

Interactively Learning Preference Constraints in Linear Bandits

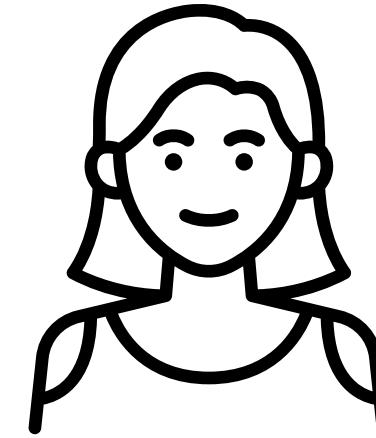
International Conference on Machine Learning (ICML), 2022

David Lindner¹, Sebastian Tschiatschek², Katja Hofmann³, Andreas Krause¹

¹ETH Zurich; ²University of Vienna; ³Microsoft Research Cambridge

Human preferences often naturally decompose into rewards and constraints

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Human preferences often naturally decompose into rewards and constraints



„Drive to the
grocery store.“

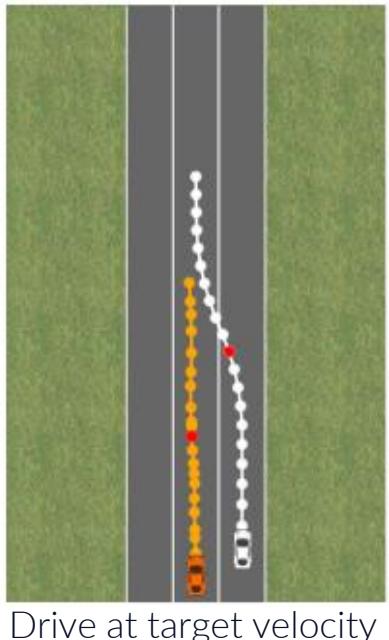
Human preferences often naturally decompose into rewards and constraints



Constraint can be more transferable and robust than rewards

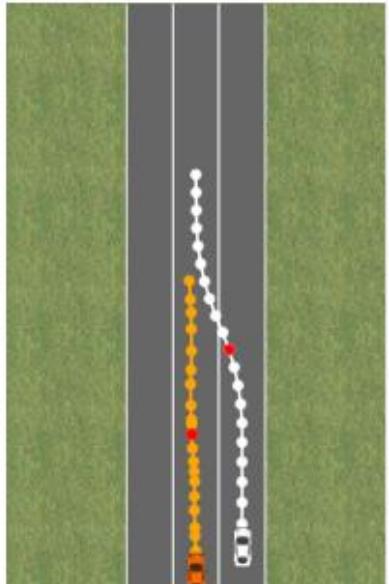
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Encode task as reward + penalty

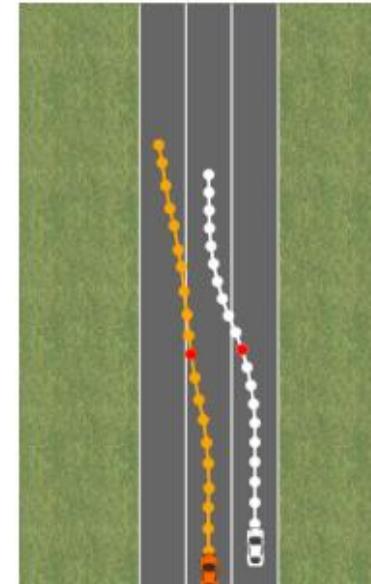


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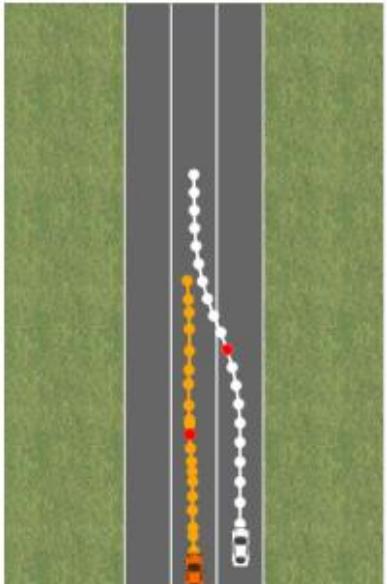


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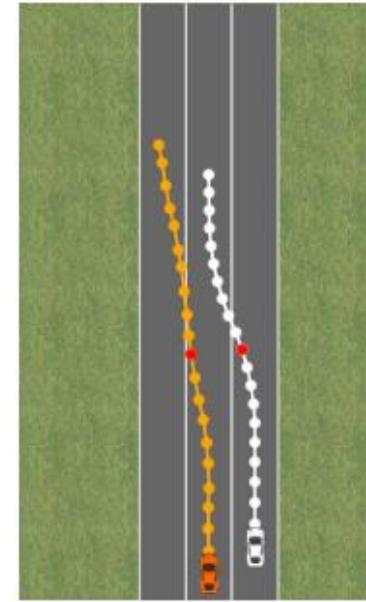
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Drive at target velocity



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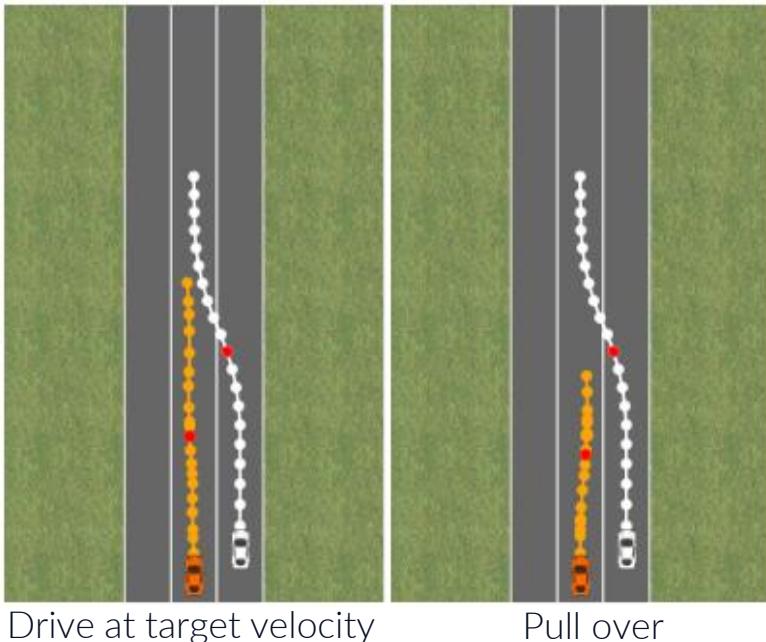


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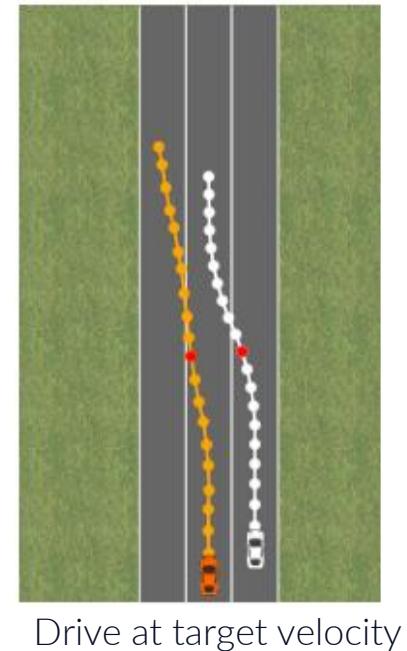


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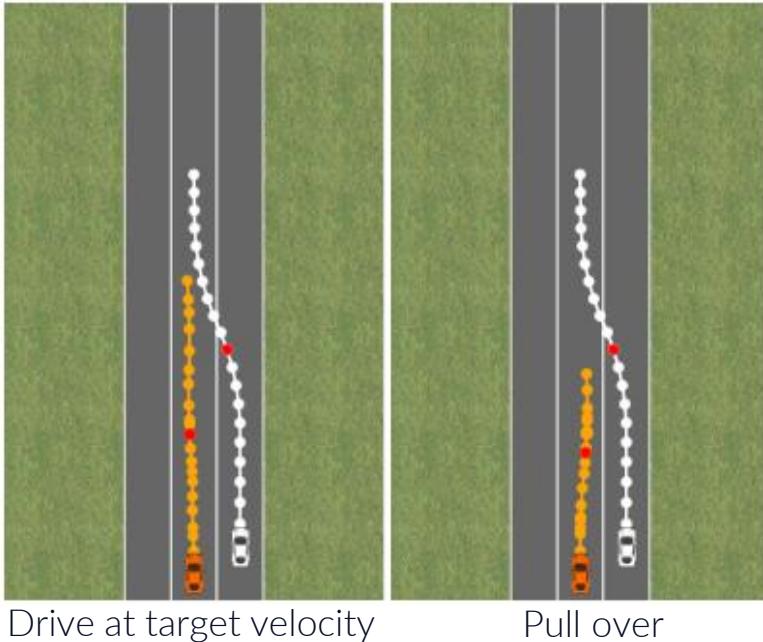


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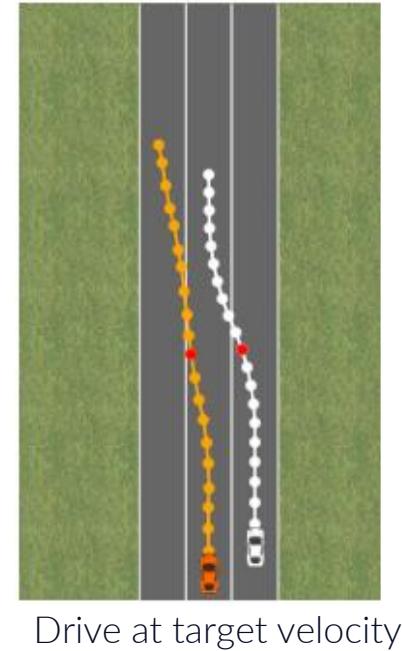


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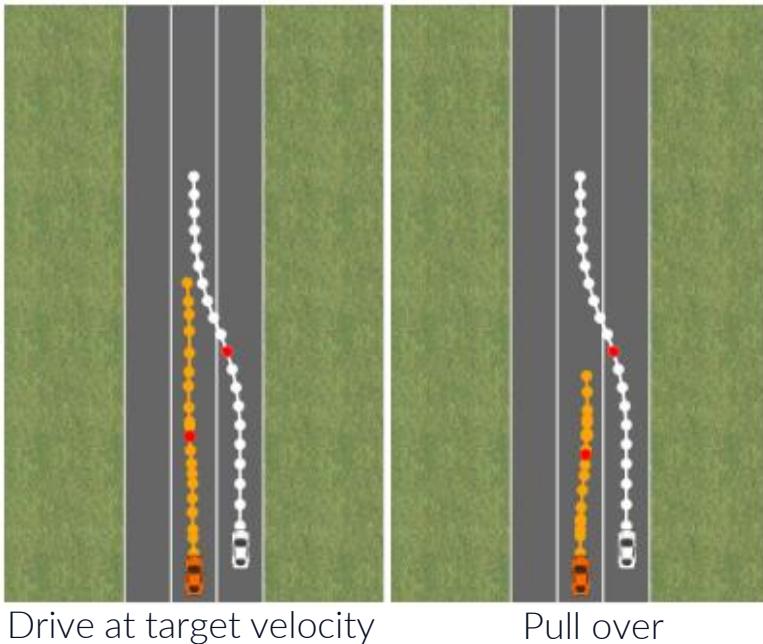


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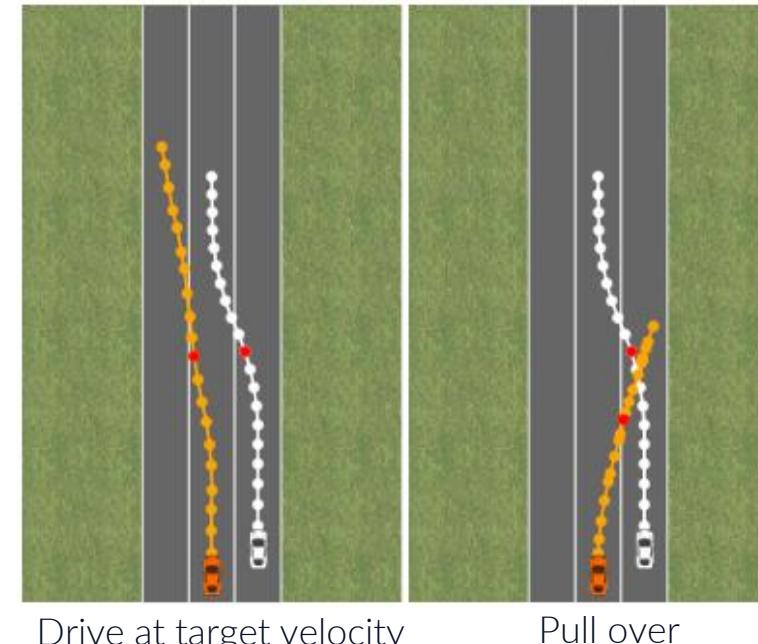


