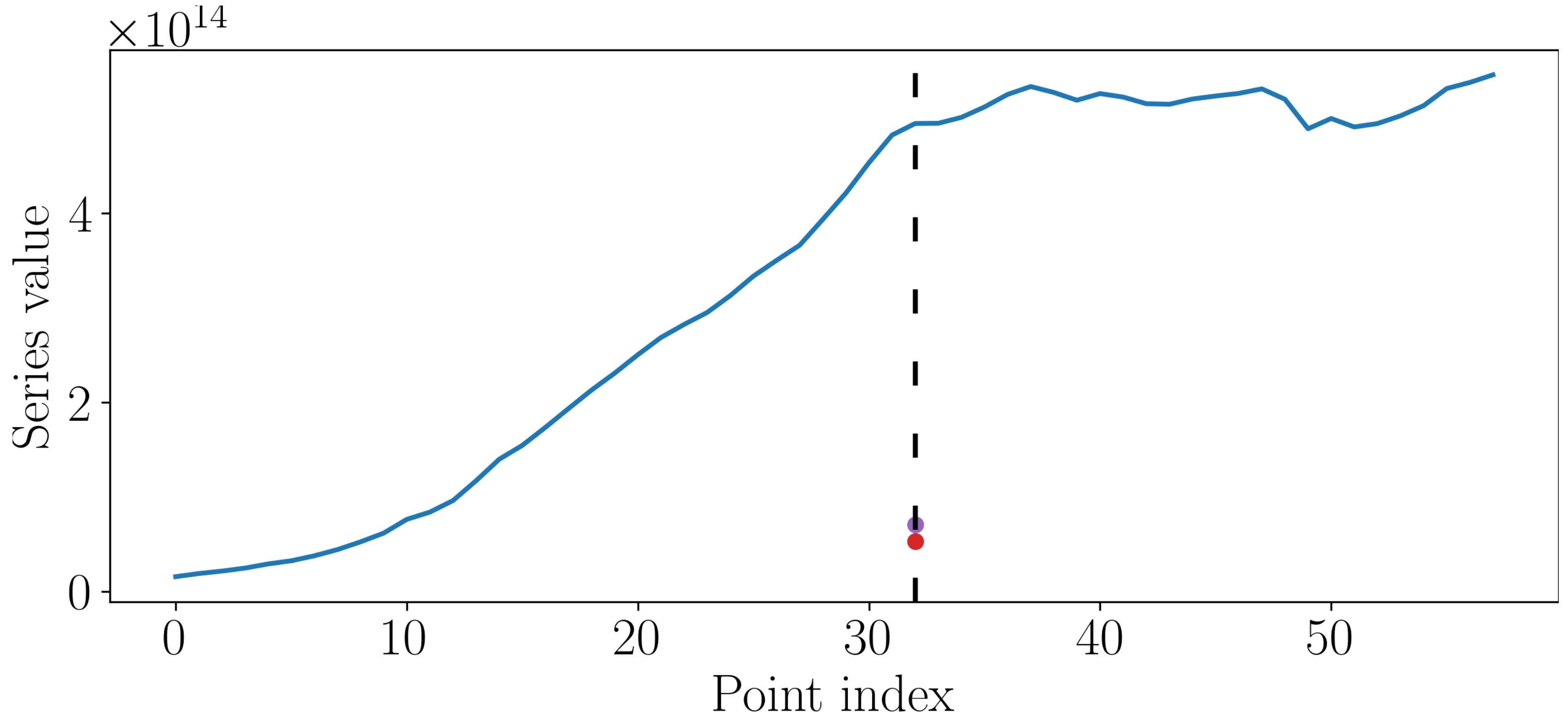
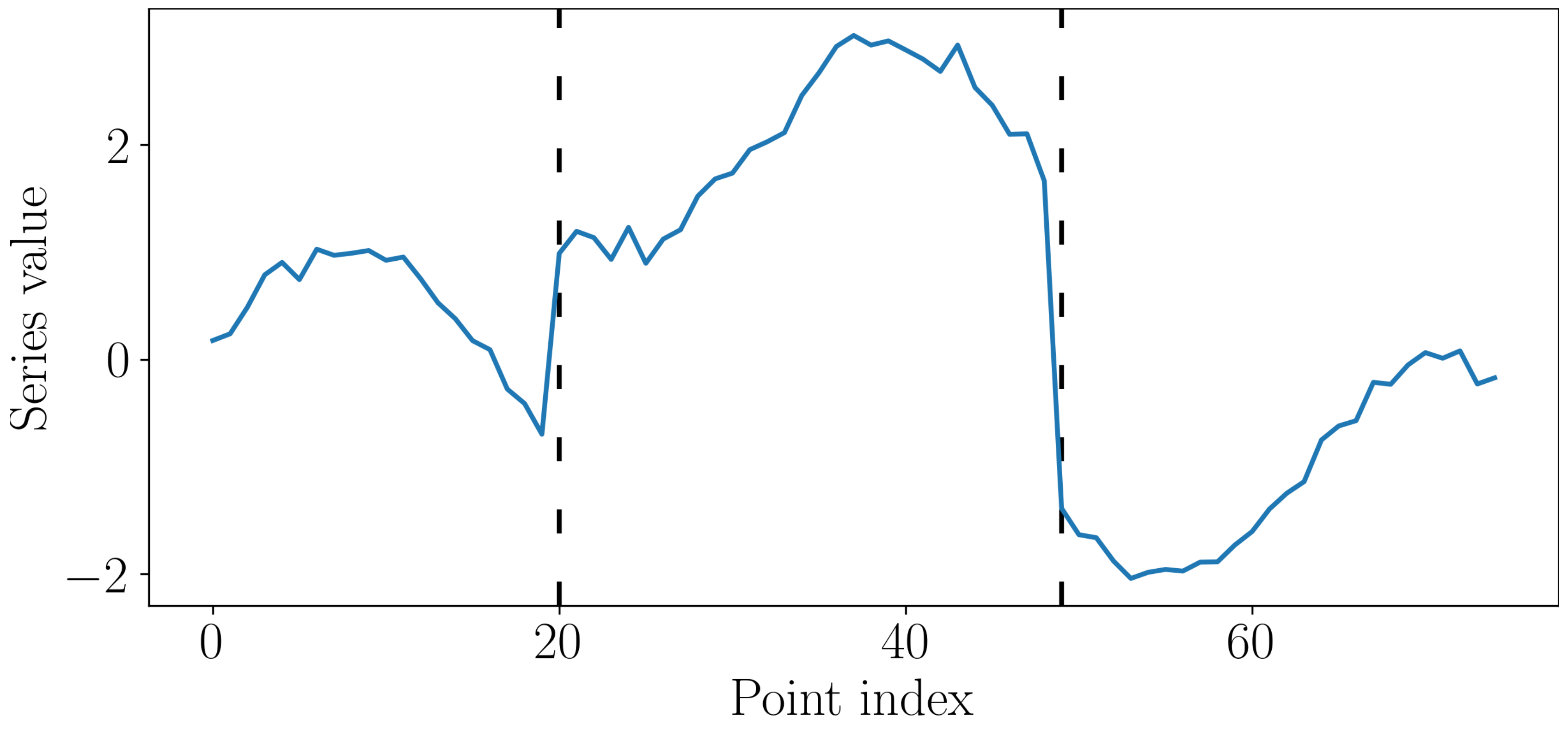
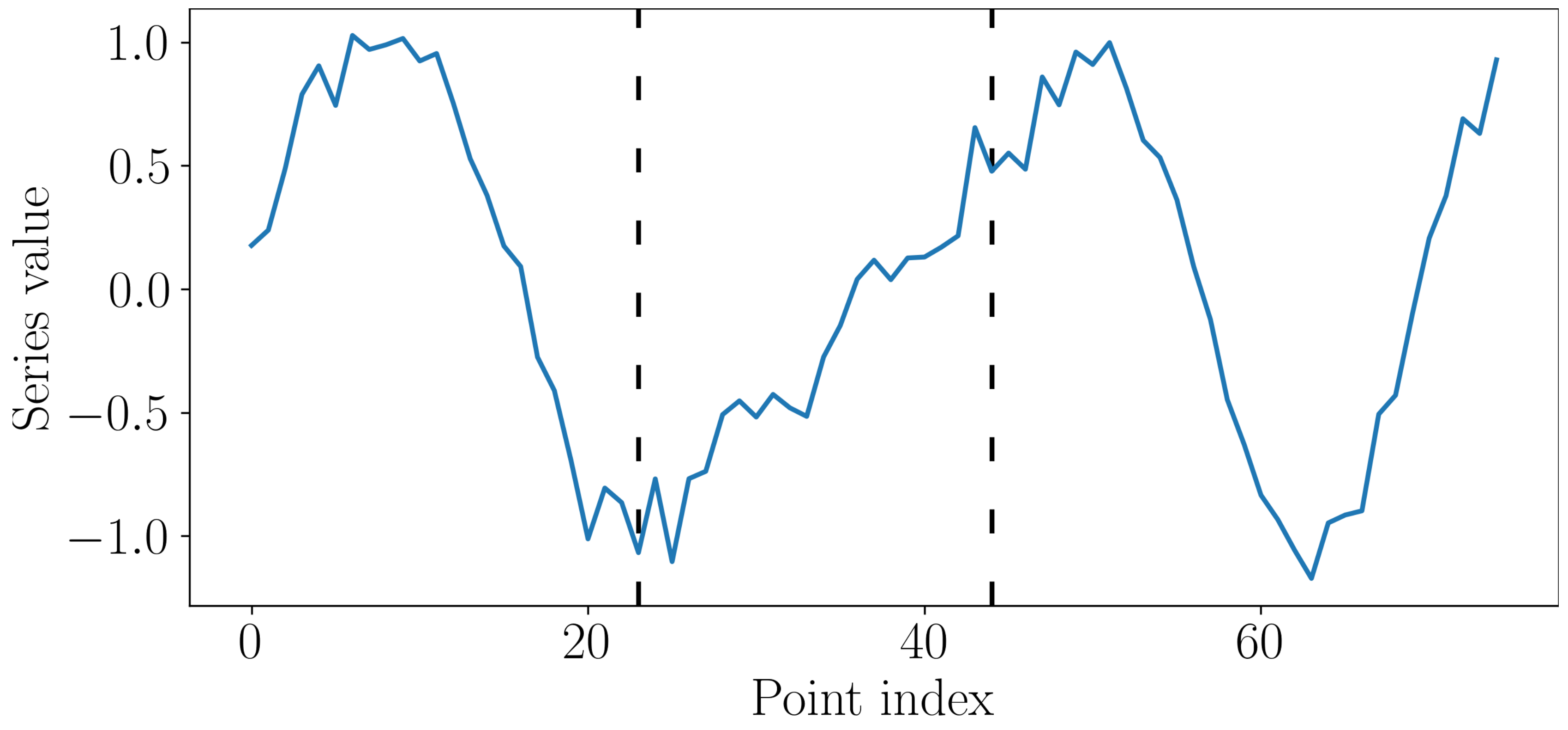


