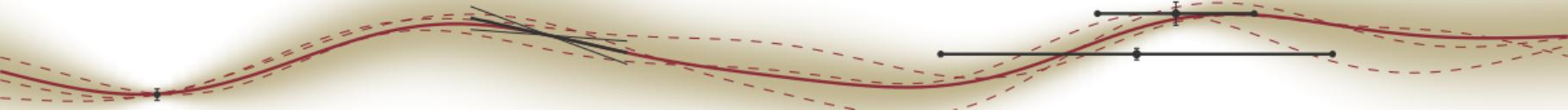


# Fenrir: Physics-Enhanced Regression in Initial Value Problems

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Latent initial value problem:

$$\varphi_\theta(t) = y_0(\theta) + \int_0^t f_\theta(\tau, \varphi_\theta(\tau)) d\tau, \quad (1a)$$

$$u(t) = H^\top \varphi_\theta(t) + v(t), \quad t \in \mathbb{T}_D \quad (1b)$$

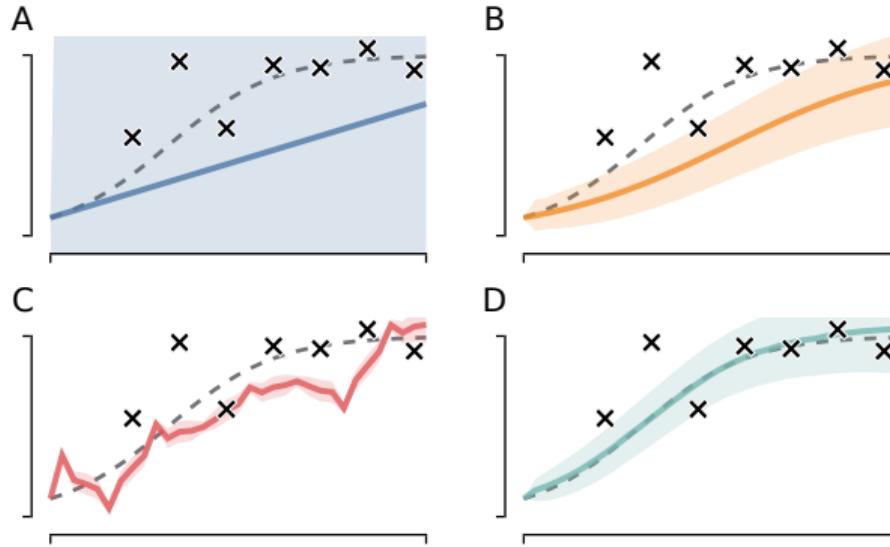
Likelihood:

$$\mathcal{L}_D(R_\theta, y) = \prod_{t \in \mathbb{T}_D} \mathcal{N} \left( u(t); H^\top y(t), R_\theta \right), \quad (2a)$$

$$\mathcal{M}(\theta) = \mathcal{L}_D(R_\theta, \varphi_\theta). \quad (2b)$$

Maximum likelihood:

$$\hat{\theta} = \arg \max_{\theta \in \Theta} \mathcal{M}(\theta). \quad (3)$$



$$\widehat{\mathcal{M}}_N(\theta, \kappa) = \int \mathcal{L}_D(R_\theta, y) \widehat{\delta}_N(y \mid \theta, \kappa) dy, \text{ where } \widehat{\delta}_N(y \mid \theta, \kappa) \approx \delta(y - \varphi_\theta).$$