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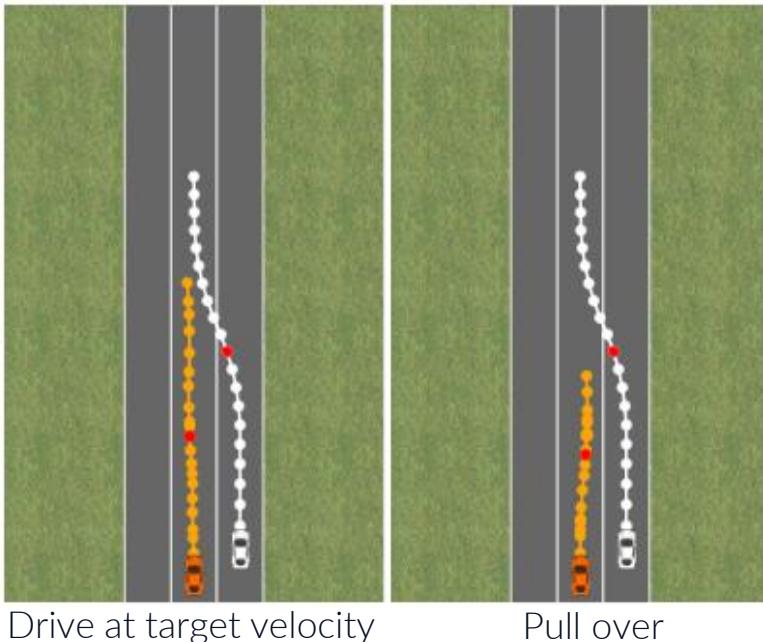


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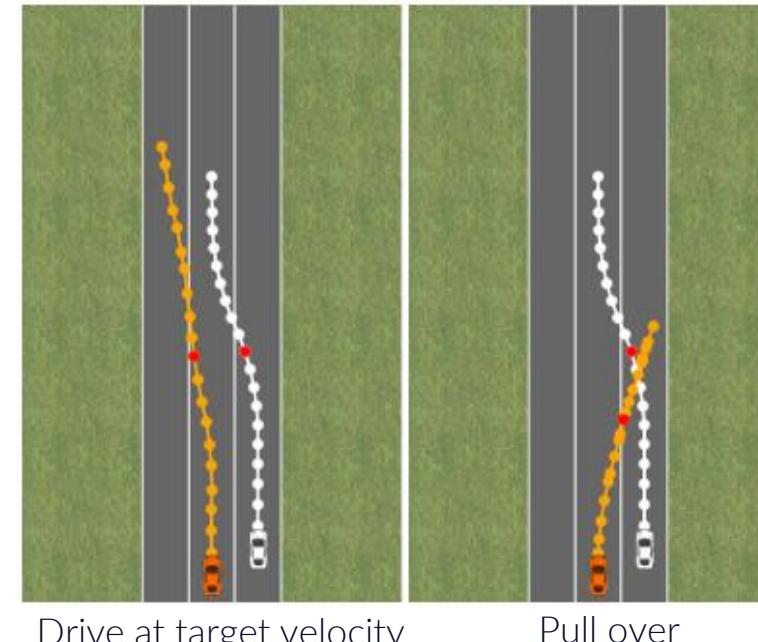


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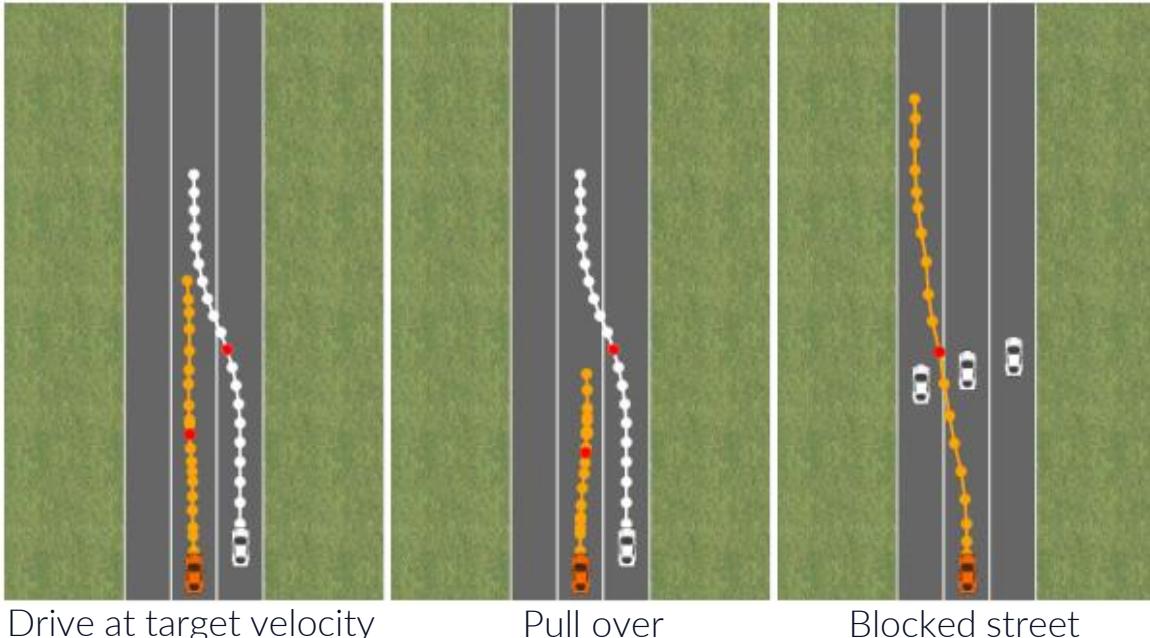


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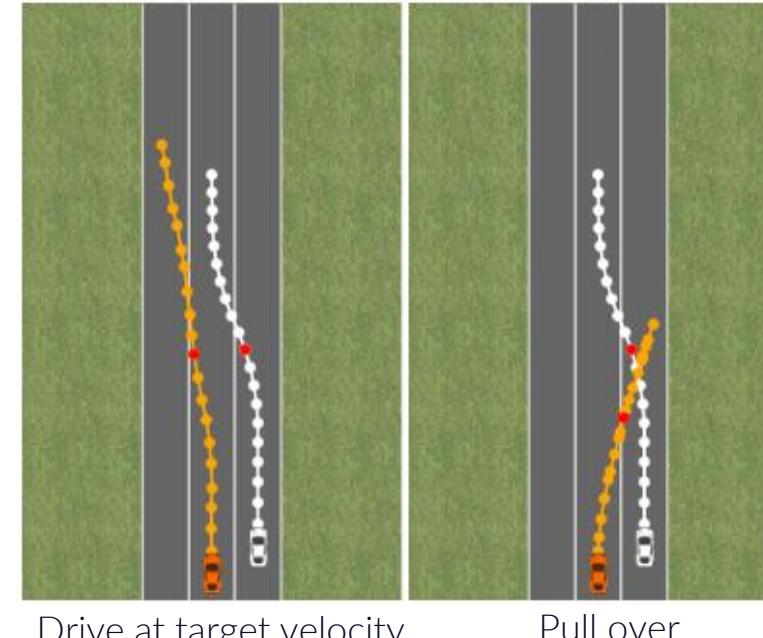


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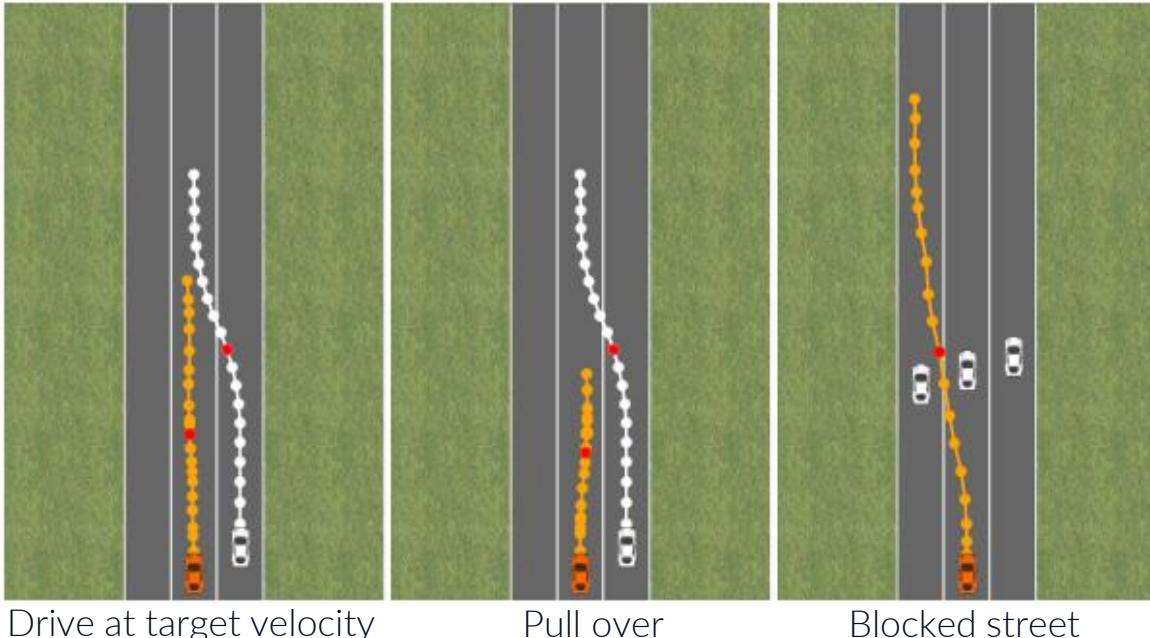


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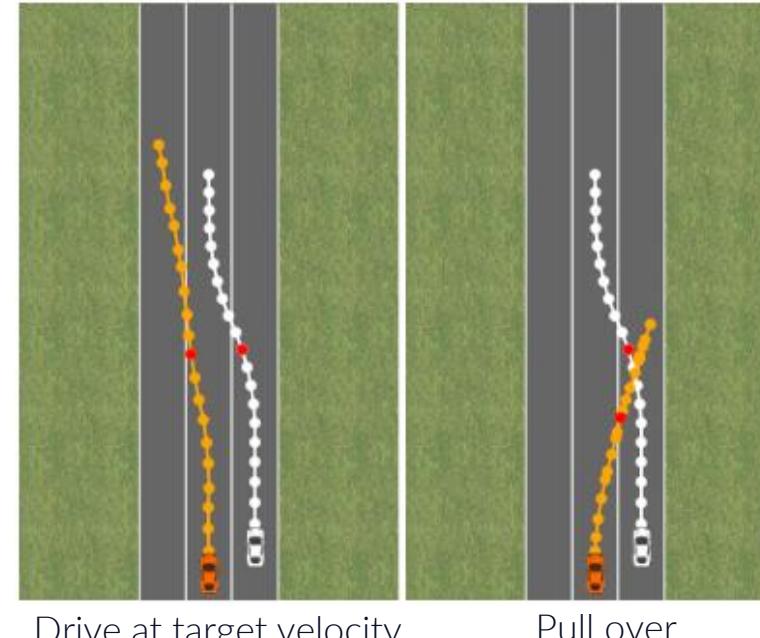


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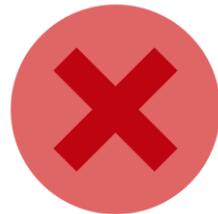
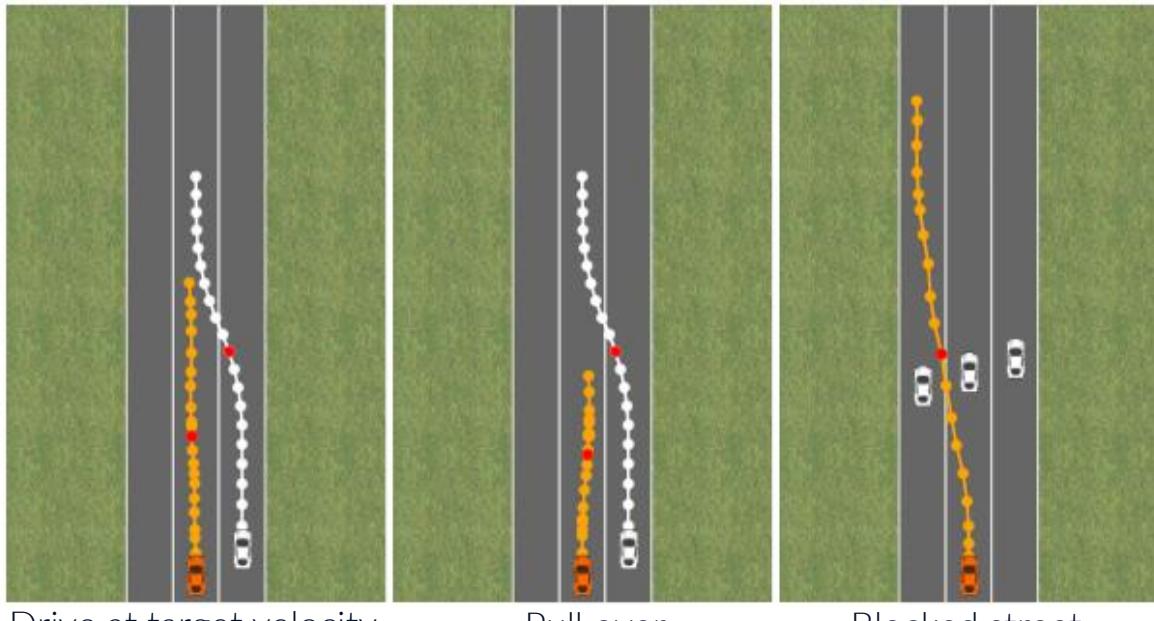