- Changes in the **kernel** and **mean** function of a **Gaussian process**

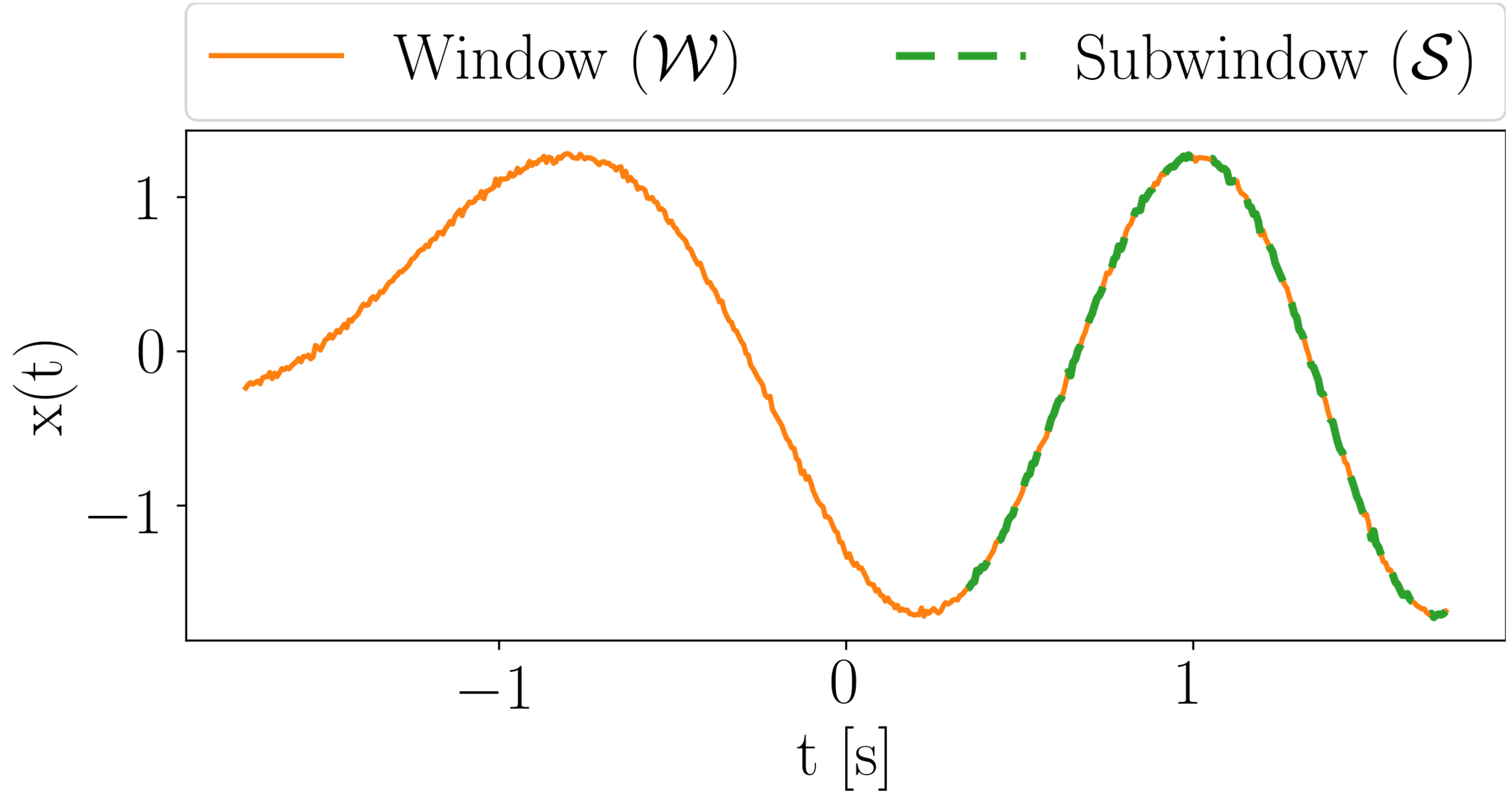
Mean



Periodicity



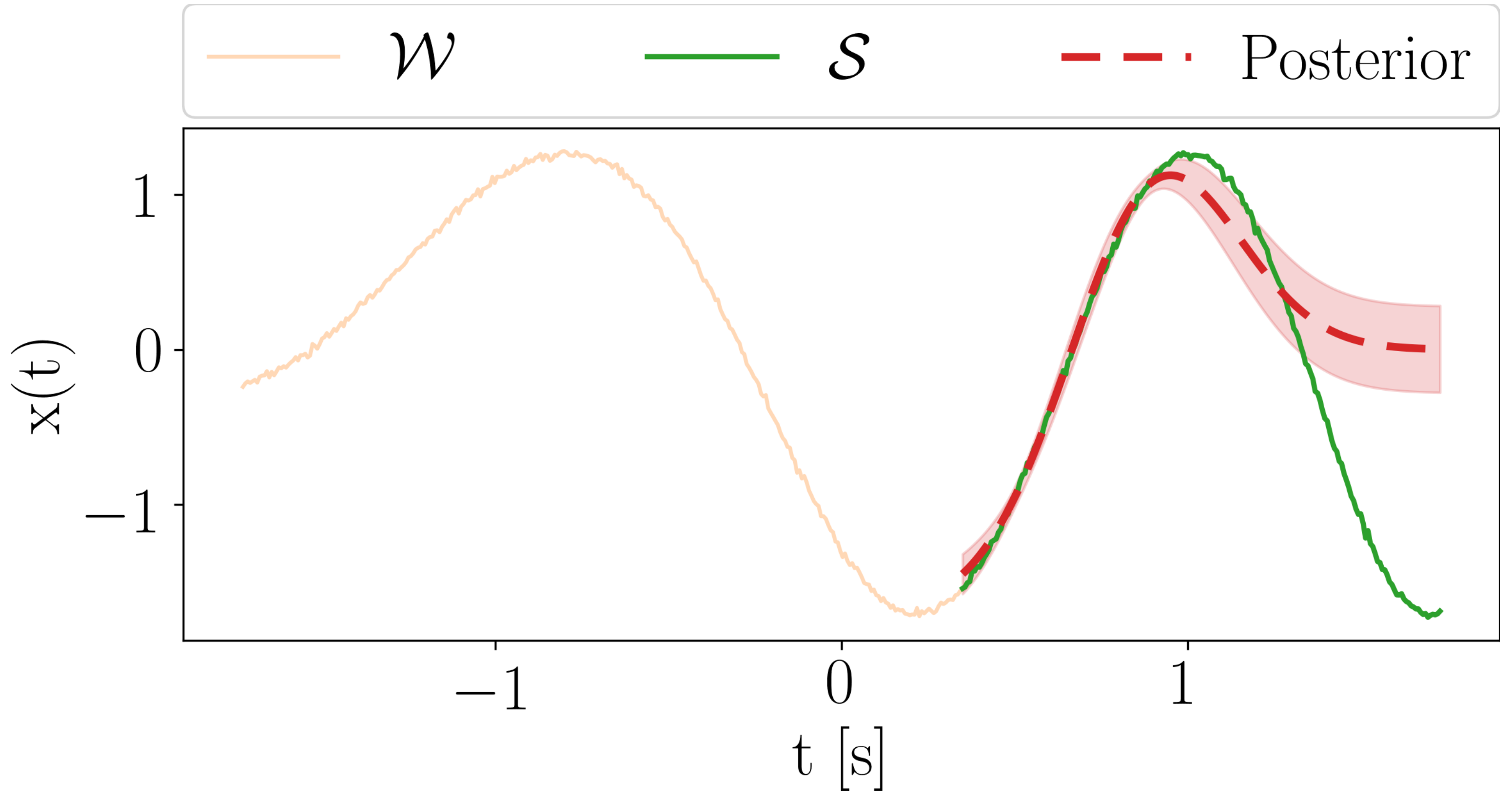
ADAGA



ADAGA

Train GP on the whole window, predict on subwindow:

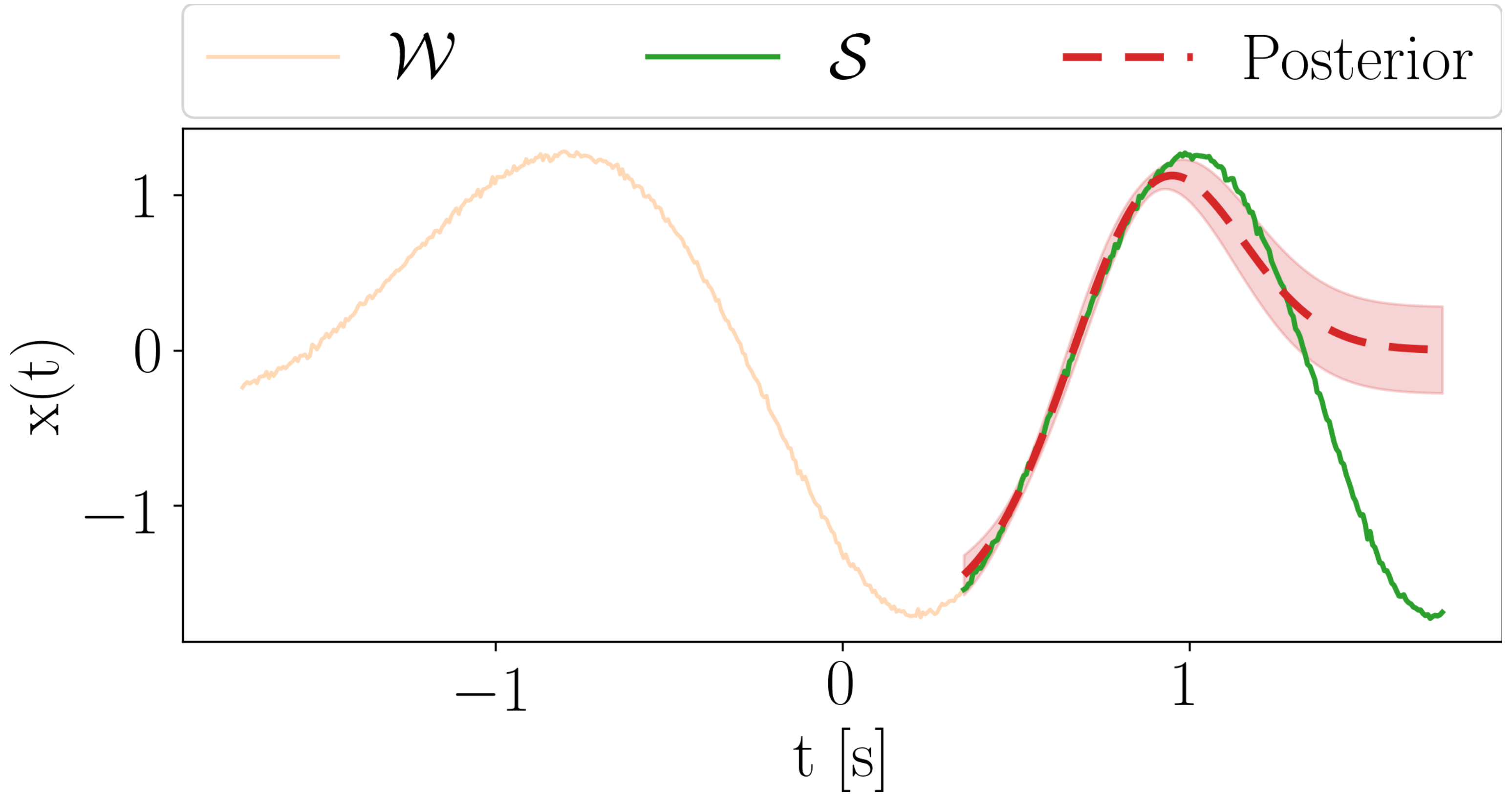
$$p(\mathbf{y}_s \mid H_0) = p(\mathbf{y}_s \mid \mathbf{t}_s, \boldsymbol{\phi}_{H_0}, \sigma^2_{H_0})$$



ADADA

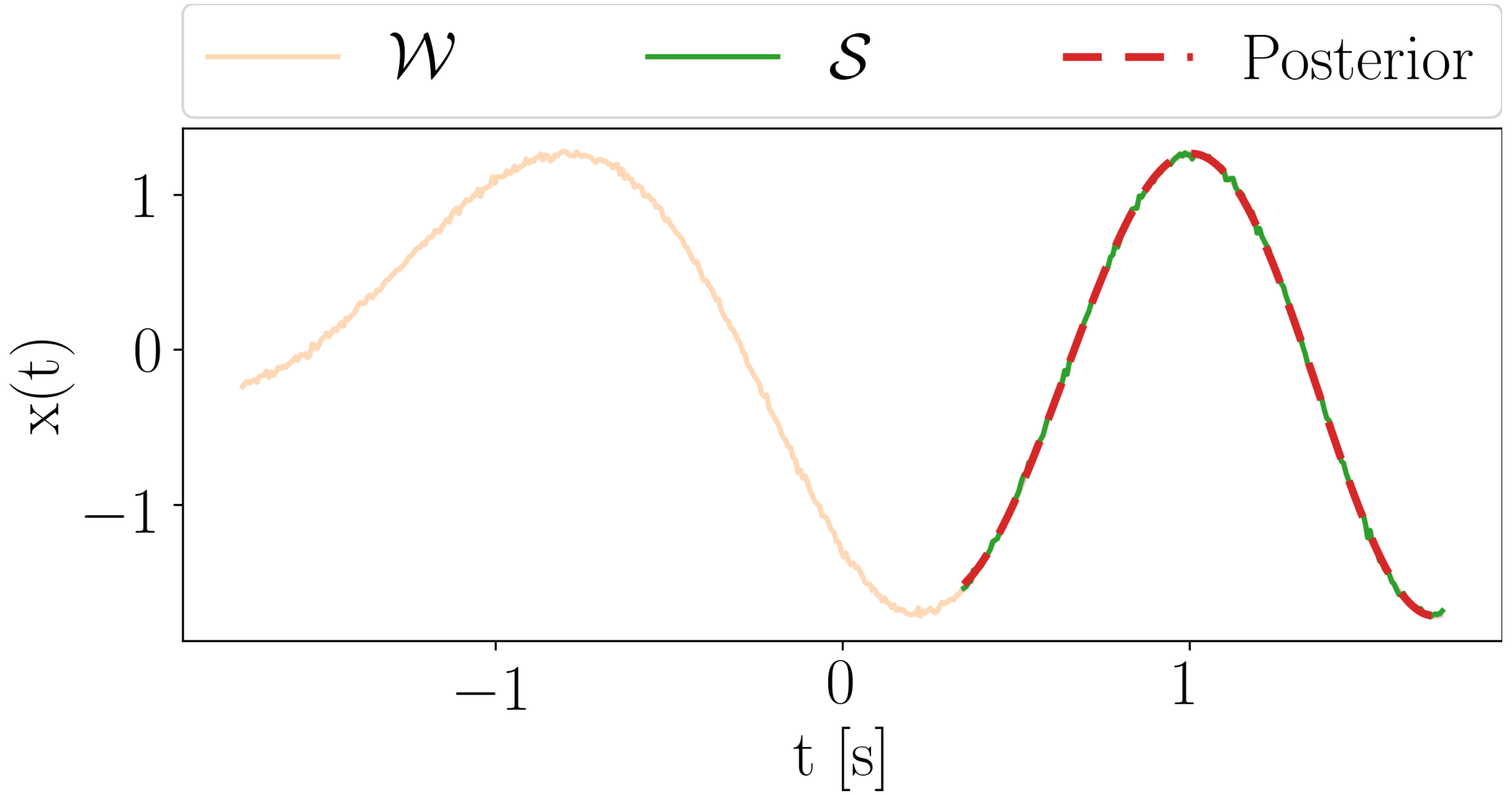
Train GP on the whole window, predict on subwindow:

$$p(\mathbf{y}_s \,|\, H_0) = p(\mathbf{y}_s \,|\, \mathbf{t}_s, \boldsymbol{\phi}_{H_0}, \sigma^2_{H_0})$$



Train a GP on the subwindow, predict on subwindow:

$$p(\mathbf{y}_s \,|\, \mathbf{t}_s, \boldsymbol{\phi}_{new}, \sigma^2_{new})$$



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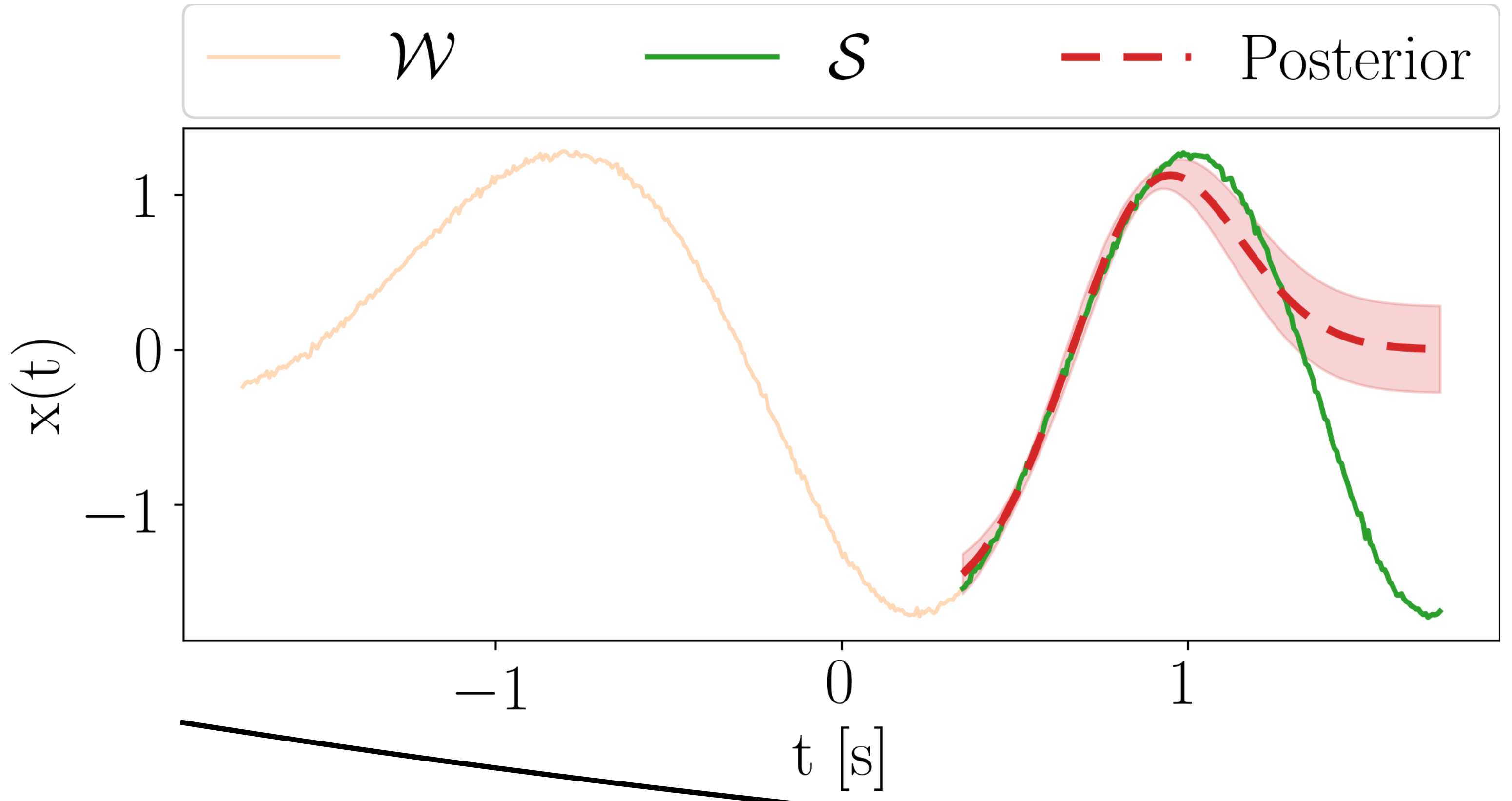

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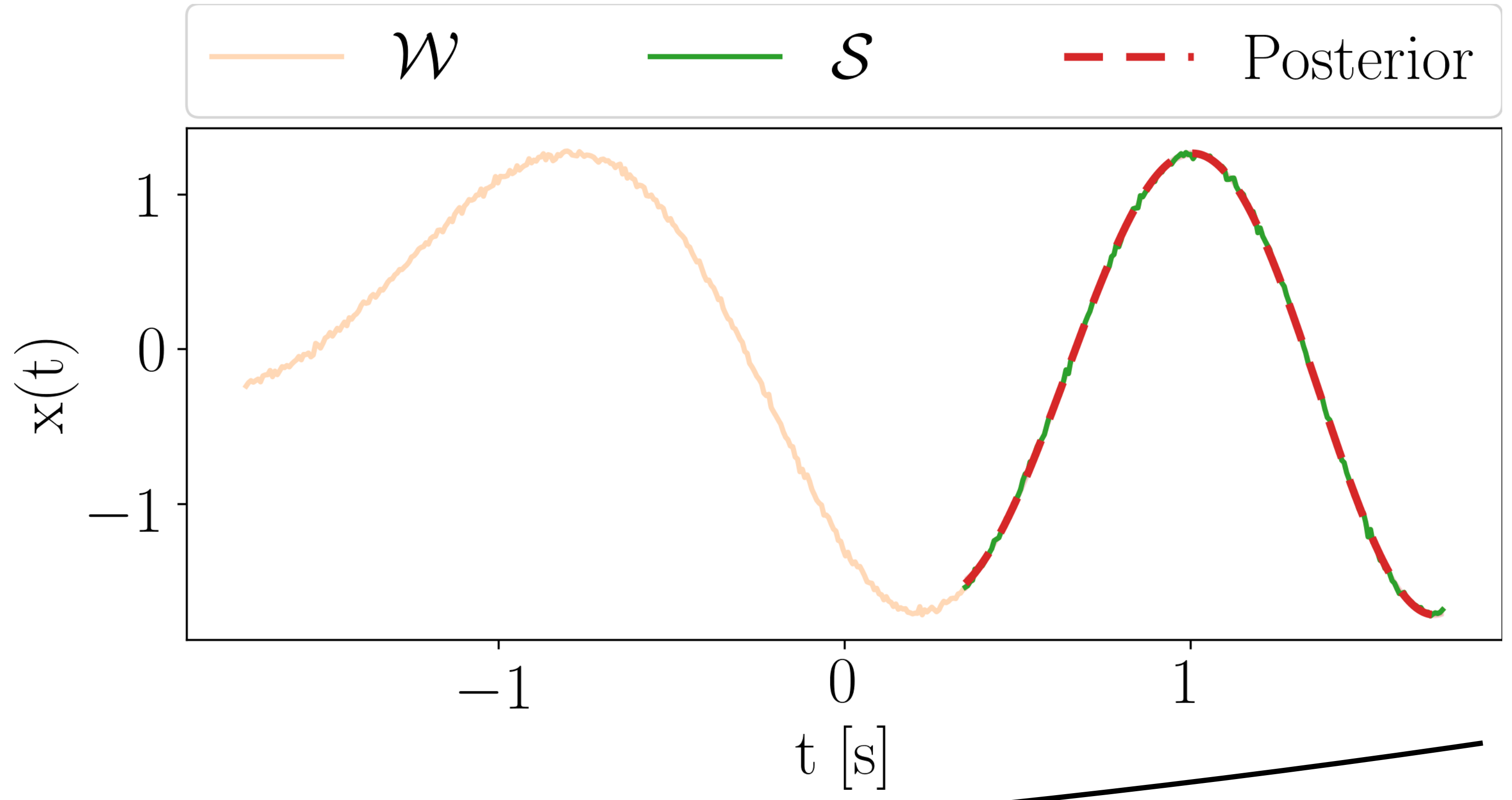