# Experiments: Fully observed states

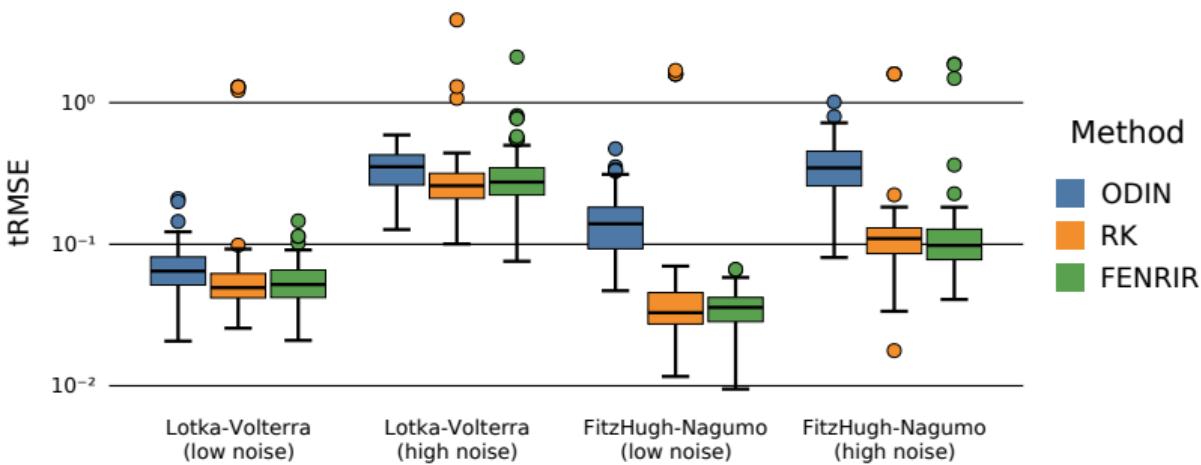
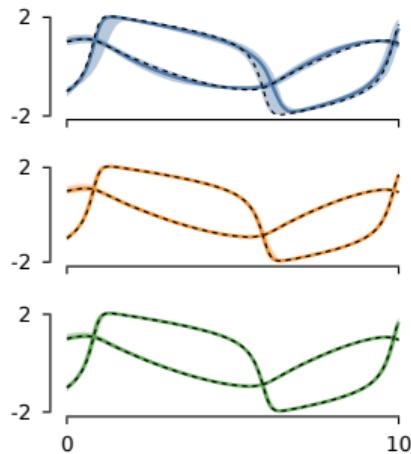


Figure: Left: reconstruction of one trajectory. Right: Reconstruction errors for a Monte-Carlo trial of 100 trajectories.

# Experiments: Partially observed states

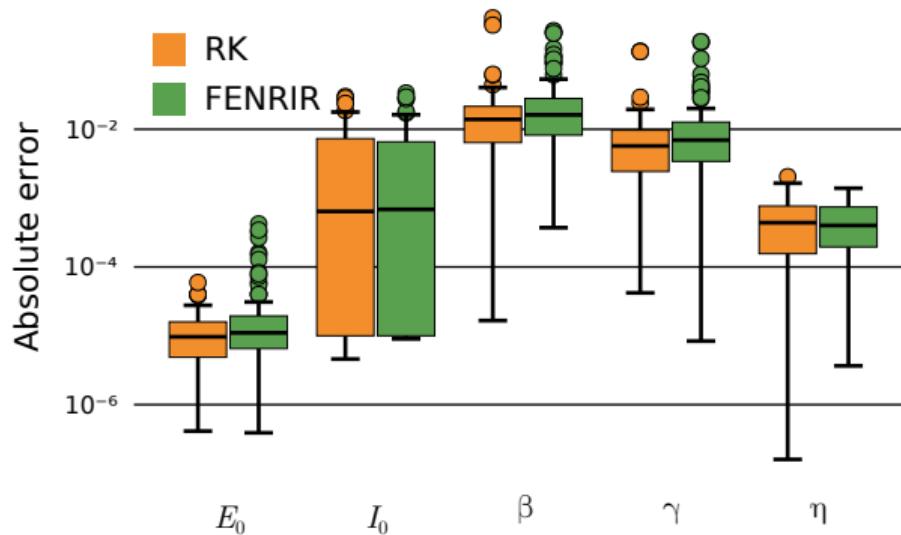
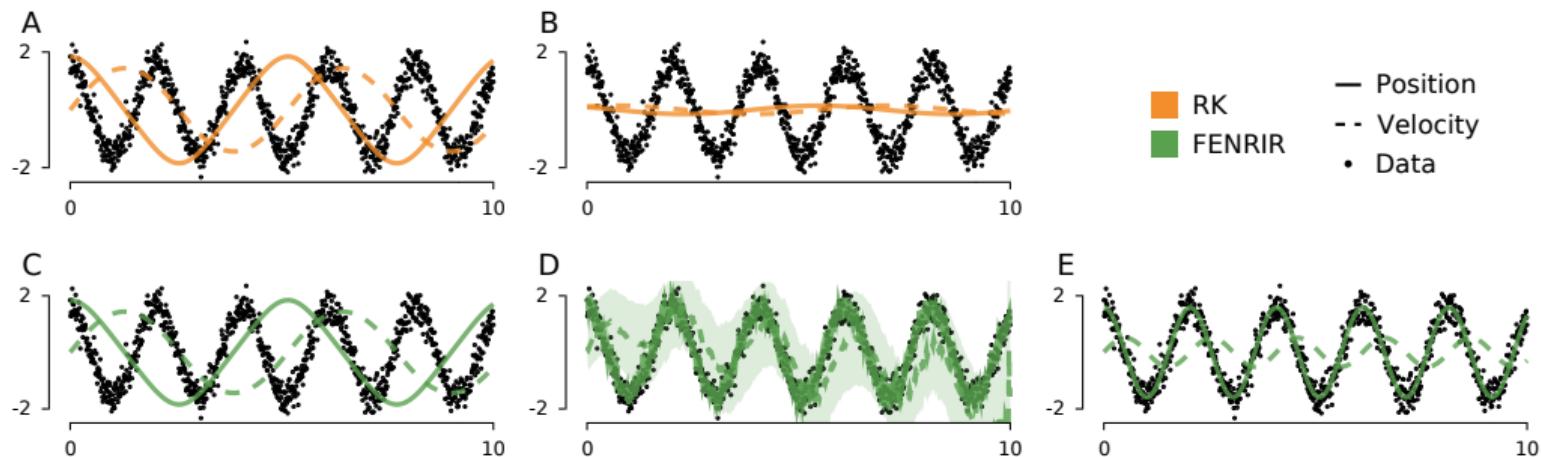