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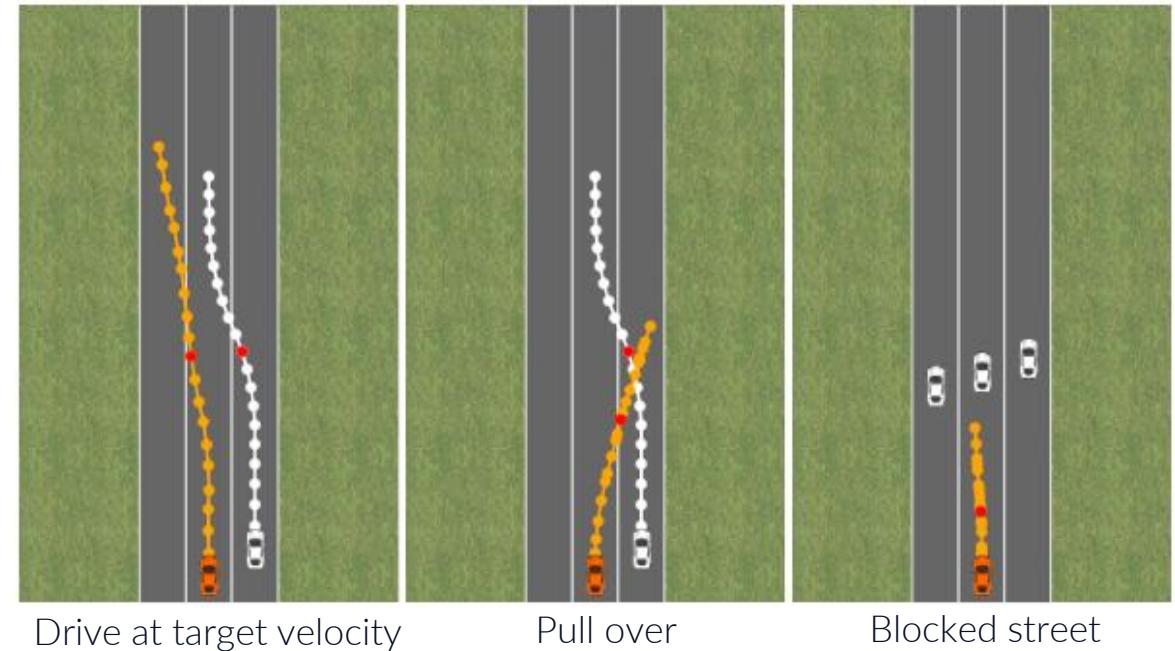


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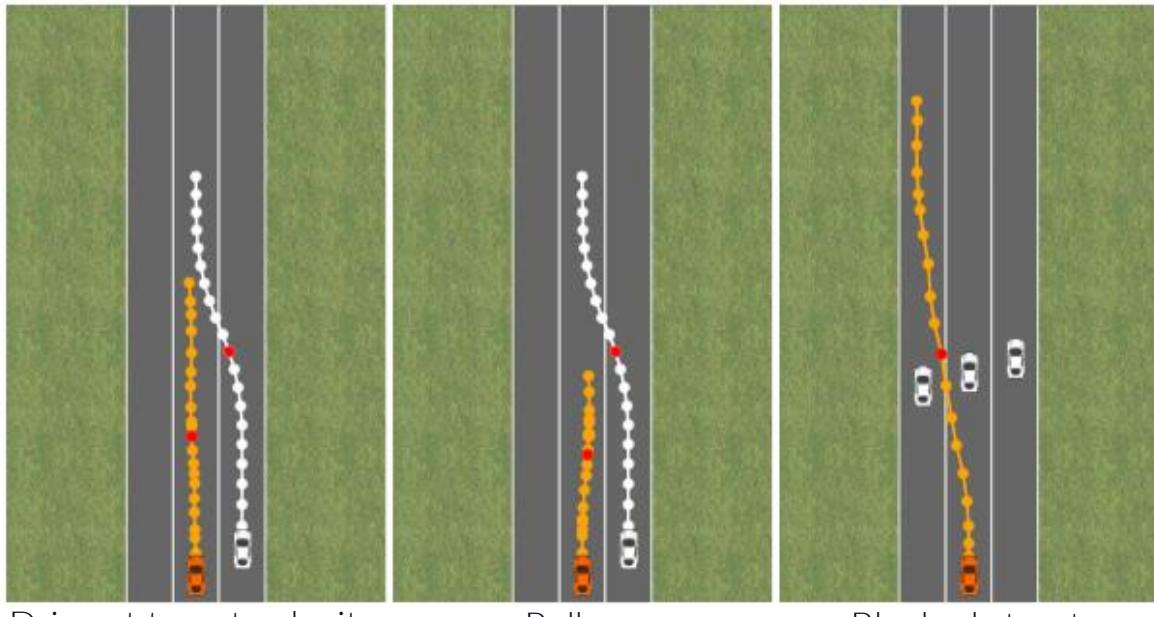


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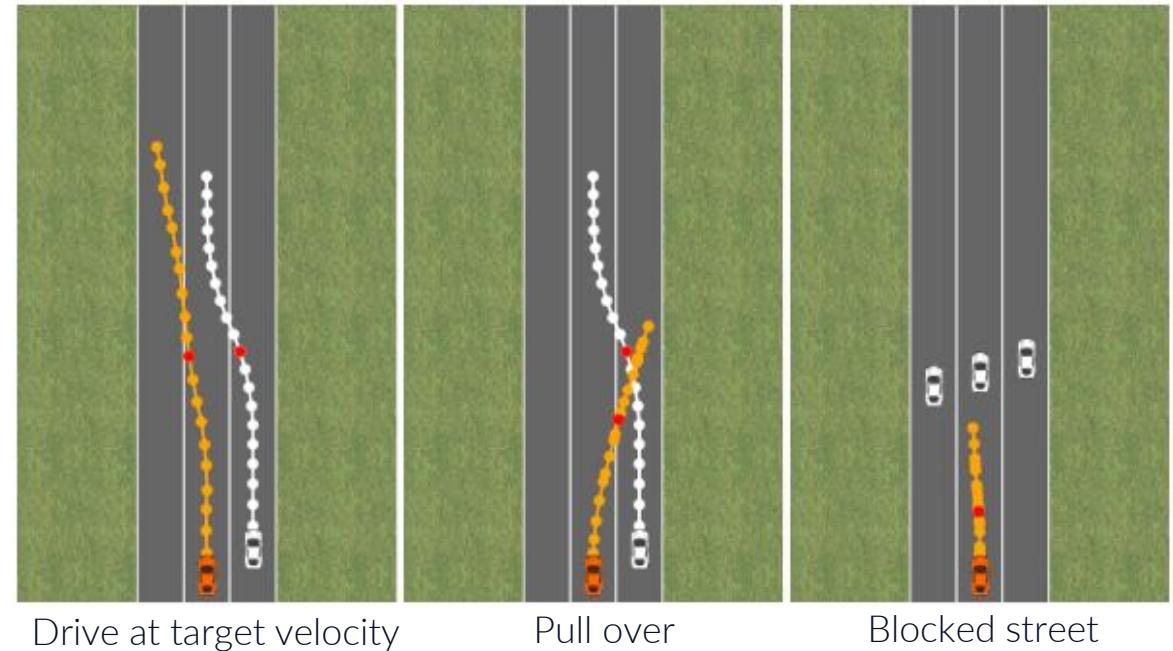


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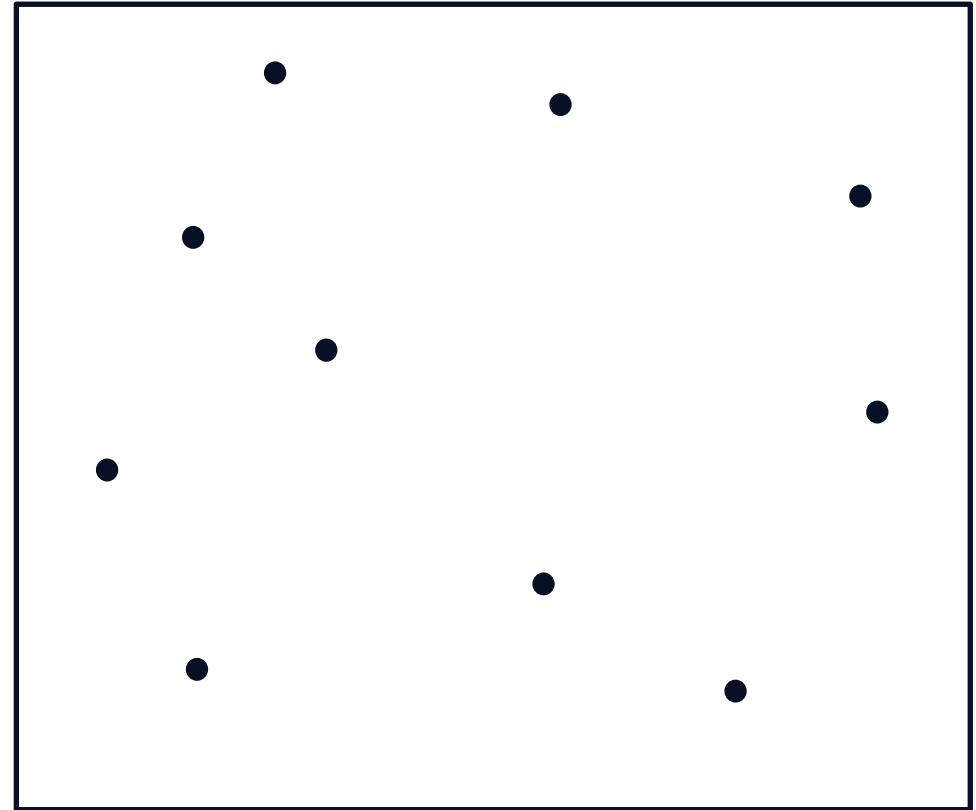
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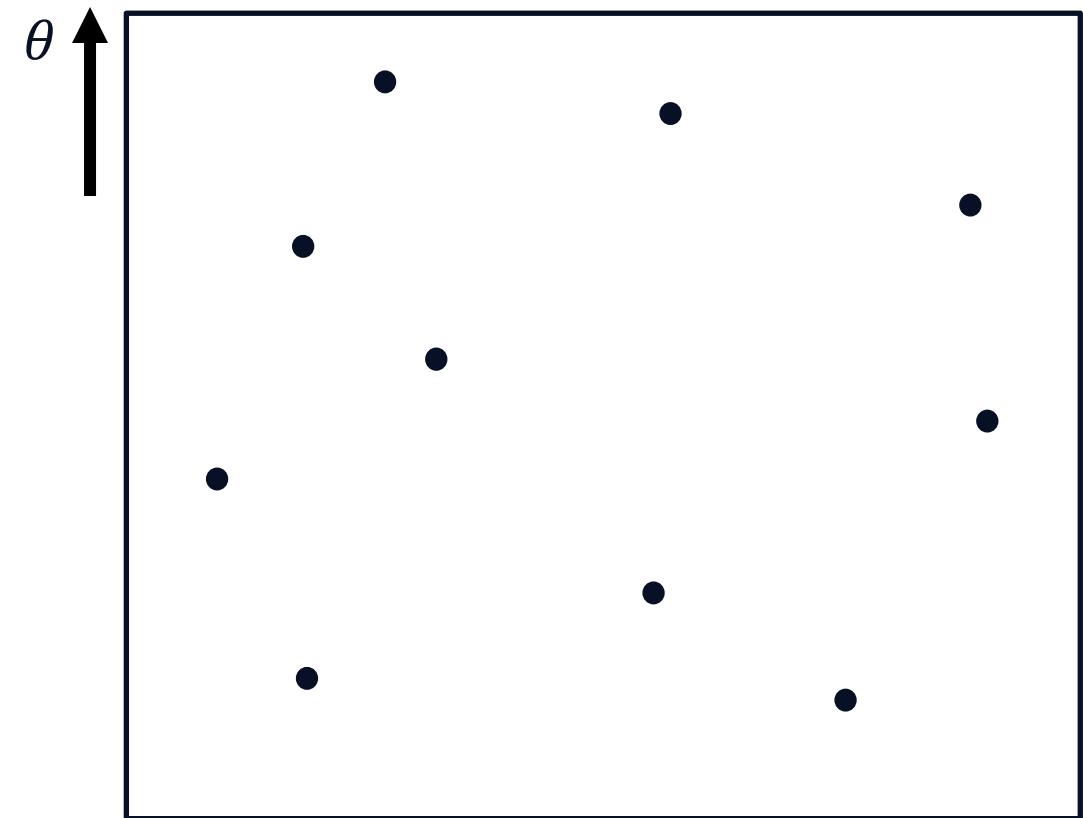
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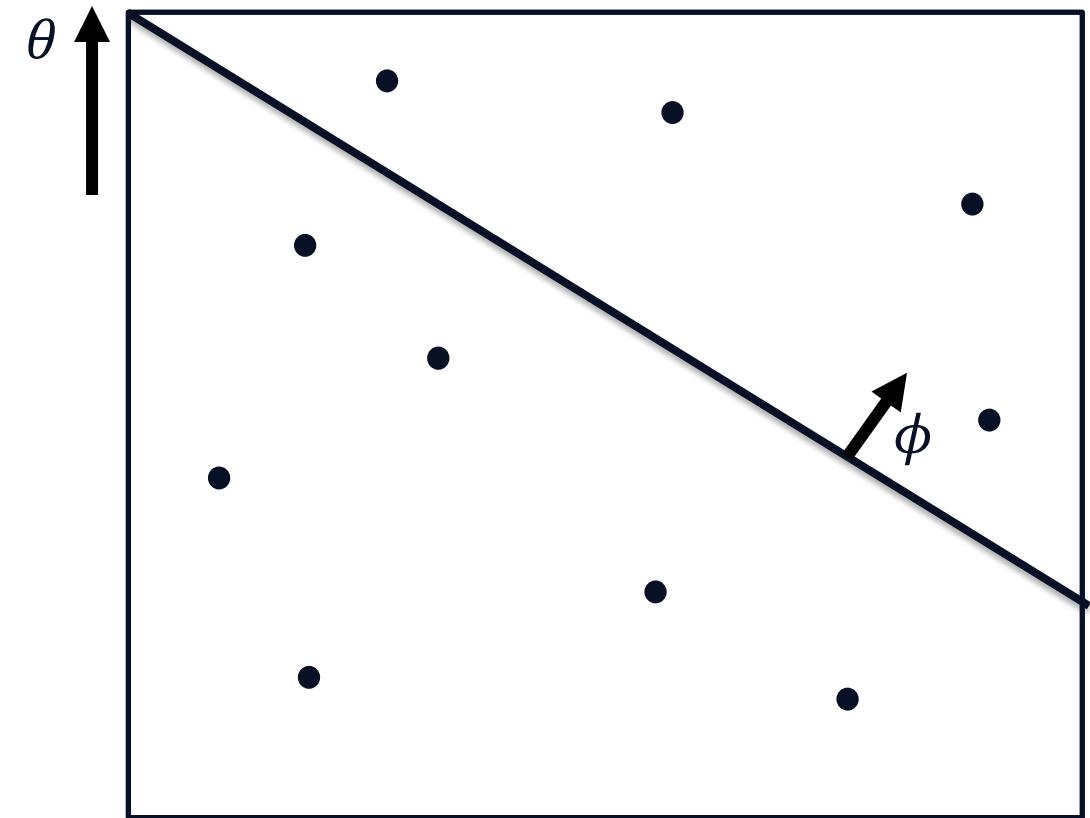