Null hypothesis: $p(\mathbf{y}_s \mid H_0) = p(\mathbf{y}_s \mid \mathbf{t}_s, \boldsymbol{\phi}_{H_0}, \sigma^2_{H_0})$

Surrogate alternative hypothesis: $p(\mathbf{y}_s \mid H_1) = \dfrac{p(\mathbf{y}_s \mid \mathbf{t}_s, \boldsymbol{\phi}_{H_0}, \sigma^2_{H_0})\, p(\mathbf{y}_s \mid \mathbf{t}_s, \boldsymbol{\phi}_{new}, \sigma^2_{new})}{Z_1}$

ADAGA

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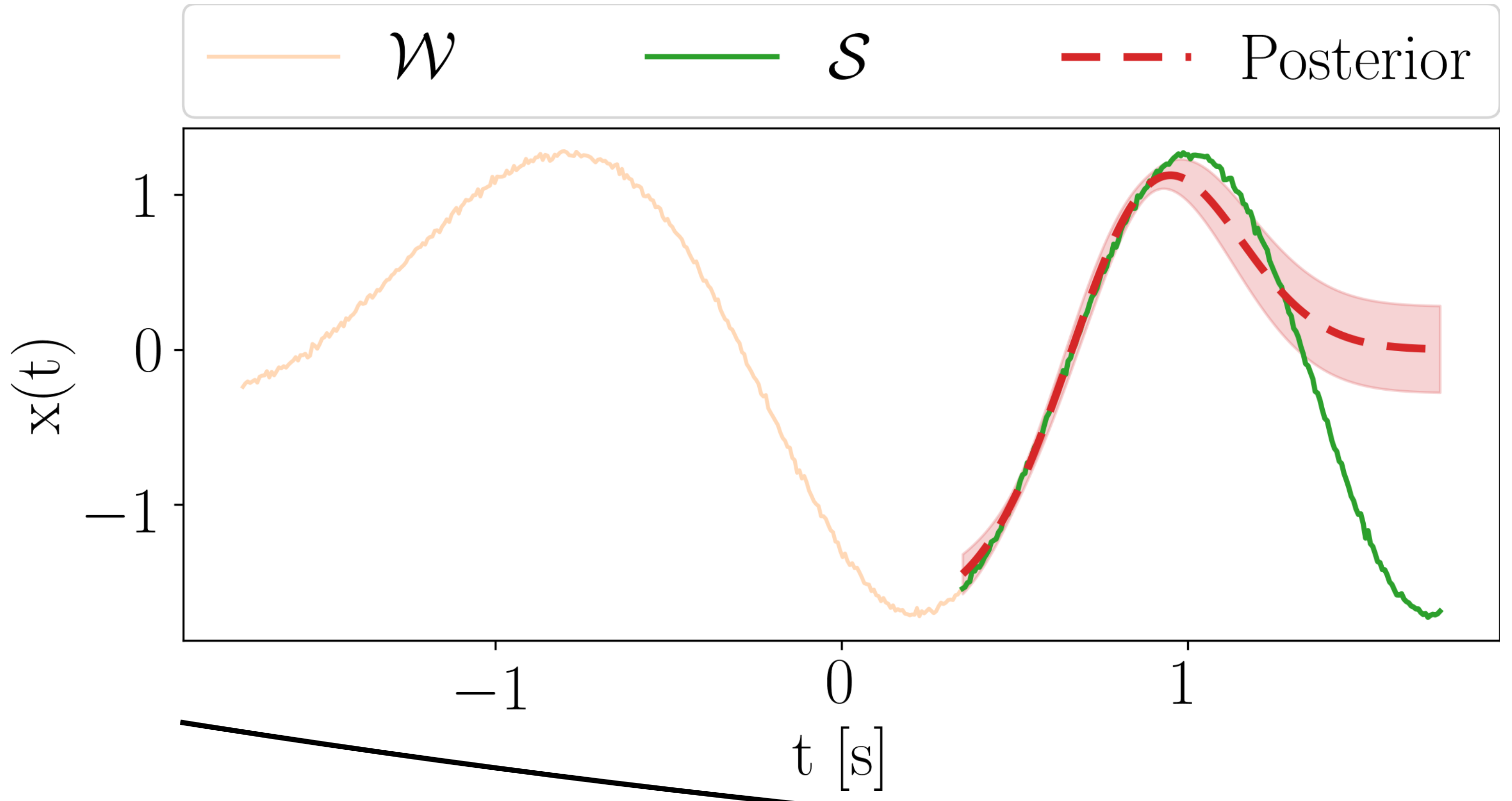

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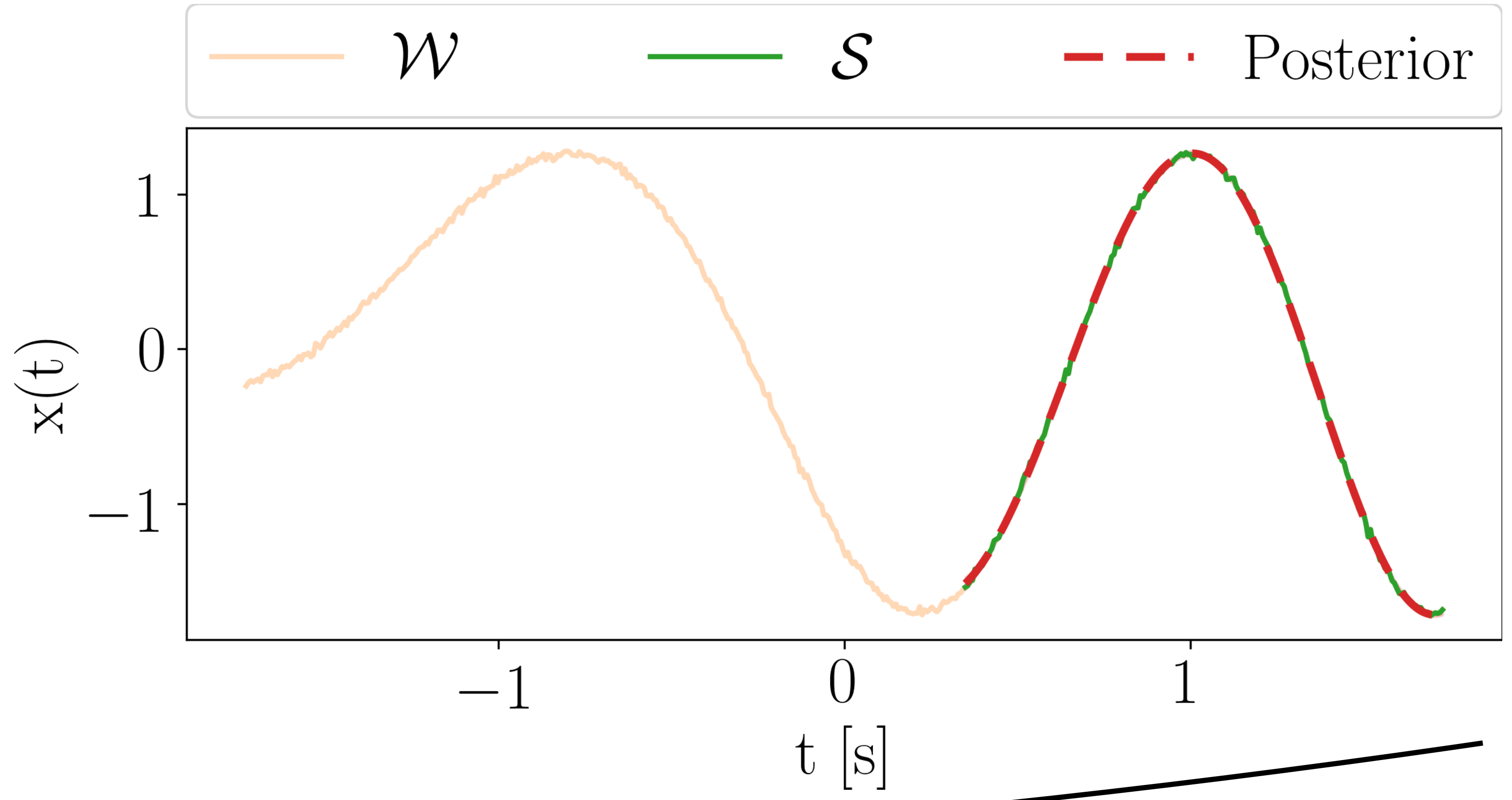

