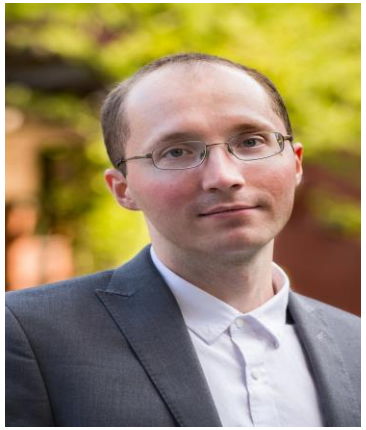
Unified Fourier-based Kernel and Nonlinearity Design for Equivariant Networks on Homogeneous Spaces

Yinshuang Xu* Jiahui Lei* Edgar Dobriban Kostas Daniilidis

University of Pennsylvania



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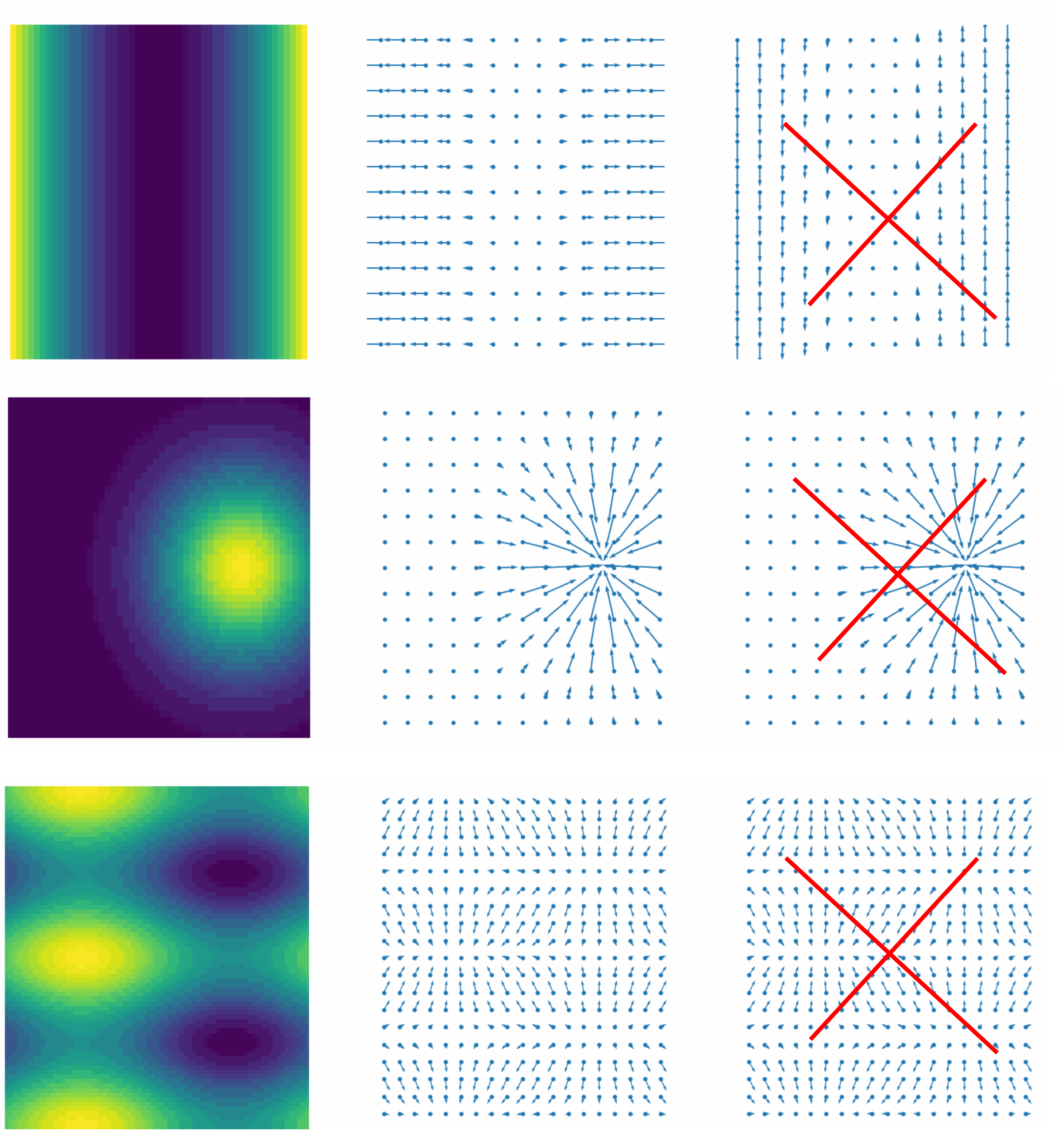
Motivation