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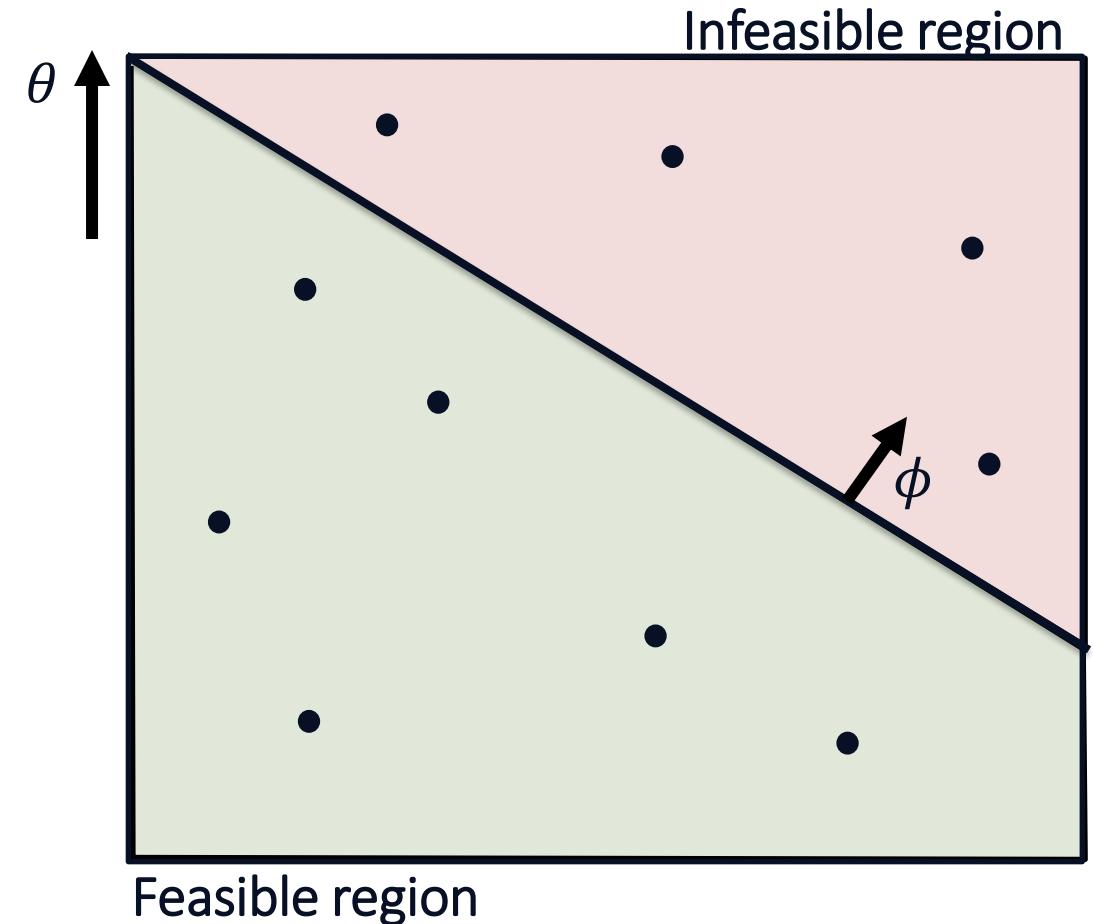


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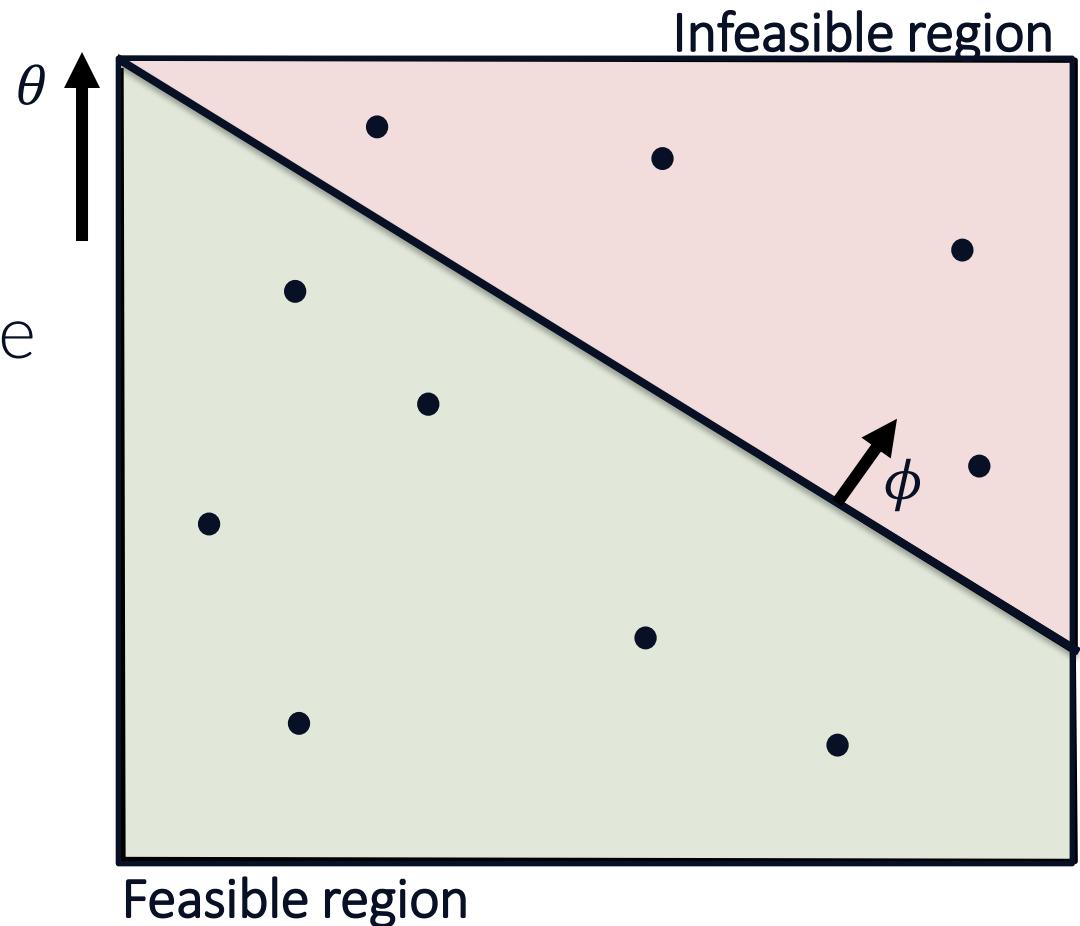
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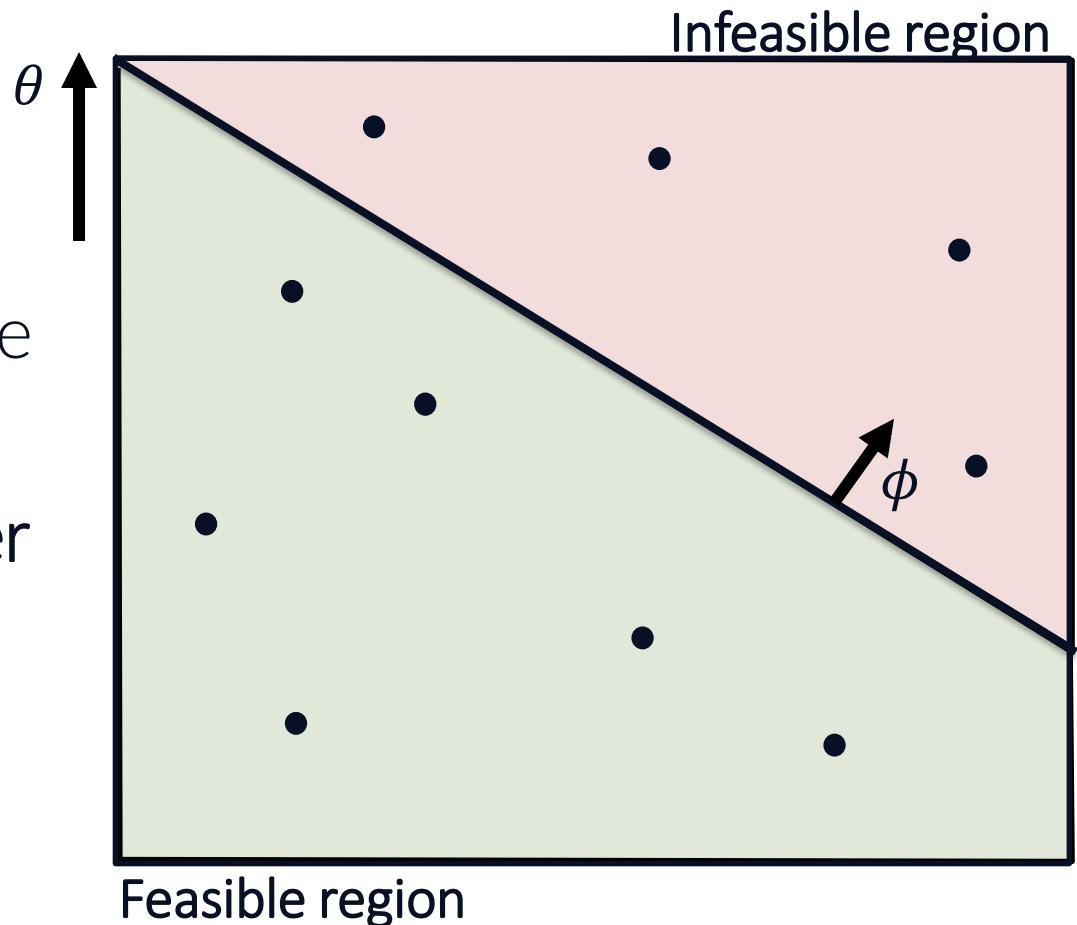
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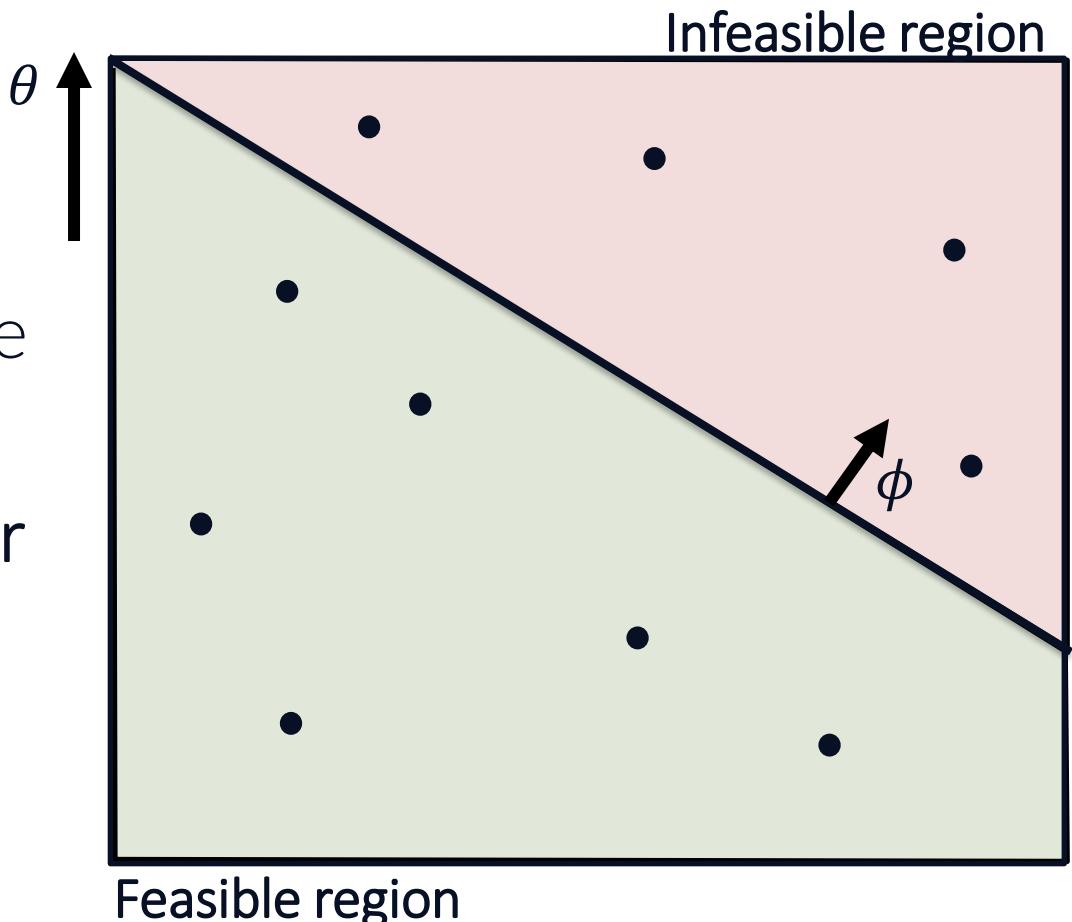
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in as few iterations as possible