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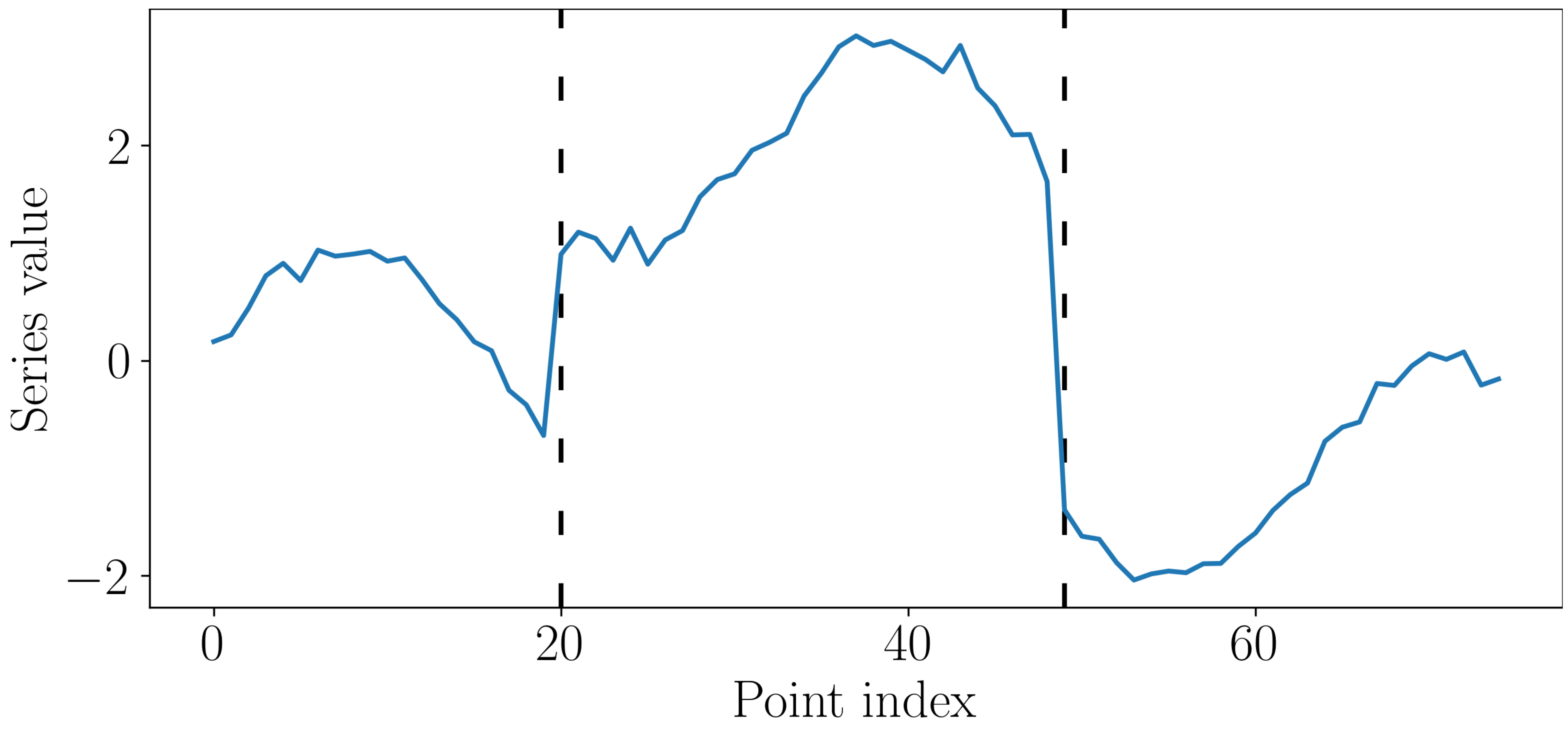
Hypothesis testing on subwindow: $\mathcal{R} = 2 \ln \dfrac{p(\mathbf{y}_s \,|\, H_1)}{p(\mathbf{y}_s \,|\, H_0)}$

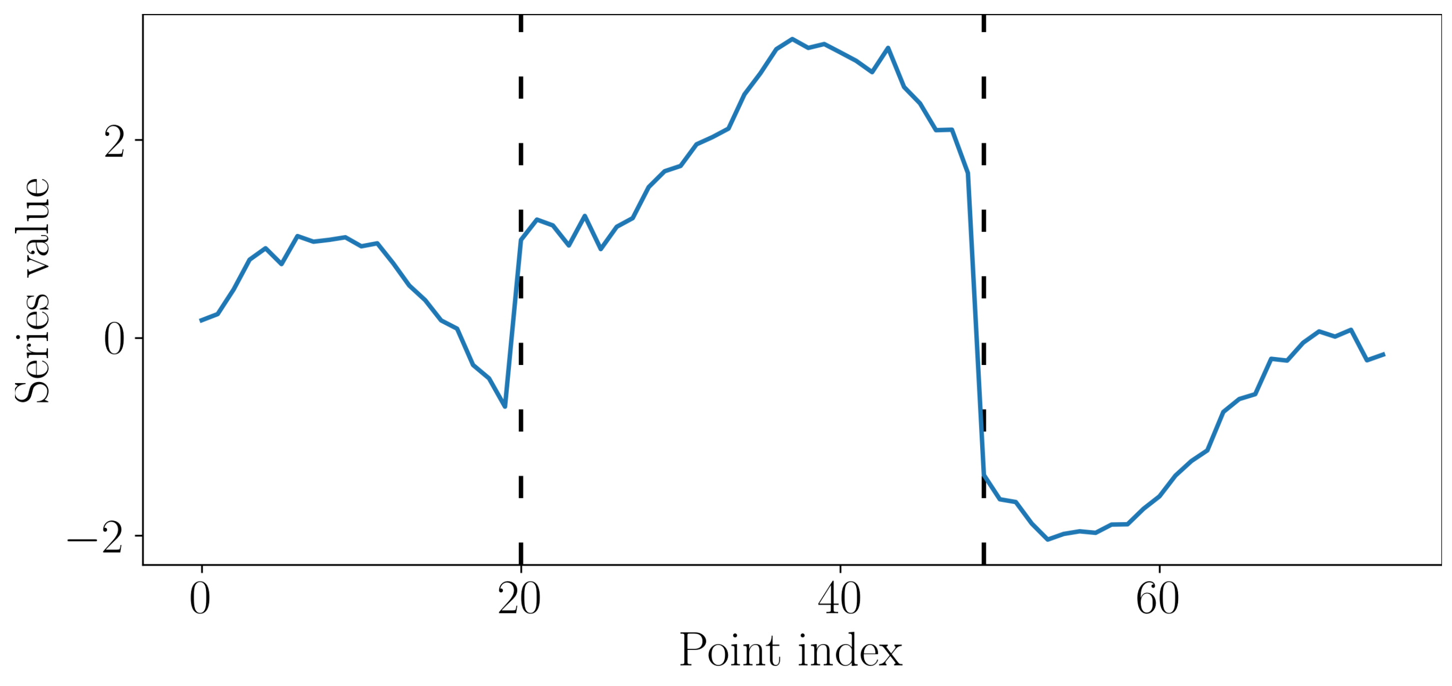
$$\mathcal{T}_I = \mu_{H_0} + \max \left\{ \sqrt{8 \ln(1/\delta) \sum_i \lambda^2_{i,H_0}},\ 8 \ln(1/\delta) \max_i \left\{ \lambda_{i,H_0} \right\} \right\}$$

$$\mathcal{T}_{II} = \mu_{H_1} - \max \left\{ \sqrt{8 \ln(1/\delta) \sum_i \lambda^2_{i,H_1}}, 8 \ln(1/\delta) \max_i \left\{ \lambda_{i,H_1} \right\} \right\}$$