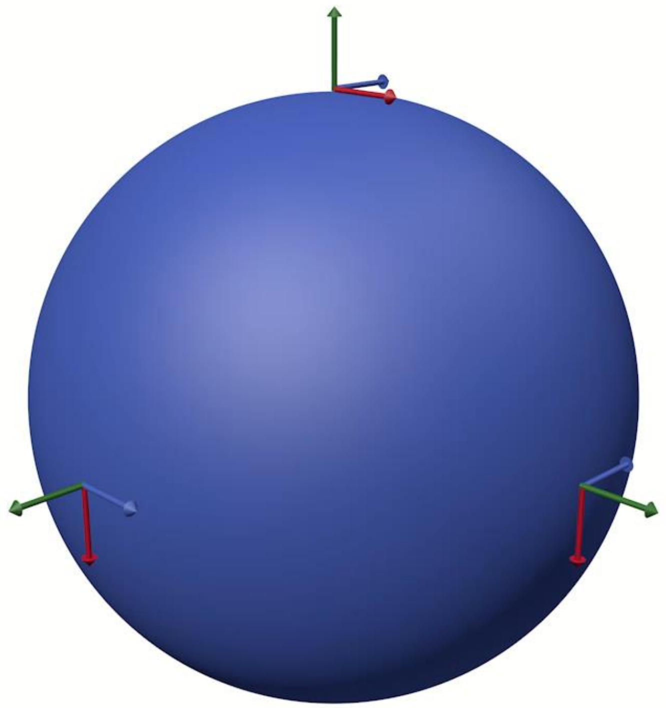


Unified way to design kernel

From Fourier perspective

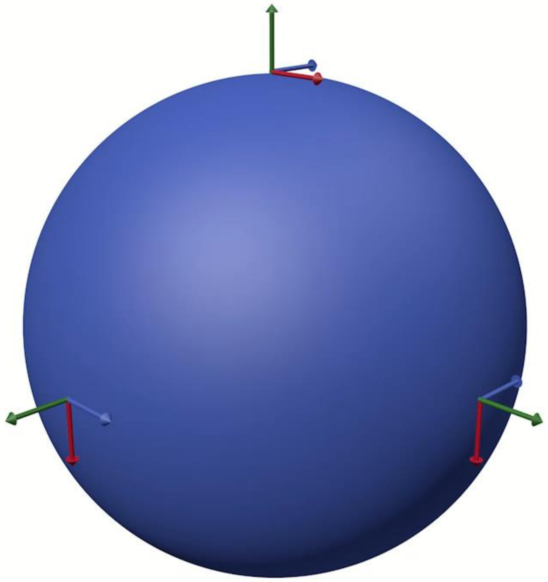
Expressive nonlinearity

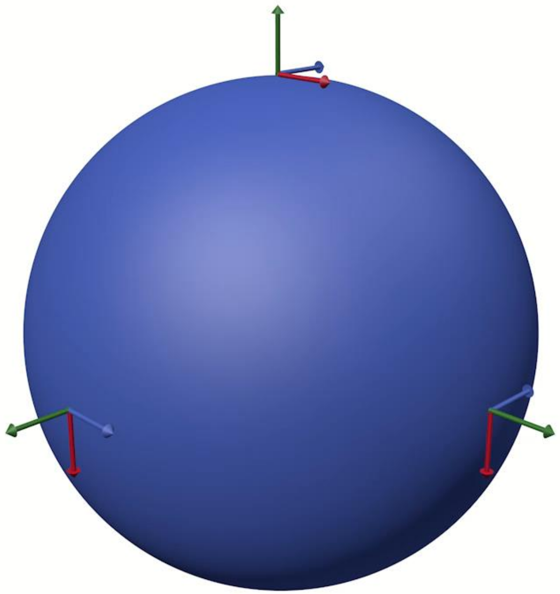
Preliminary



Preliminary

$$gs(x) = s(gx)\mathrm{h}(g, x) \quad s : G/H \to G$$





Preliminary

Lifting to Mackey function:

$$f{\uparrow^G}(g) = (\Lambda f)(g) = \rho(\mathrm{h}(g)^{-1})f(gH)$$

$$f{\uparrow^G}(gh) = \rho(h^{-1})f{\uparrow^G}(g)$$

Fourier Transform:

$$\hat{f}(p) = \mathcal{F}(f)(p) = \int_G f(g)\overline{U(g,p)}dg$$

$$f(g) = \int_{\hat{G}} \mathrm{tr}(\hat{f}(p)^T U(g,p))d\nu(p)$$

Preliminary

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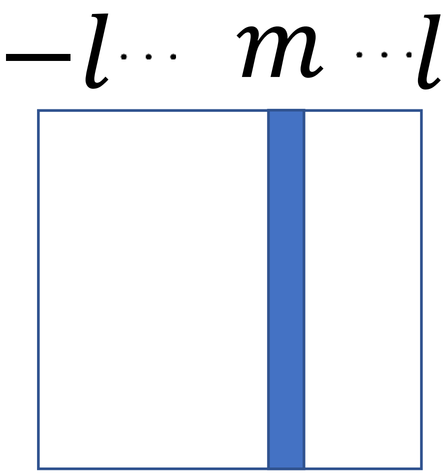
Sparsity in Spectrum

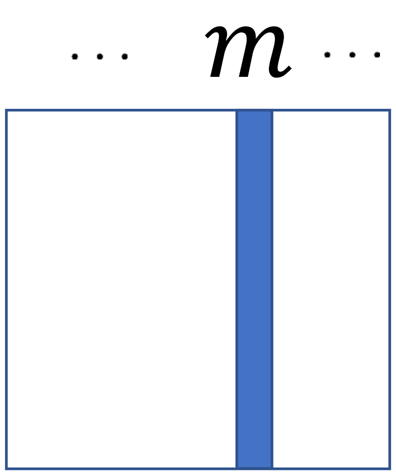
Assume G is a unimodular group and its stabilizer subgroup is a compact Lie group. A Mackey function $f{\uparrow}^G\colon G \to V$ lifted via $f{\uparrow}^G(g) = \rho(\mathrm{h}(g)^{-1})f(gH)$ from a field $f : G/H \to V$ has the following sparsity pattern in the Fourier domain: $\left[\widehat{f{\uparrow}^G}\right](p)_{\star,j}$ could be nonzero only if the block at column j in the decomposition of $U(\cdot,p)|_H$ is equivalent to the dual representation of ρ.

Examples:

$SO(3)$:

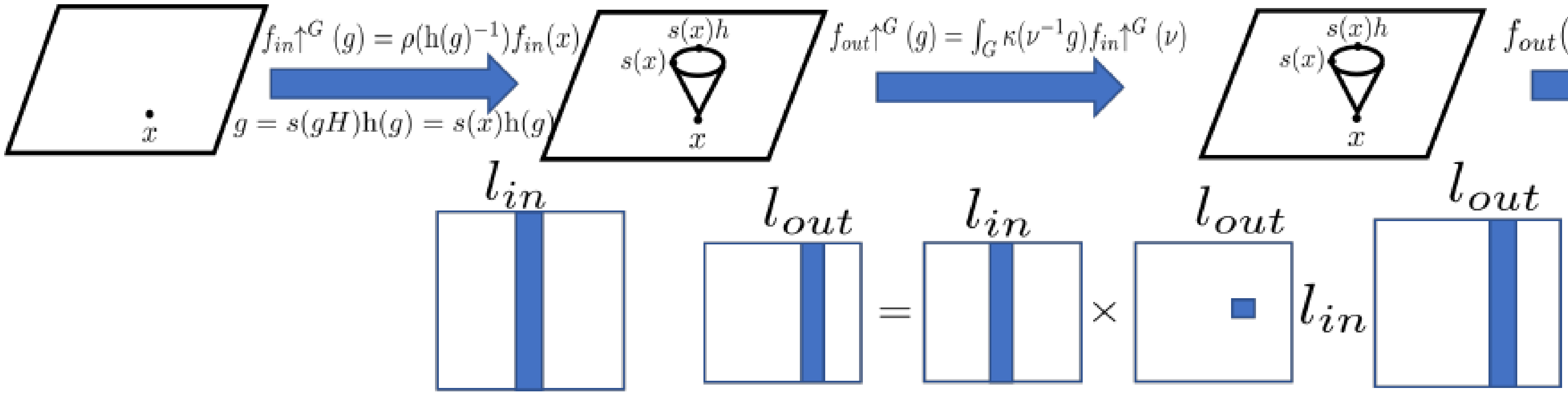


$SE(2)$:



Sparsity in Spectrum

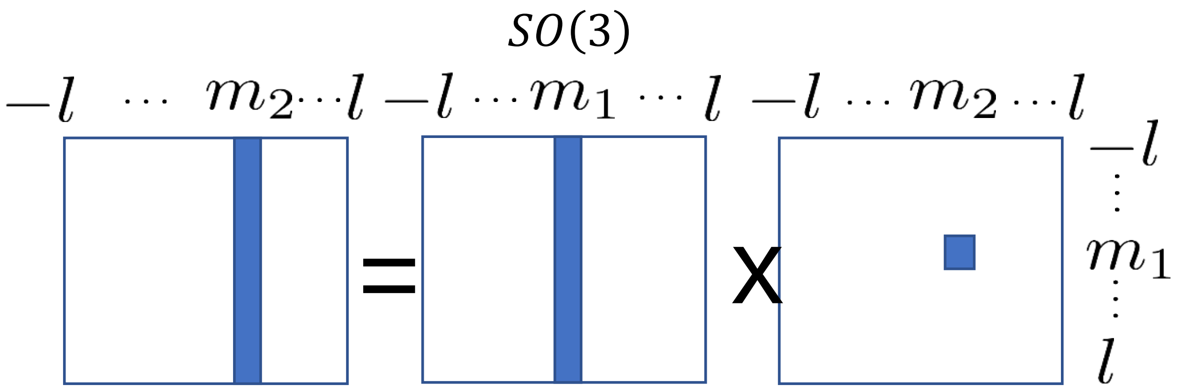
$$f_{out}(x) = (\Lambda_2^{-1}(\kappa * (\Lambda_1 f_{in})))(x) = \int_G \kappa(g^{-1}s(x))(\Lambda_1 f_{in})(g)dg$$

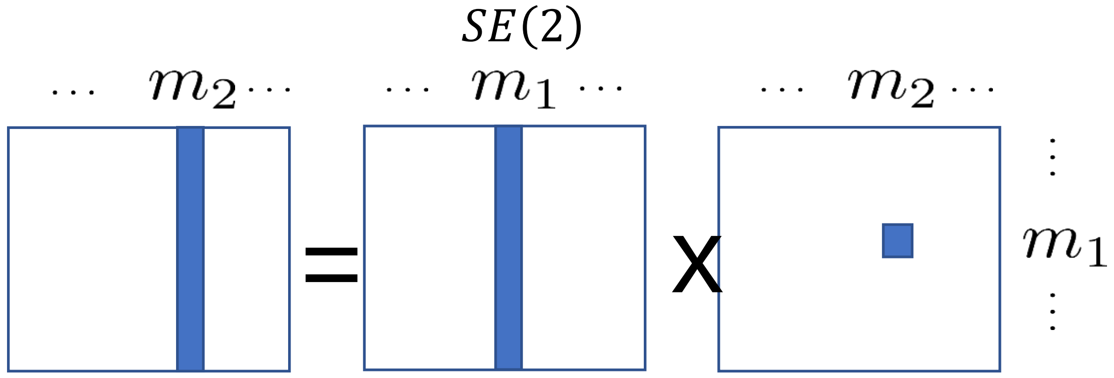


$$\kappa(h_2 g h_1) = \rho_2(h_2)\kappa(g)\rho_1(h_1)$$

Cohen et al. Neurips'19

Examples:



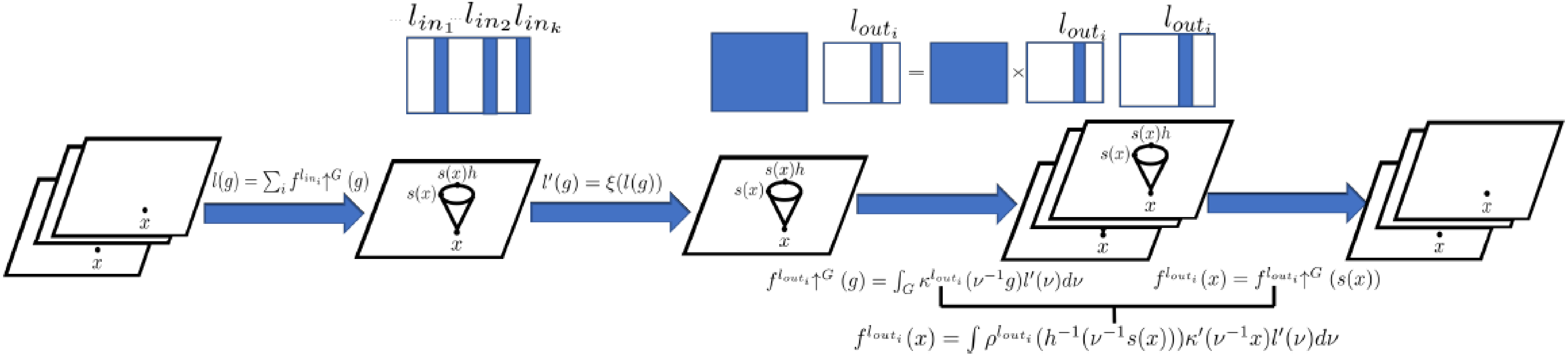


Nonlinearity

$$l(g) = \left[\overline{\Lambda}(\bigoplus f^{l_i})\right](g) = \sum_i \left[\Lambda_i(f^{l_i})\right](g)$$
$$= \sum_i f^{l_i}\!\uparrow^G(g) = \sum_i \rho^{l_i}(\mathrm{h}(g)^{-1}) f^{l_i}(gH),$$

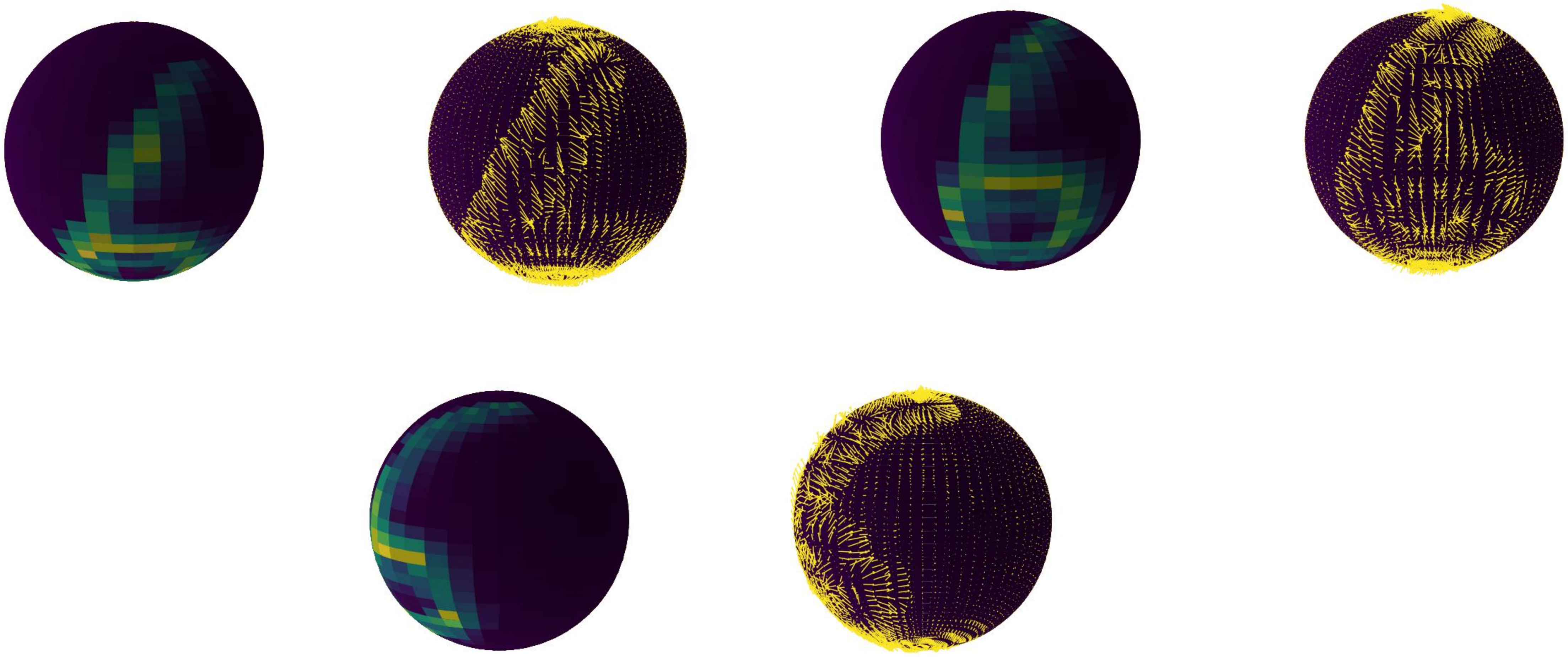
$$\bigoplus_j \mathrm{Ind}_H^G \rho^{l_{out_j}} \circ \sigma = \sigma \circ \bigoplus_i \mathrm{Ind}_H^G \rho^{l_{in_i}}$$

