# Robustness in Multi-Objective Submodular Optimization: a Quantile Approach

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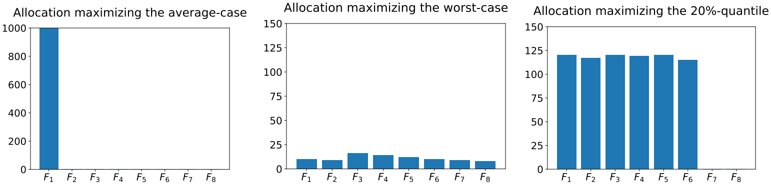
# Multi-ojective submodular optimization (setup)

## Multi-objective submodular optimization

- We have a collection of submodular functions $F_1(S), \dots, F_d(S)$
- Aggregating the functions $F_{avg} = \frac{1}{d}\sum_{i=1}^{d} F_i(S)$ and $F_{wc} = \min_{i=1\dots d} F_i(S)$?
- In this work, we propose to maximize the quantile of the objectives

$$S_K^\star \in \underset{S \subseteq \mathcal{V}:|S| \leqslant K}{\operatorname{argmax}} \; Q_p(F_1(S), \dots, F_d(S)). \tag{1}$$



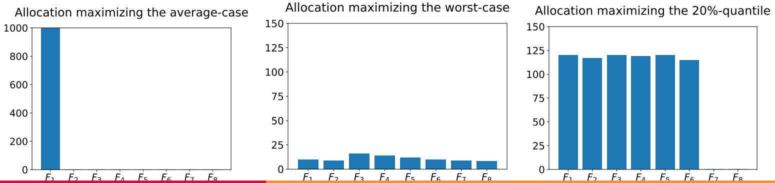
# Optimization of the quantile of the systems

## Pros of optimizing quantiles

- Successfully used in applications to mitigate the outliers effect
- Ensures that most objectives have good values
- Novel criterion/tool on top of average/worst case

## Cons of optimizing the quantile

- Optimization the quantile is NP-Hard
- Impossible to obtain an approximate algorithm



# Soft-approximation of the quantile: Biased Expectations

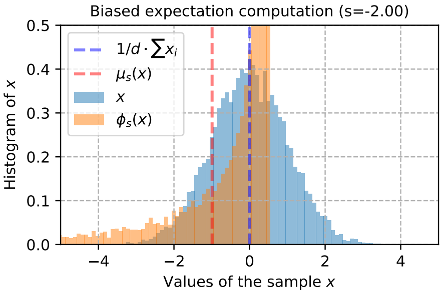
Introdution of a **novel criterion (Biased Expectations)** to approximate the quantiles.

- For any $s \in \mathbb{R}$, we define the **Biased Expectation** of a sample $x = (x_1, \dots, x_d) \in \mathbb{R}^d$ as follows:

$$\mu_s(x) = \phi_s^{-1}\left(\frac{1}{d}\sum_{i=1}^{d}\phi_s(x_i)\right)$$

where $\phi_s(x) = (e^{sx} - 1)/s$.

- **Properties of Biased Expectations**:
$\lim_{s\to-\infty}\mu_s(x) = \min_{i=1\dots n} x_i$
$\lim_{s\to 0}\mu_s(x) = \frac{1}{d}\sum_{i=1}^{d} x_i$
$\lim_{s\to+\infty}\mu_s(x) = \max_{i=1\dots d} x_i$



# Convergence with soft-quantile Approximation

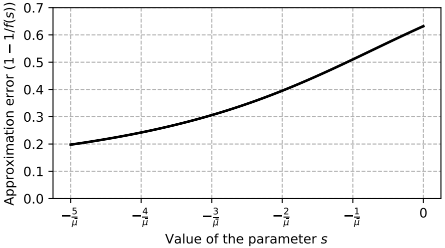
**Algorithm 1** SOFTSAT

1: **Require:** Cardinality constraint $K$, functions $F(\cdot) = [F_1(\cdot), \dots, F_d(\cdot)]$, parameter $s \in \mathbb{R}$
2: $\hat{S}_0 \leftarrow \varnothing$
3: **for** $t = 0$ to $K-1$ **do**
4: $\quad e_{t+1} \leftarrow \arg\max_{e \in \mathcal{V}/\hat{S}_t} \mu_s(F(\hat{S}_t \cup \{e\}))$
5: $\quad \hat{S}_{t+1} \leftarrow \hat{S}_t \cup \{e_{t+1}\}$
6: **end for**
7: **return** $\hat{S}_K$

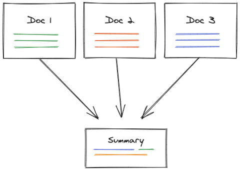
One can show that for any $s < 0$, we have:

$$\mu_s(F(\hat{S}_K)) \geqslant \left(1 - \frac{1}{f(s)}\right) \cdot \max_{|S| \leqslant K} \mu_s(F(S))$$

where $f(s) = -s\bar{\mu}/\log(1/e e^{-s\bar{\mu}} + (1-1/e))$

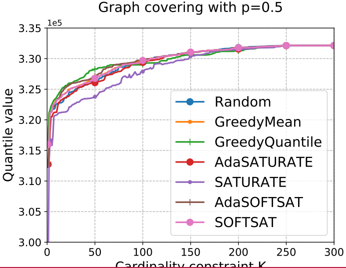


# Applications to different problems




## News Feed

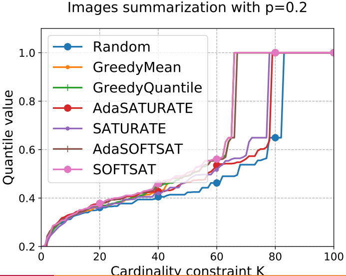
$$F_i(S) = \sum_{e \in S} f_i(e)$$

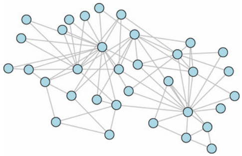




## Images Summarization

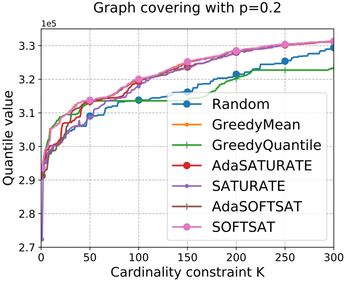
$$F_i(S) = 1 - \min_{e \in S} D(i, e)$$



## Graph Covering

$$F_i(S) = 1 - \min_{e \in S} D(i, e)$$



Thank you for watching!