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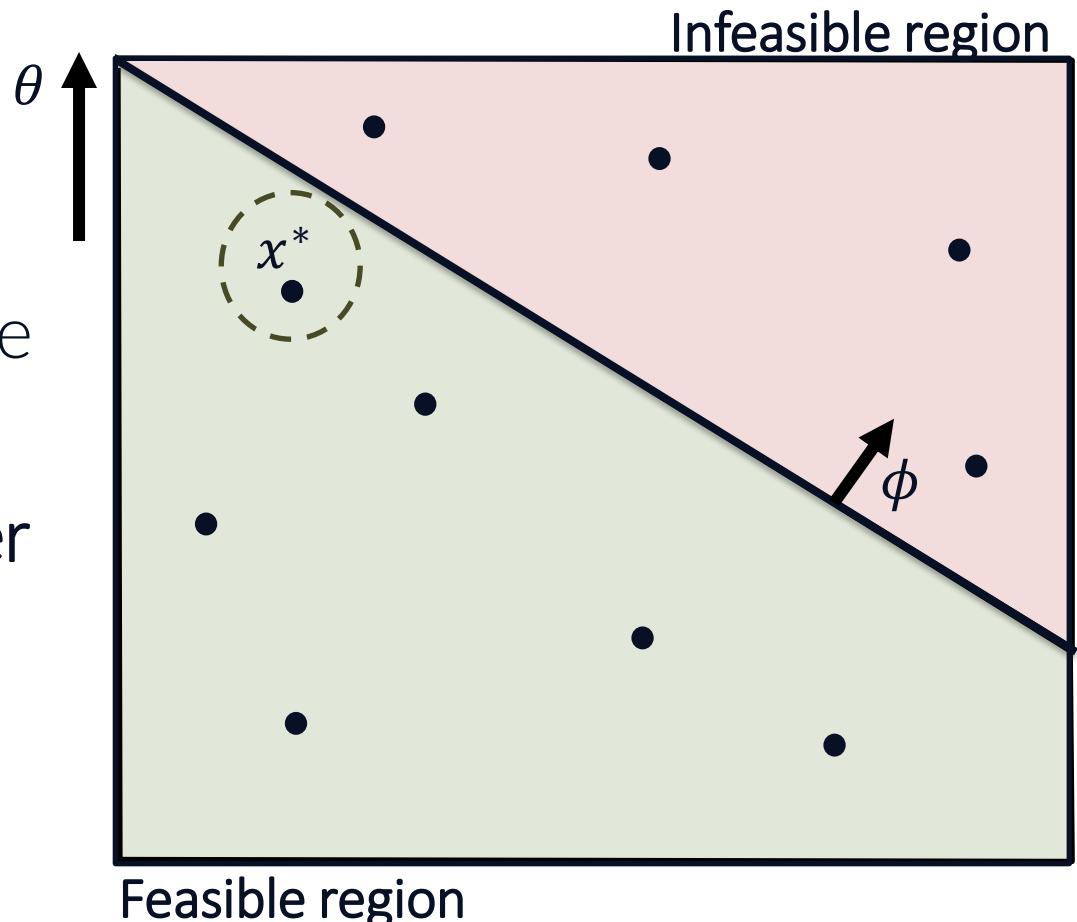
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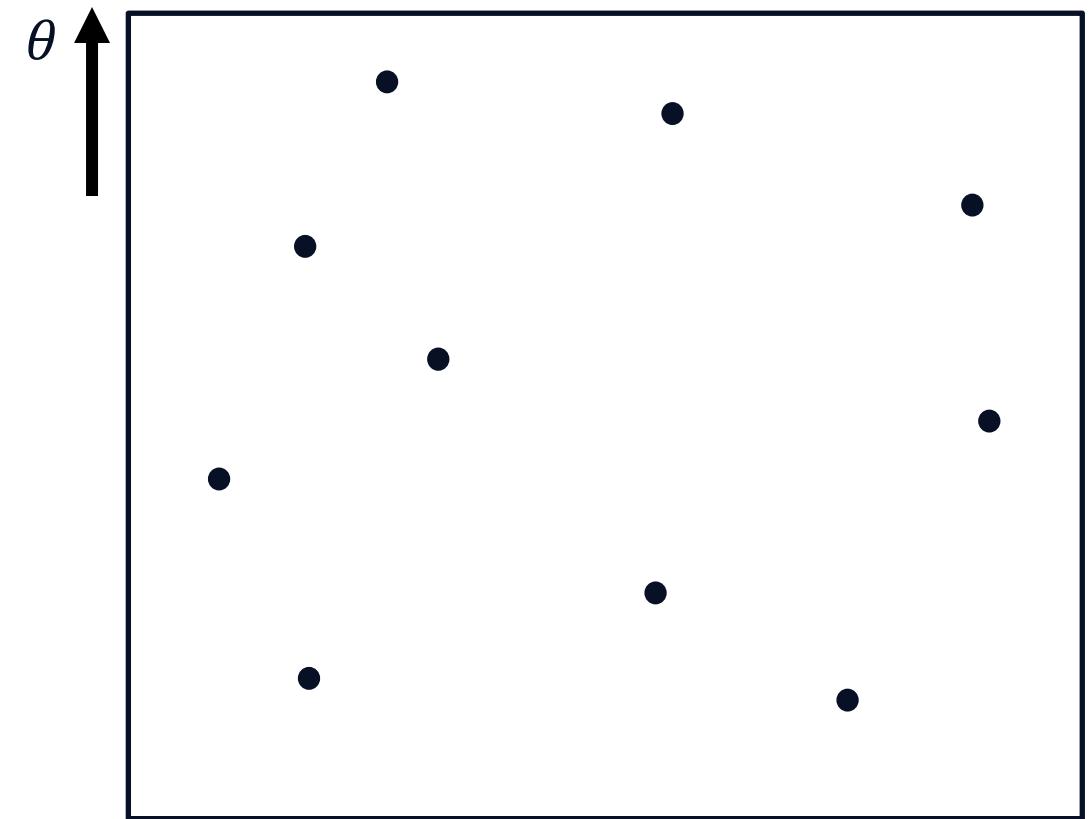
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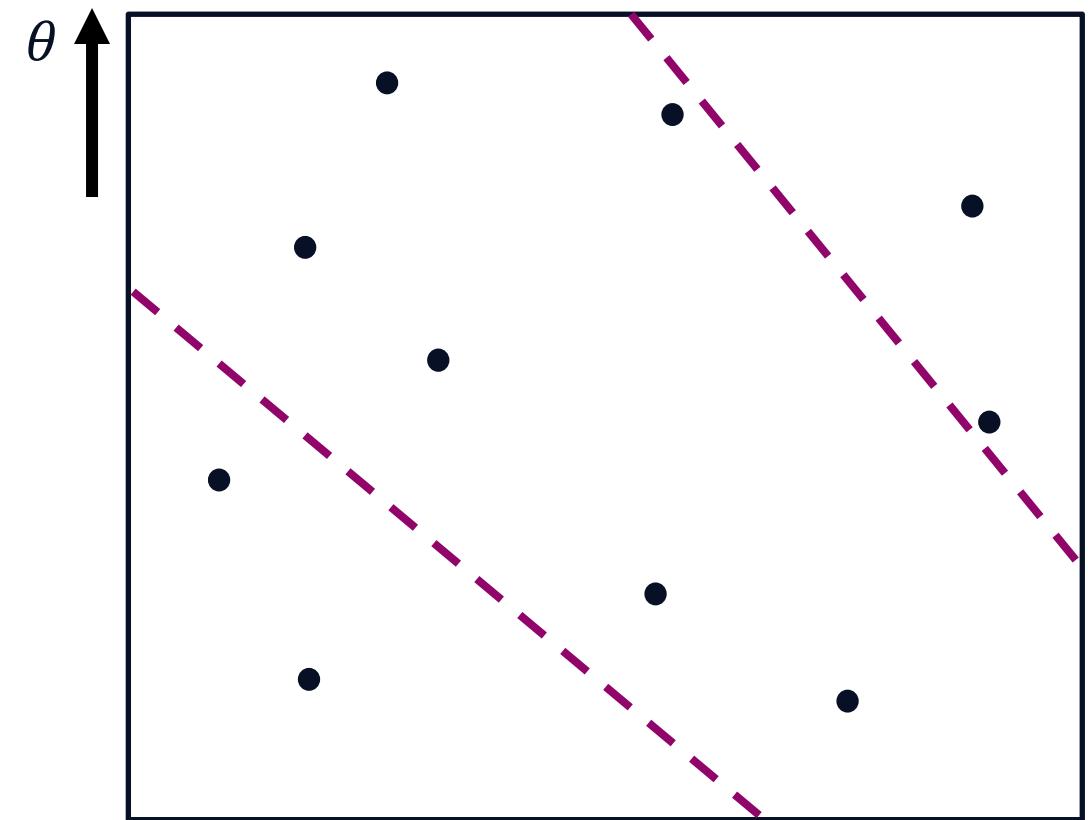