$$f(x) = \int \kappa(g^{-1}s(x))l(g)dg.$$



Experiments

Vector field prediction on the sphere



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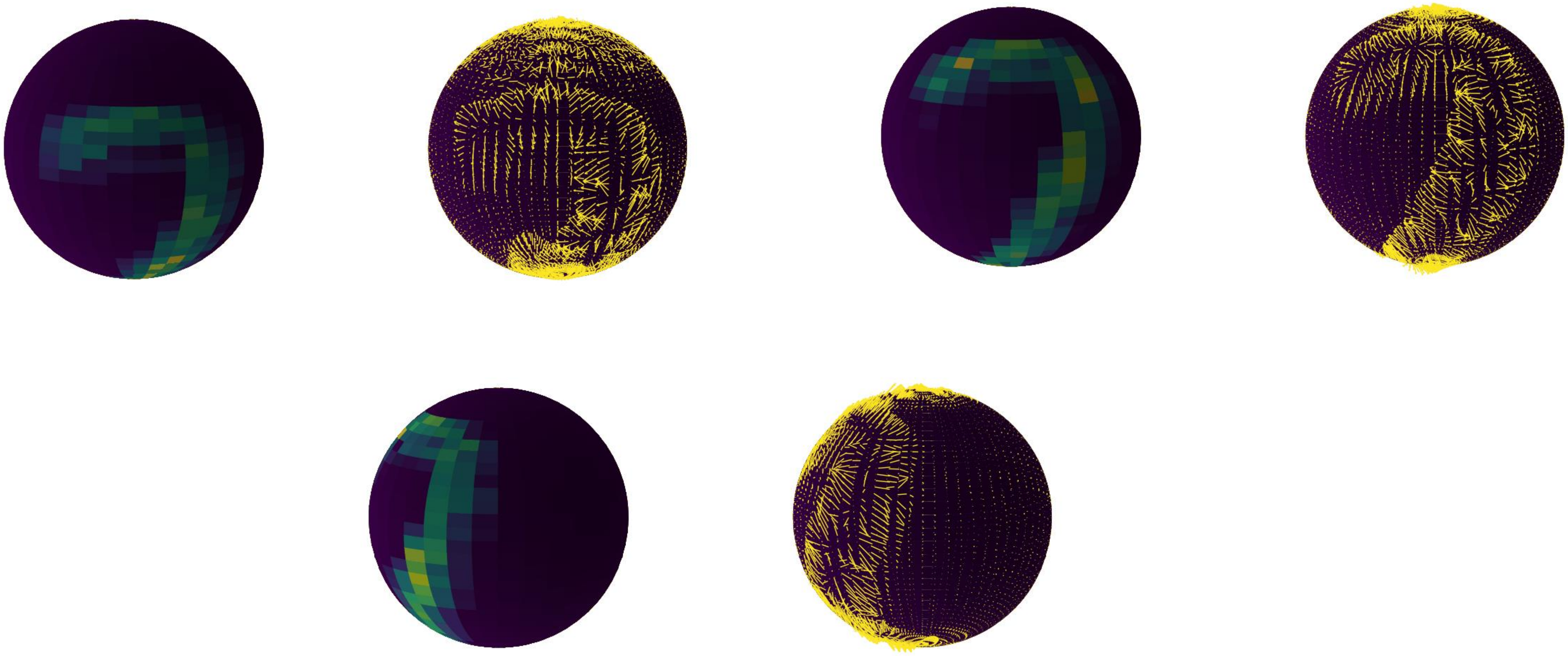
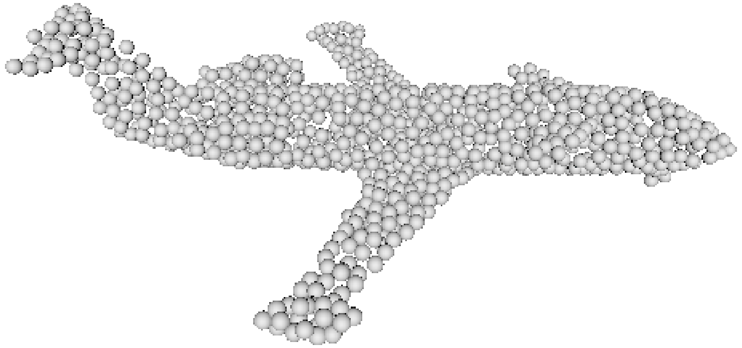
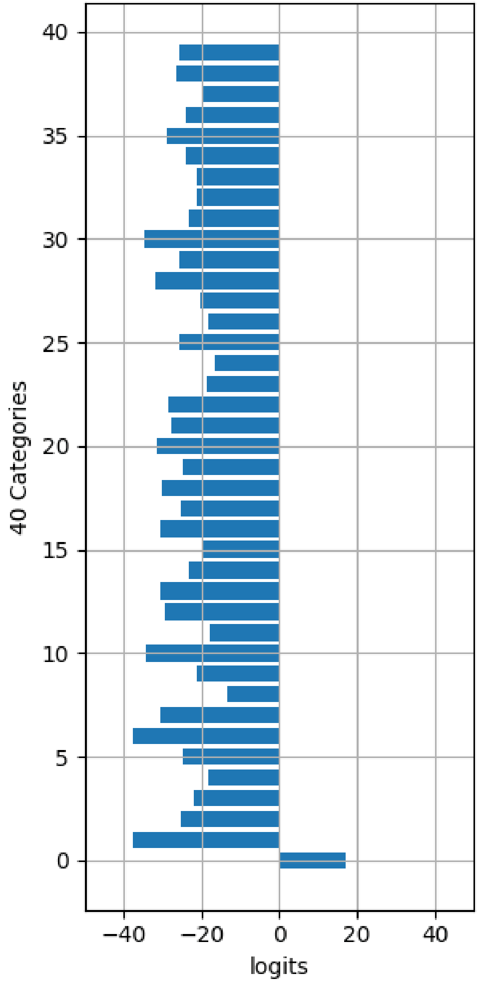