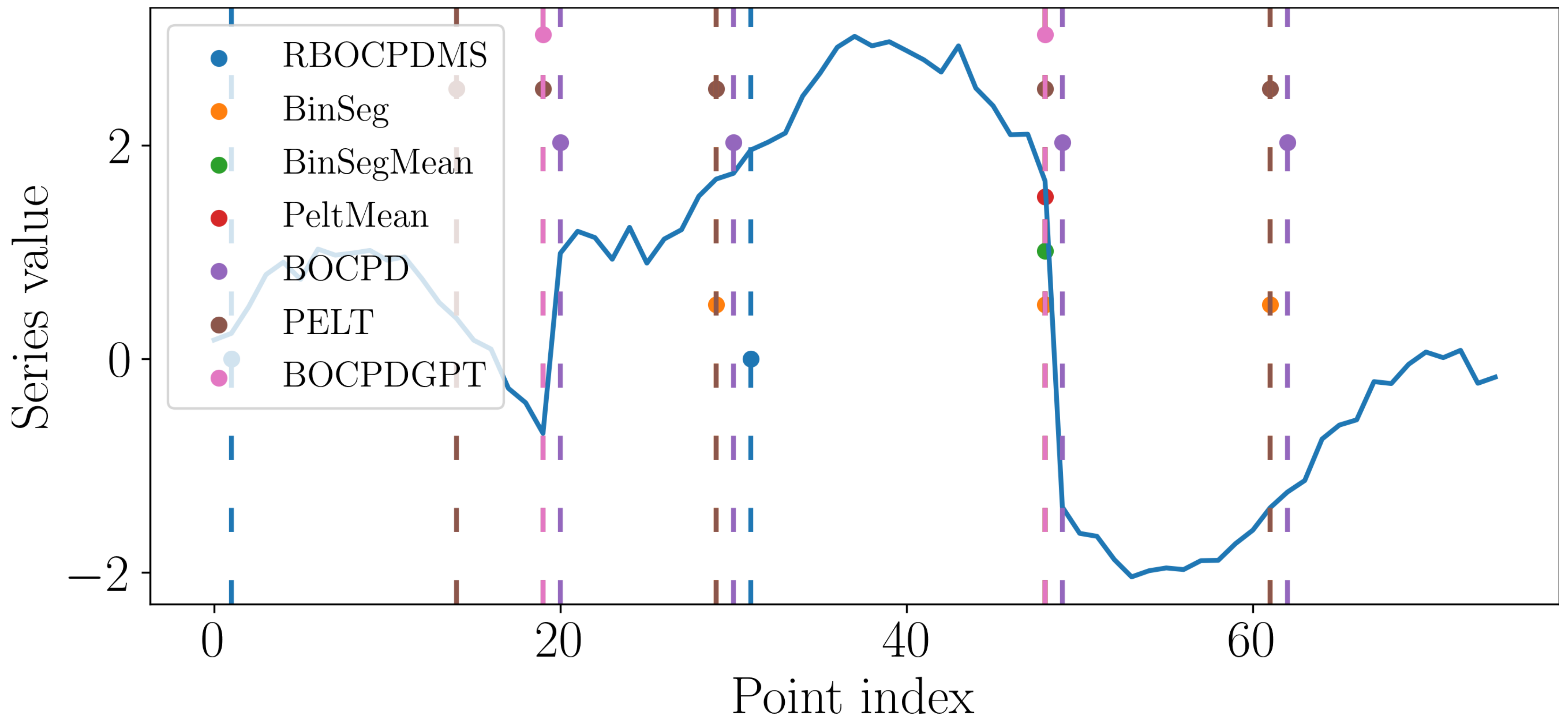
Synthetic series



Synthetic series: results



Benchmarks



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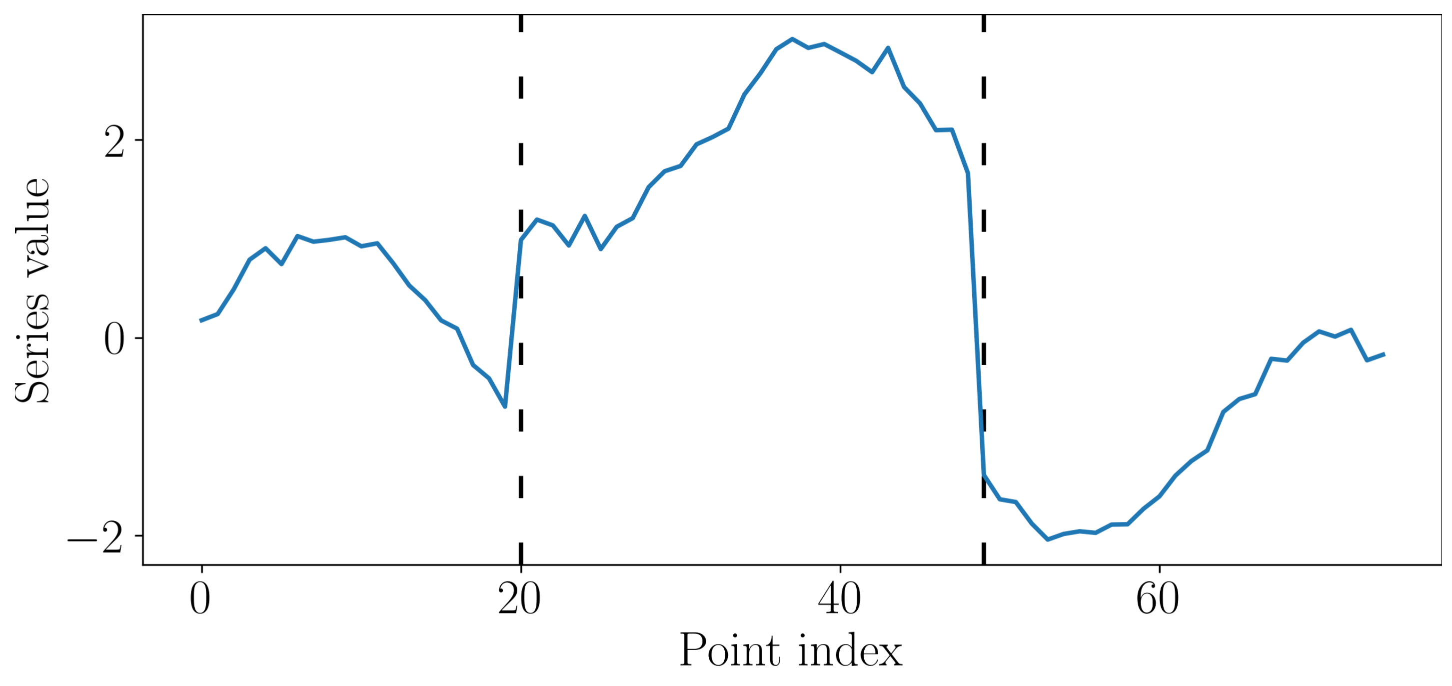