Adaptive constraint learning (ACOL)



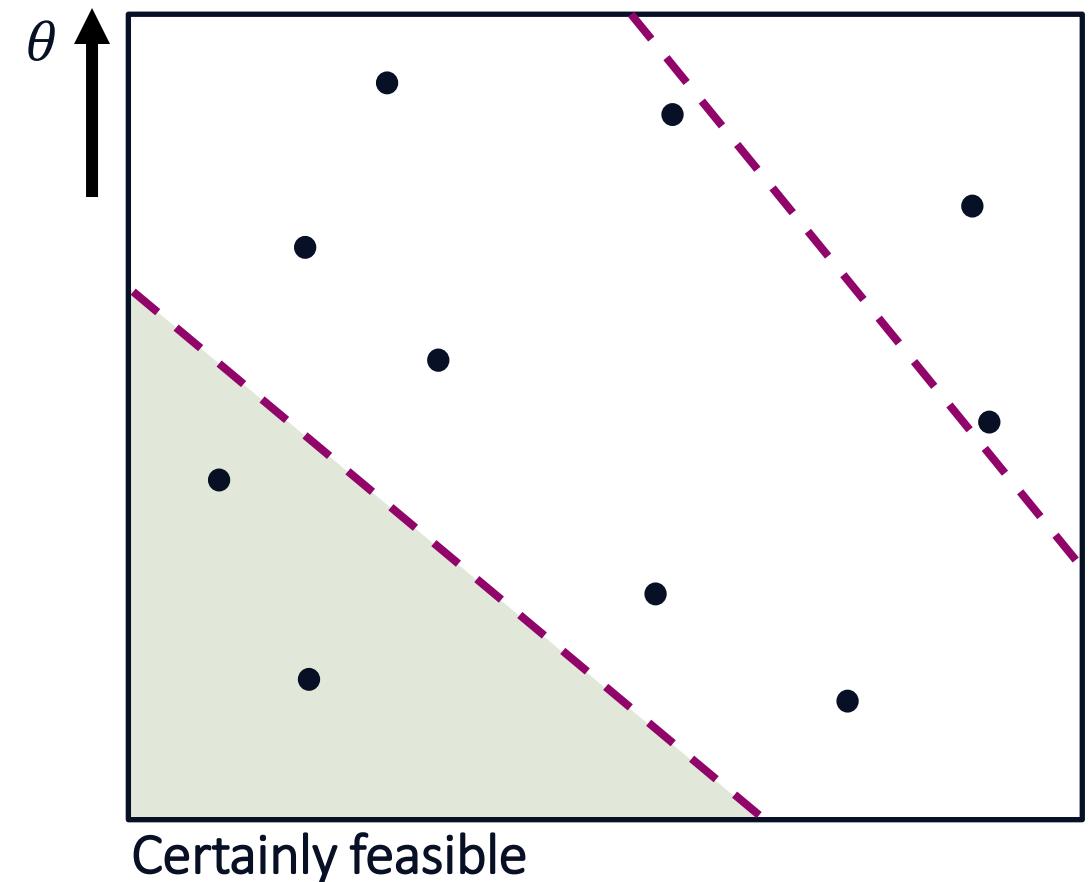
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1. Estimate confidence regions around the estimated constraint



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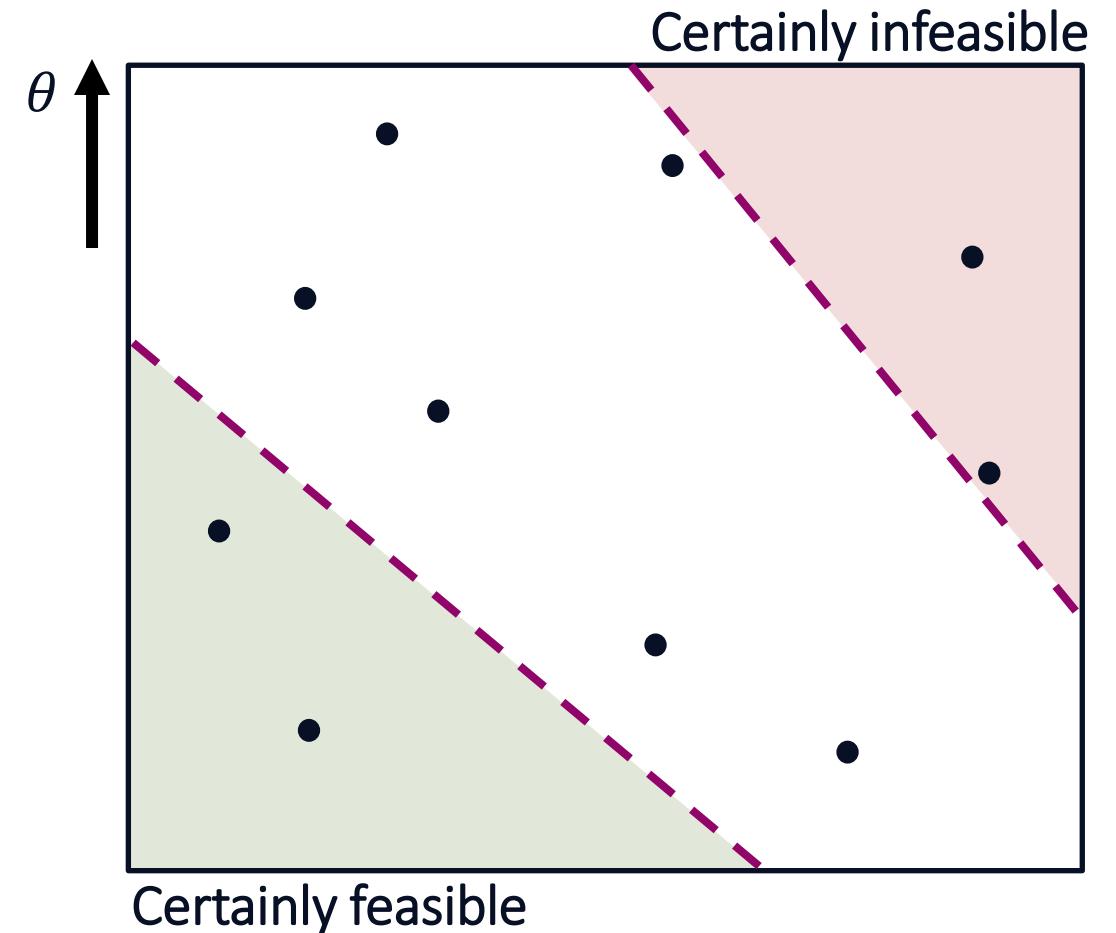
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2. Determine certainly feasible arms