Experiments

Modelnet40 Classification





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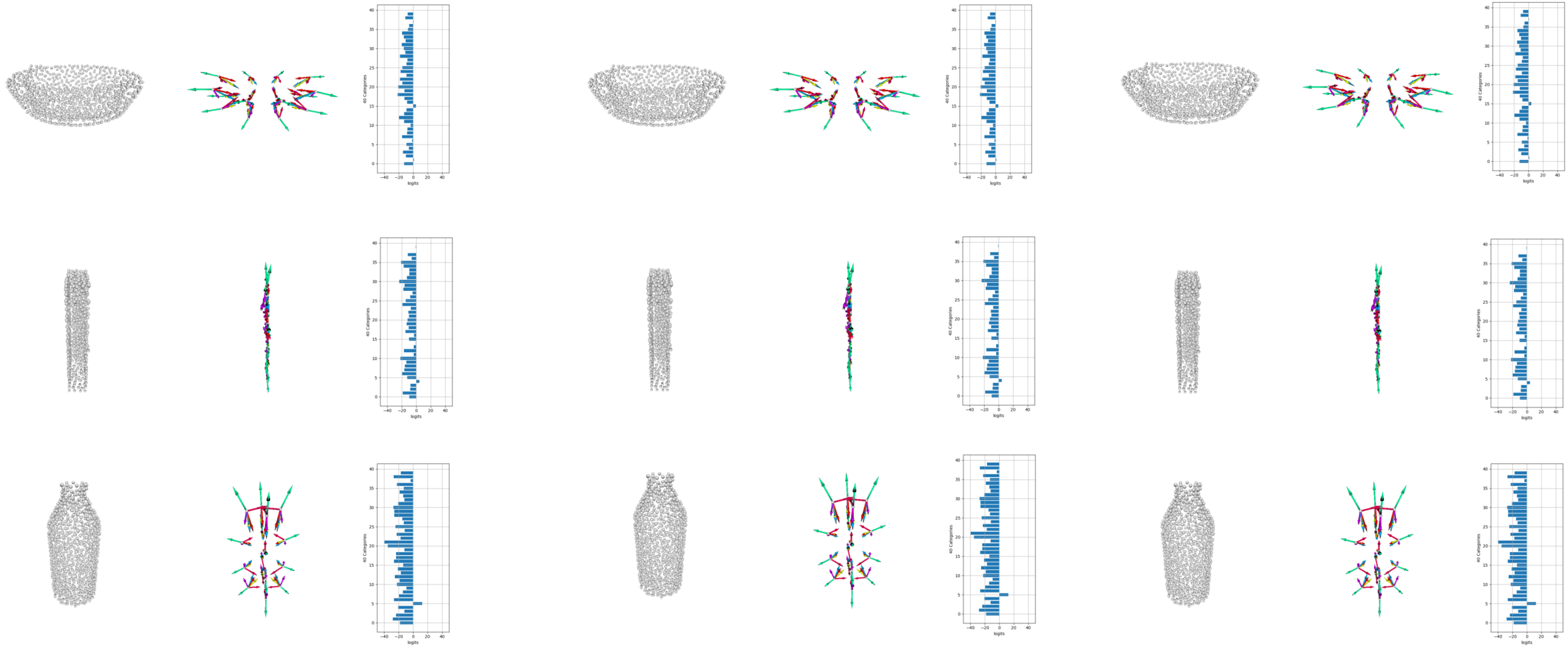