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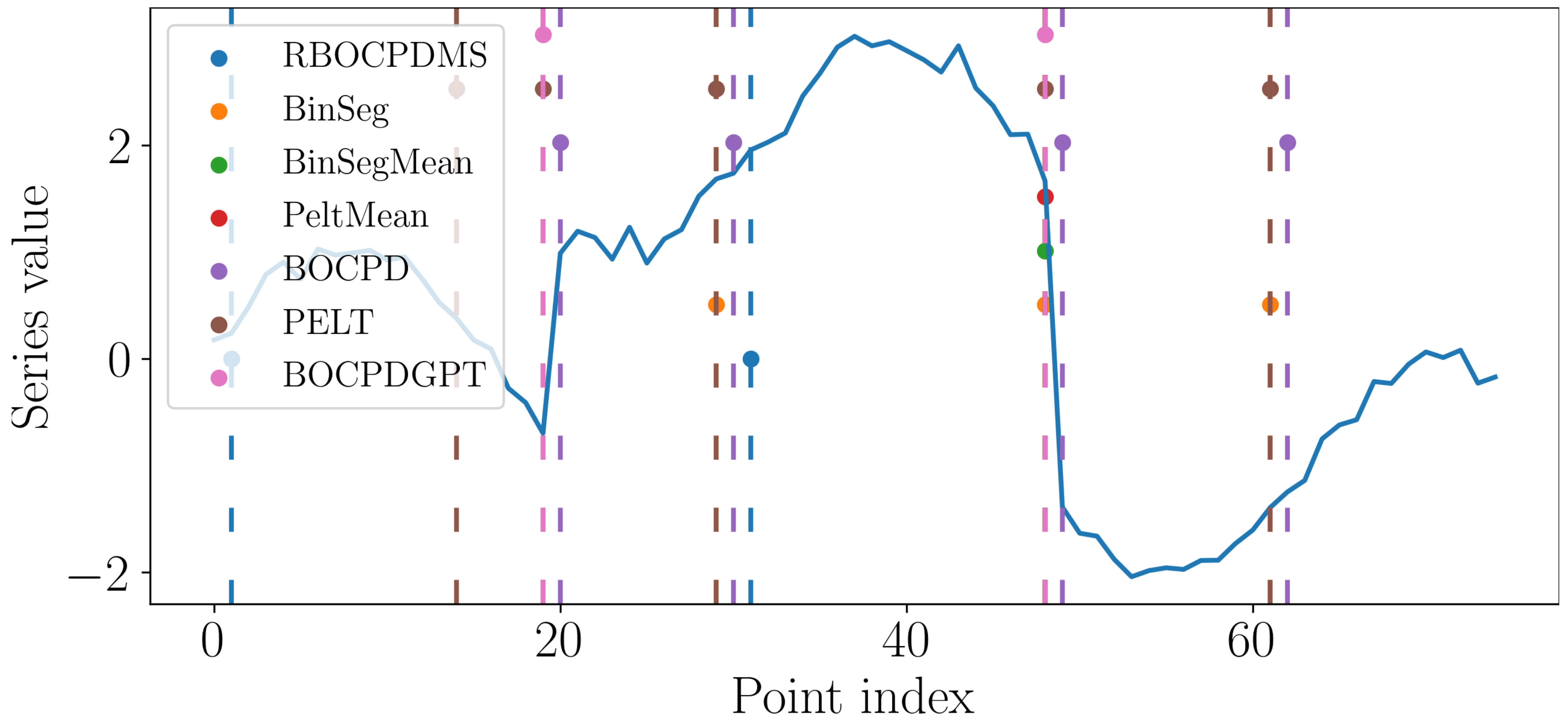
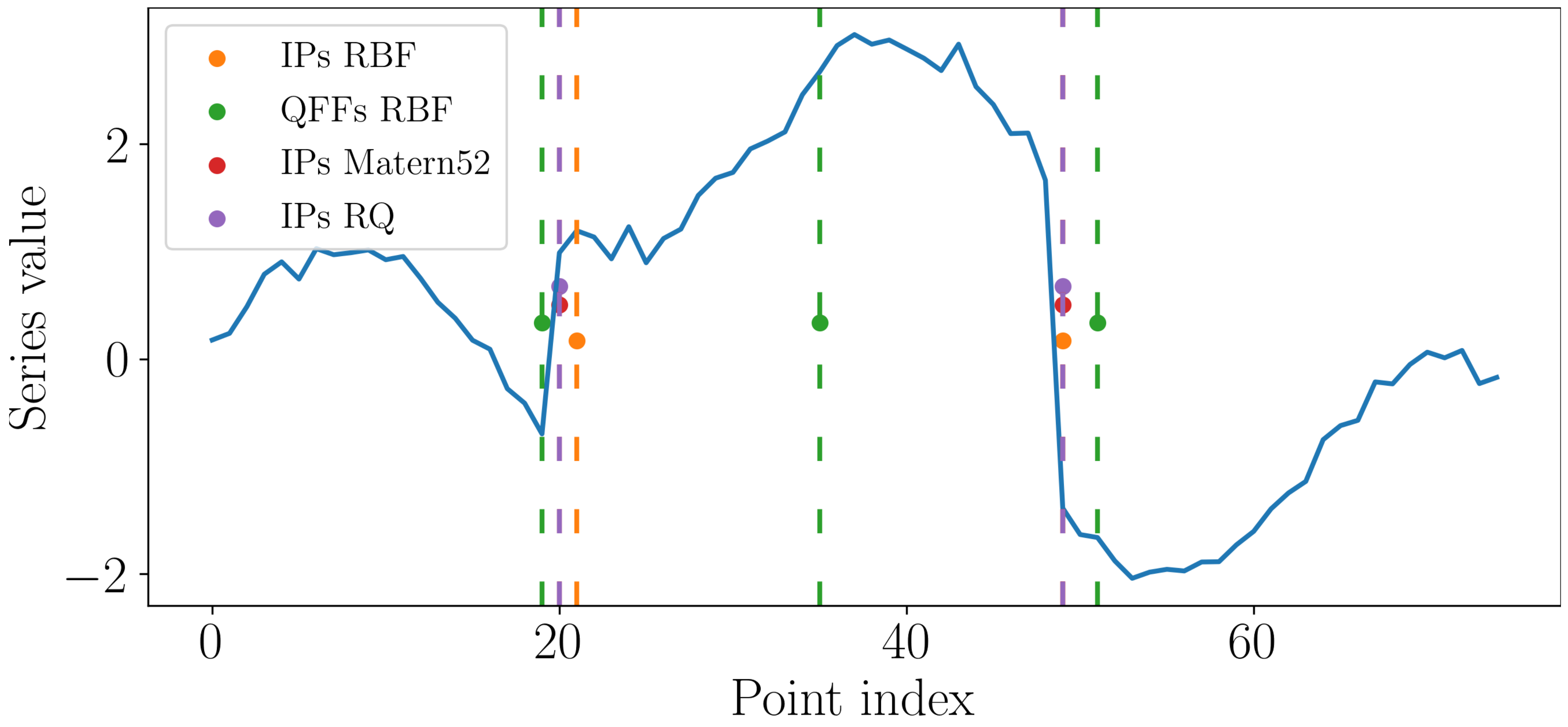


ADAGA



F-1 score: real series

ALGORITHM	RUN LOG	BUSINV	OZONE	GDP IRAN	GDP ARGENTINA	GDP JAPAN	AVERAGE
ADAGA (exact, linear)	0.57	**0.77**	–	–	–	–	0.67
ADAGA (IPs, linear)	0.60	0.63	–	–	–	–	0.62
ADAGA (QFFs, RBF)	–	–	0.97	**0.87**	0.82	**0.89**	**0.89**
ADAGA (IPs, RBF)	–	–	0.78	0.80	0.89	0.62	0.77
ADAGA (IPs, Matern52)	–	–	0.97	0.80	0.82	**0.89**	0.87
ADAGA (IPs, RQ)	–	–	0.97	0.80	0.82	0.62	0.8
BINSEG (mean)	0.43	0.37	0.65	0.49	0.89	0.62	0.57
BINSEG (mean & var)	0.35	0.24	0.56	0.39	0.8	0.57	0.49
PELT (mean)	0.31	0.37	**1.0**	0.49	0.89	0.62	0.61
PELT (mean & var)	0.45	0.20	0.60	0.44	0.67	0.50	0.48
BOCPD	0.52	0.27	0.75	0.39	0.80	0.80	0.59
RBOCPDMS	0.42	0.27	0.78	0.49	0.58	0.47	0.50
GPTS-CP (linear+const)	**0.84**	0.62	–	–	–	–	0.73
GPTS-CP (RQ+const)	–	–	0.65	**0.87**	**0.95**	0.66	0.78
ZERO	0.45	0.59	0.72	0.65	0.82	**0.89**	0.69