„Certainly“ $\hat{=}$ with high probability

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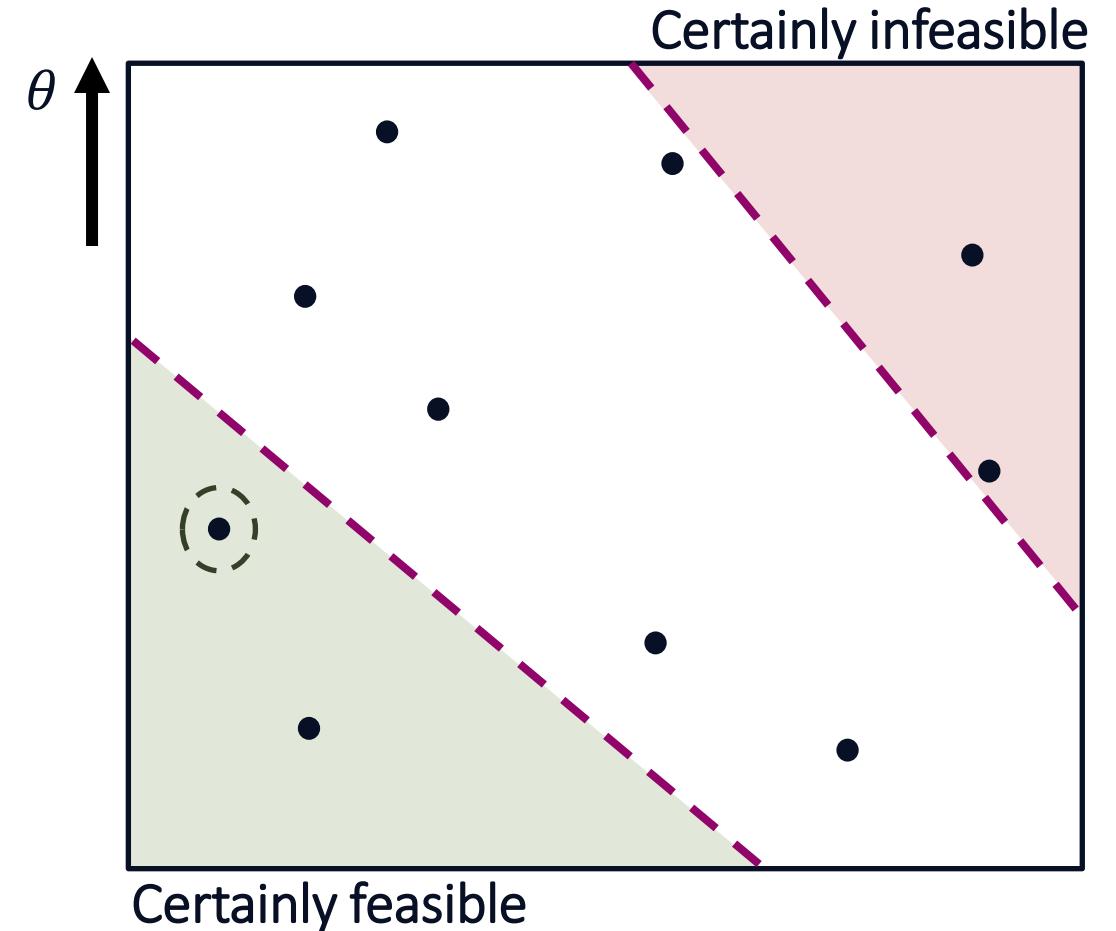
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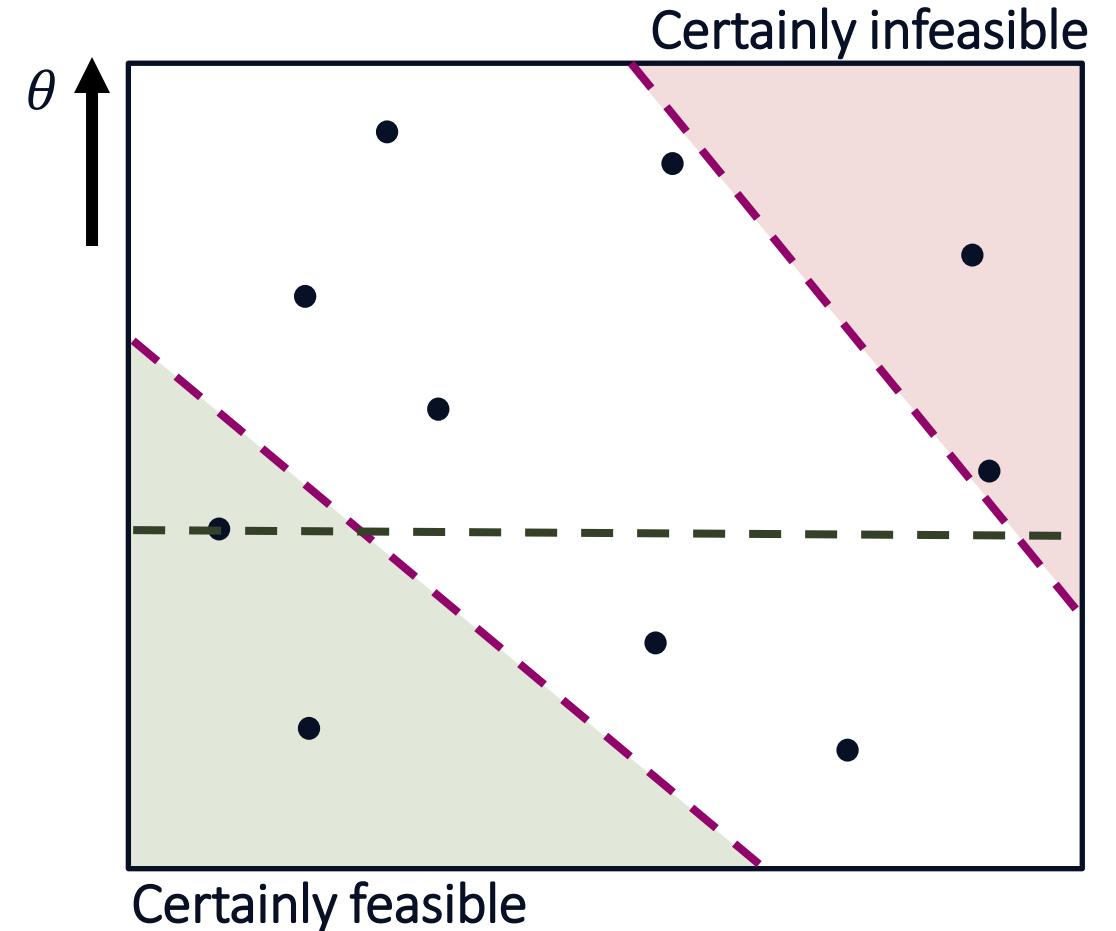
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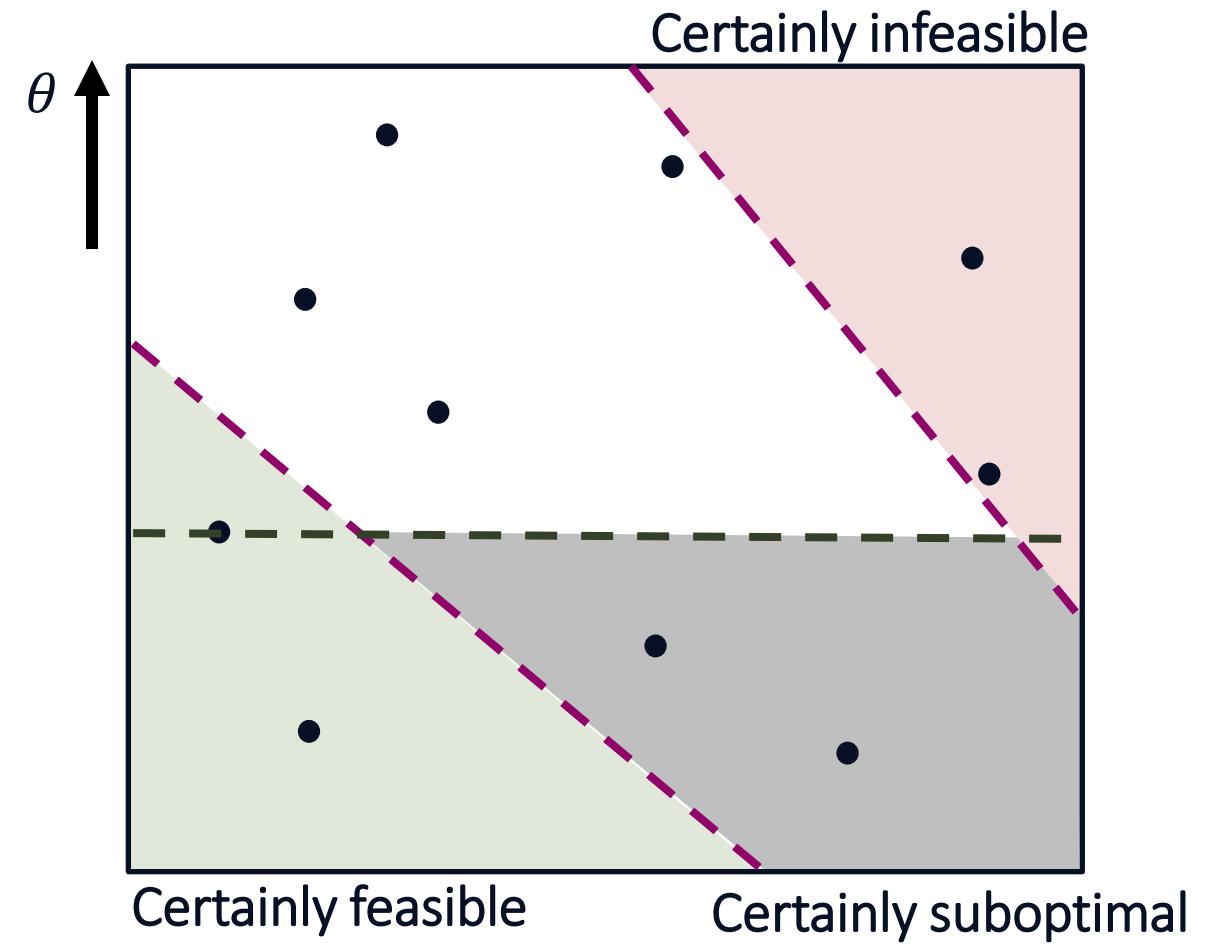
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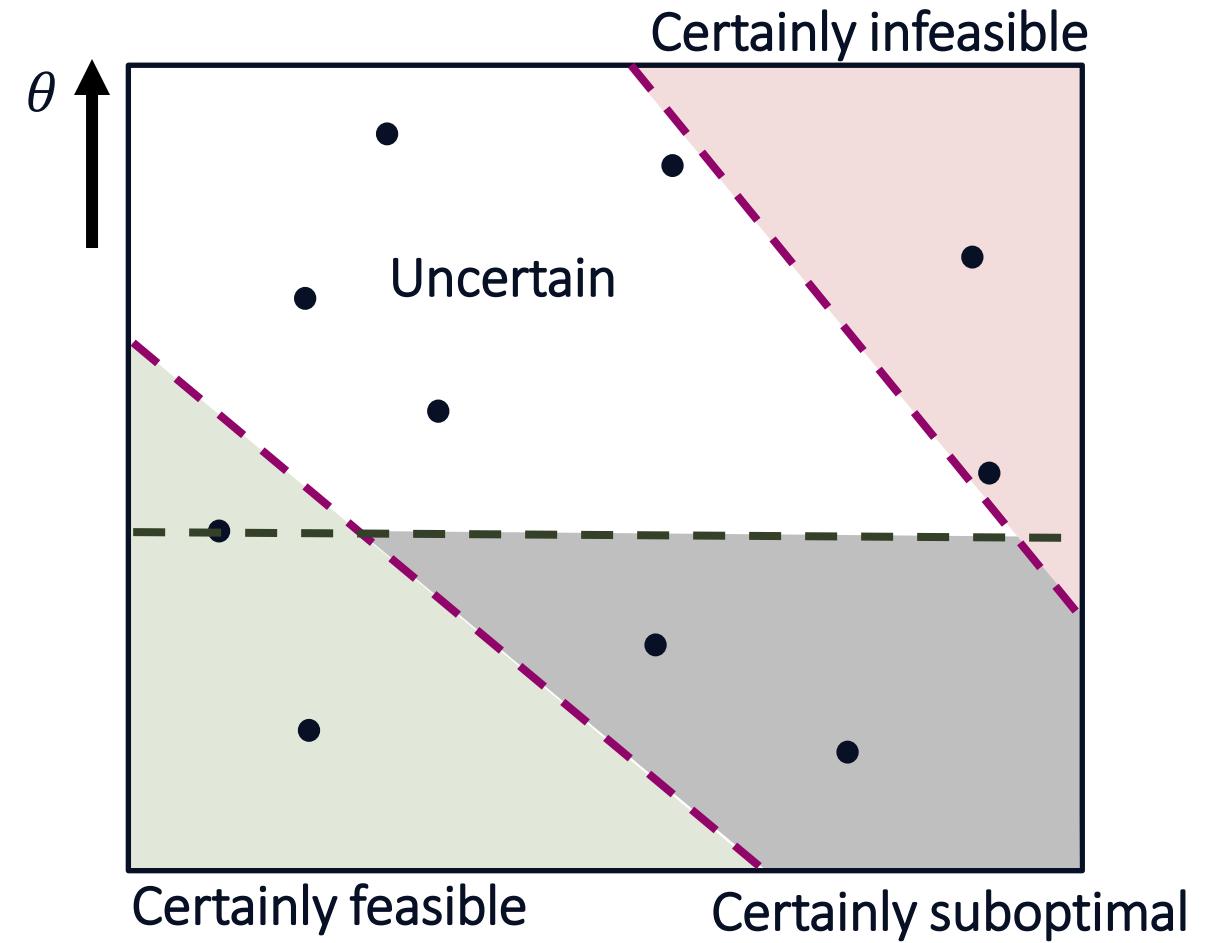
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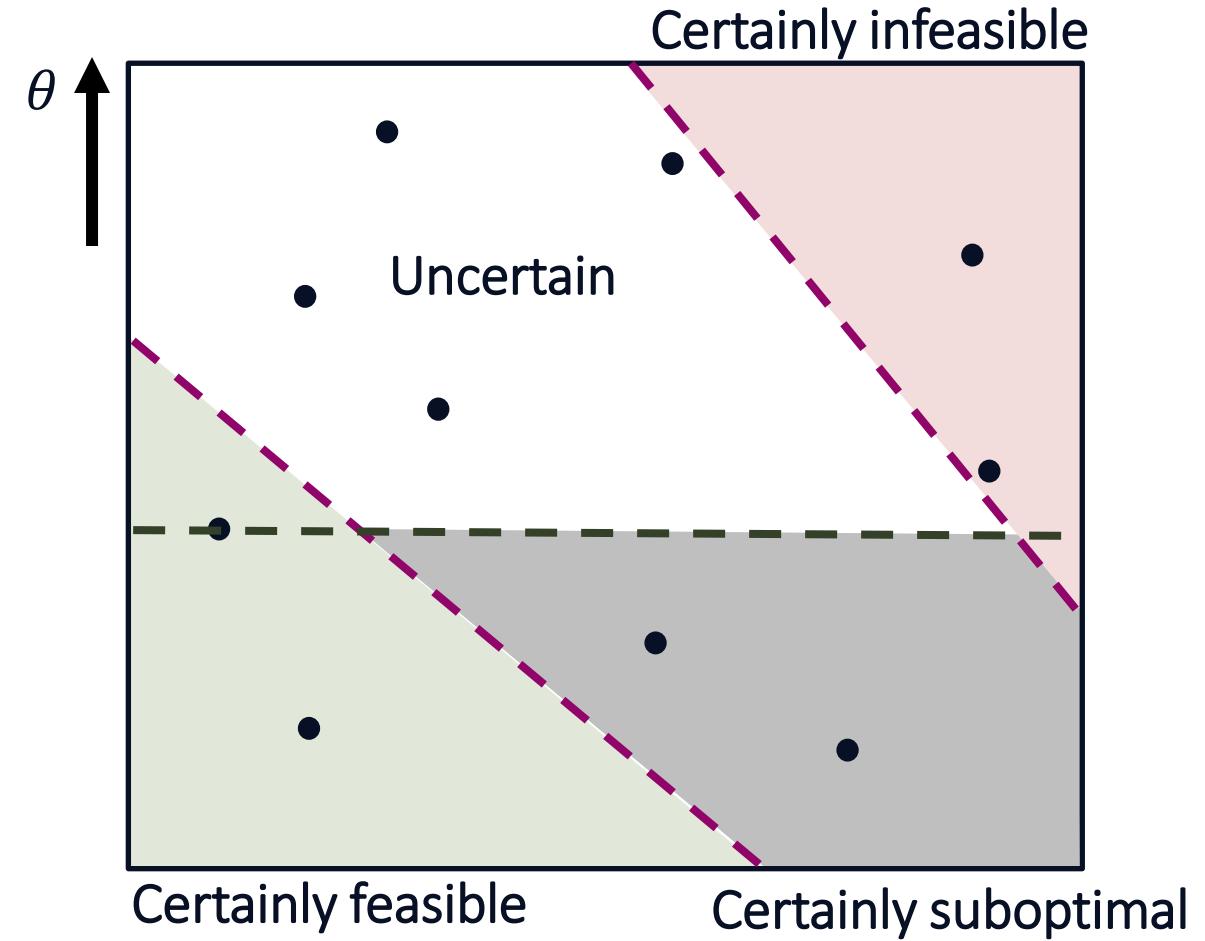


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6. Select arms to reduce uncertainty about uncertain arms