Figure: Parameter errors in a Monte-Carlo trial of the SEIR model.



# Experiments: Fast oscillations

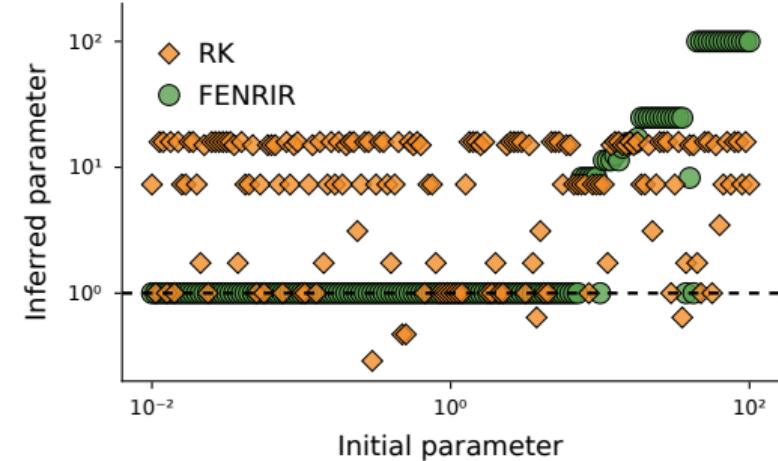
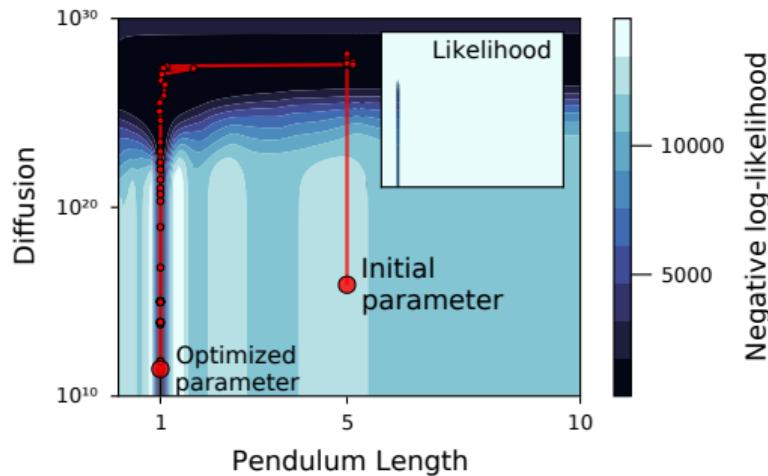
From initial guess to final estimate





# Experiments: Fast oscillations

What is going on?



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