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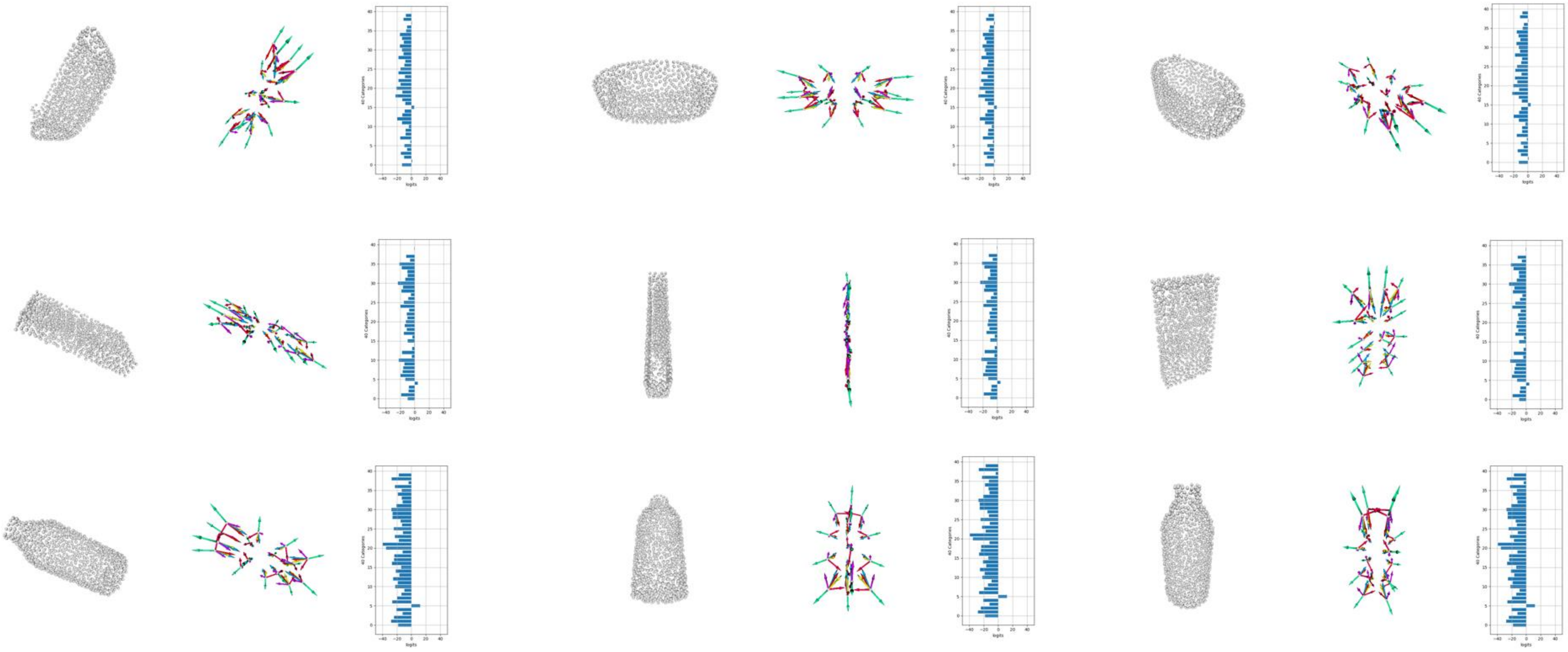

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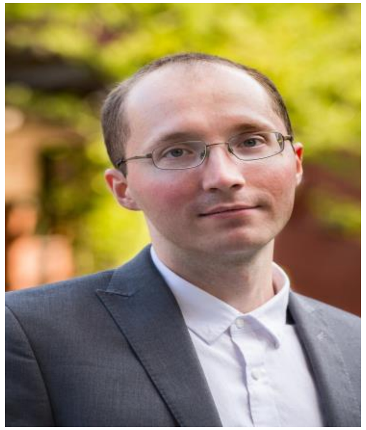


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