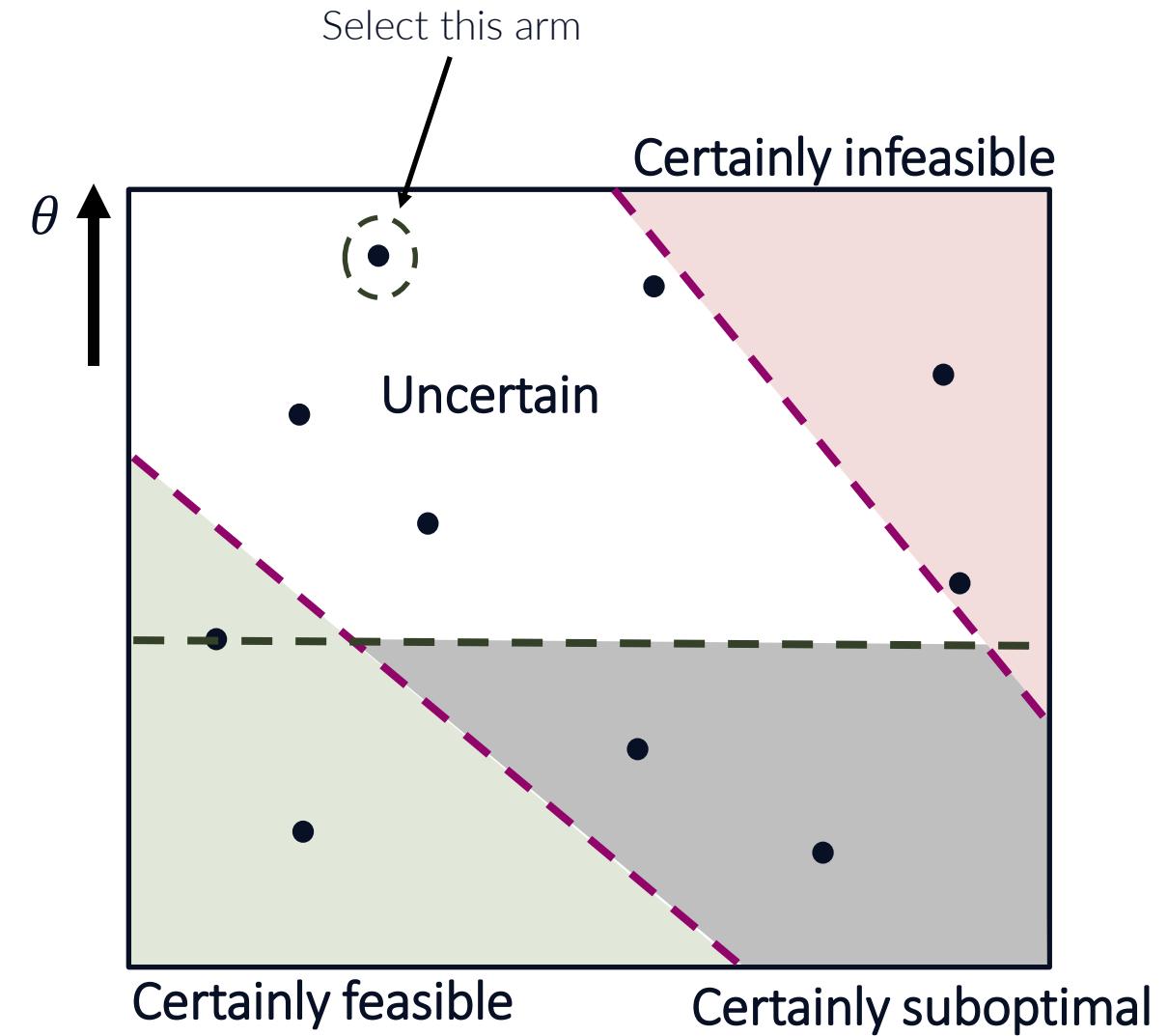
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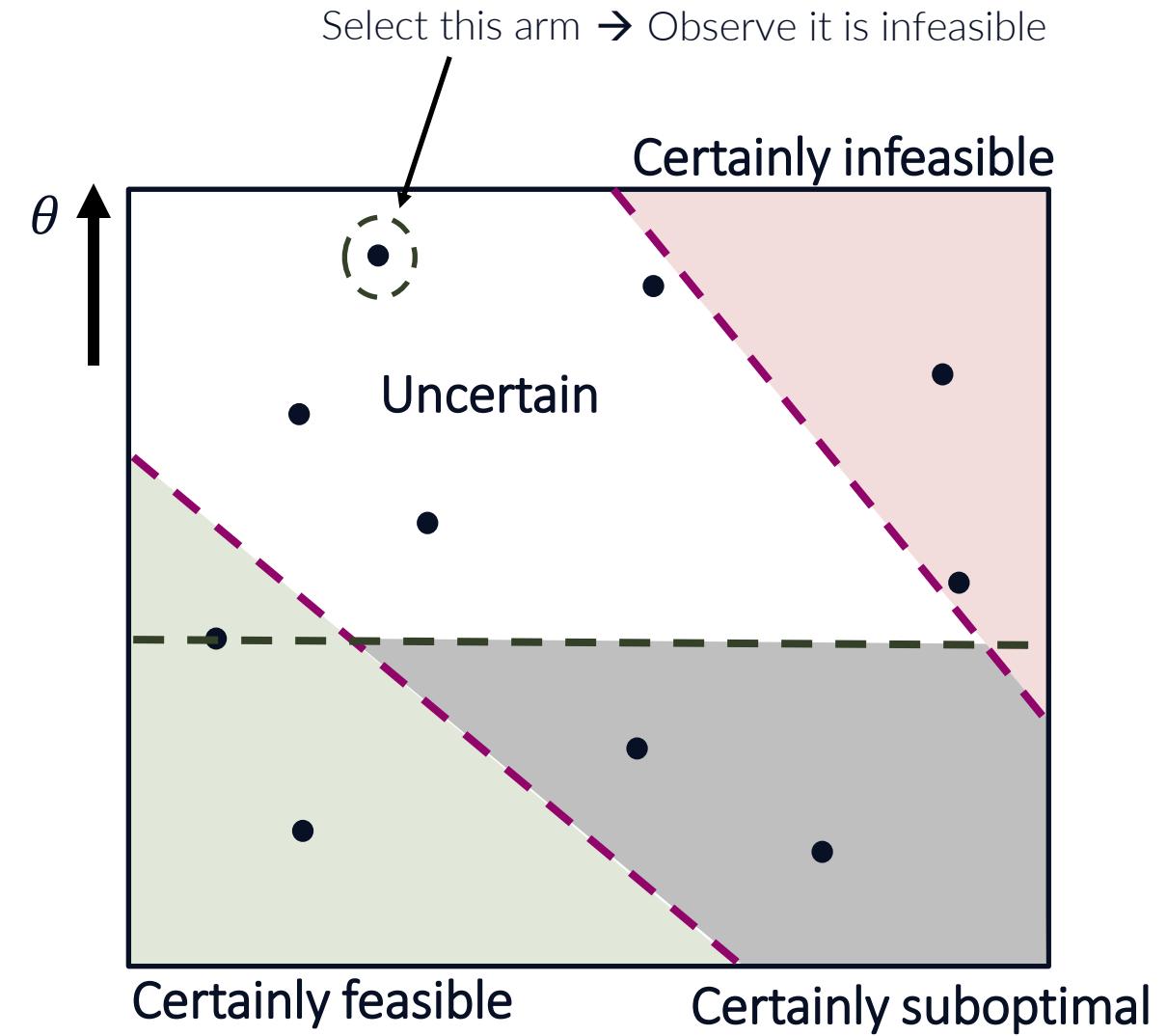
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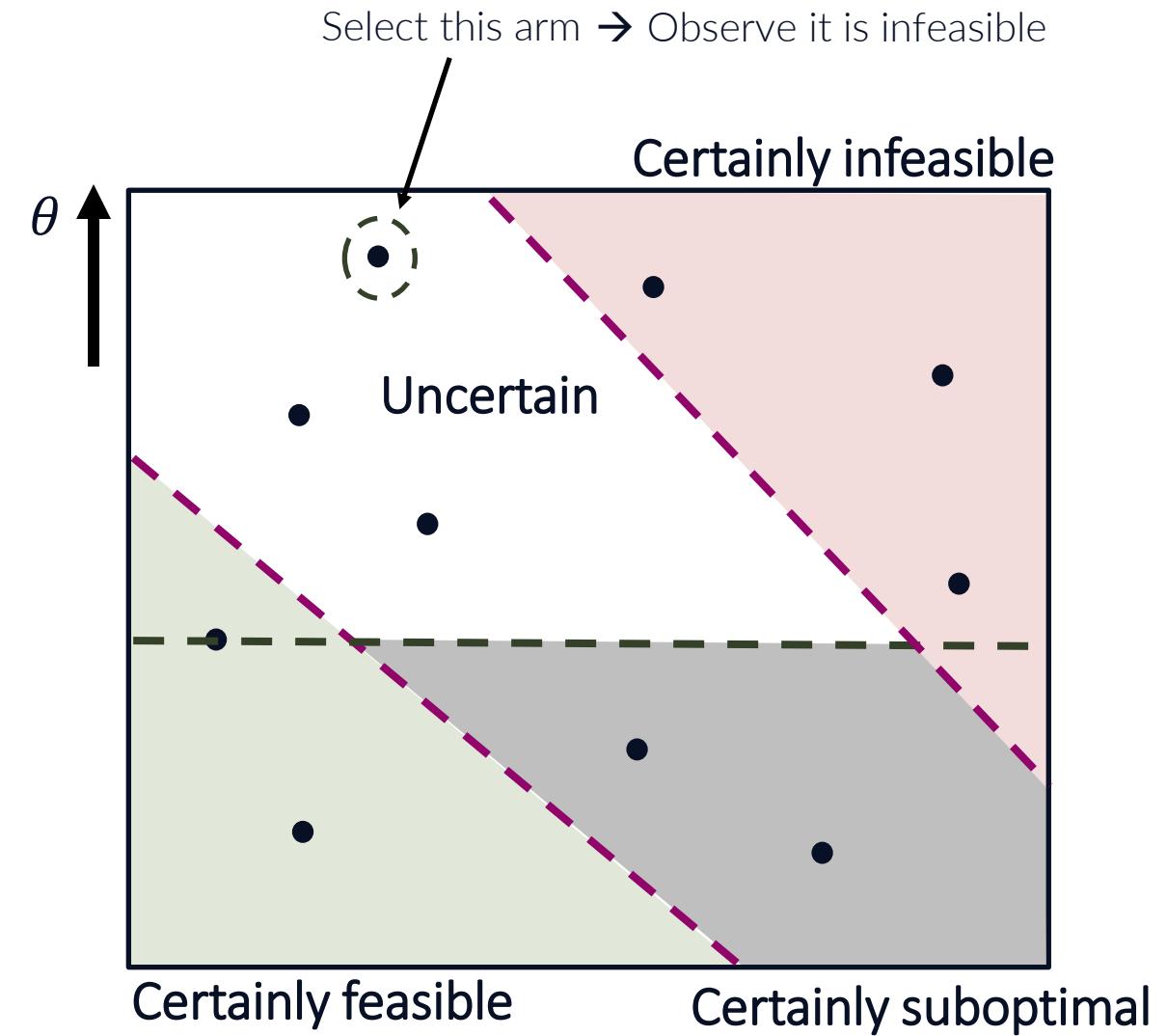
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CBAI lower bound (informal)

The number of samples necessary to solve a CBAI problem is lower bounded by

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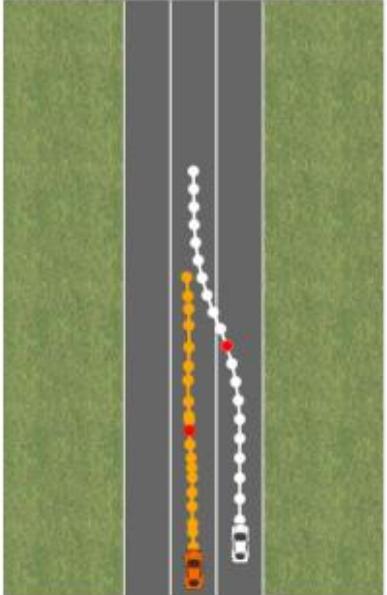
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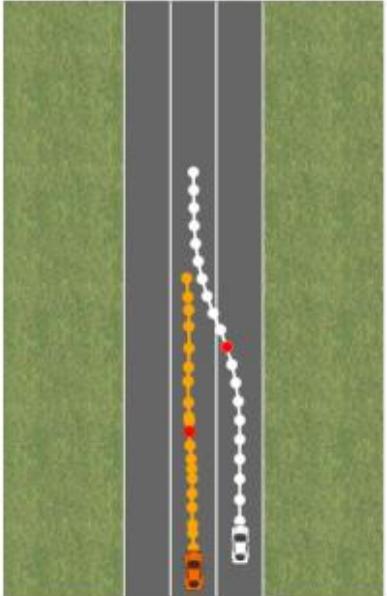
Adaptive constraint learning works well in simulations

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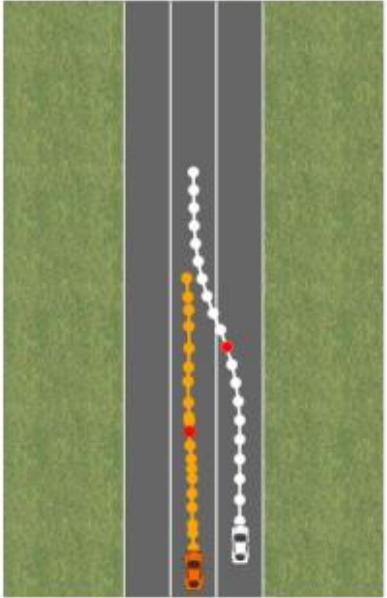
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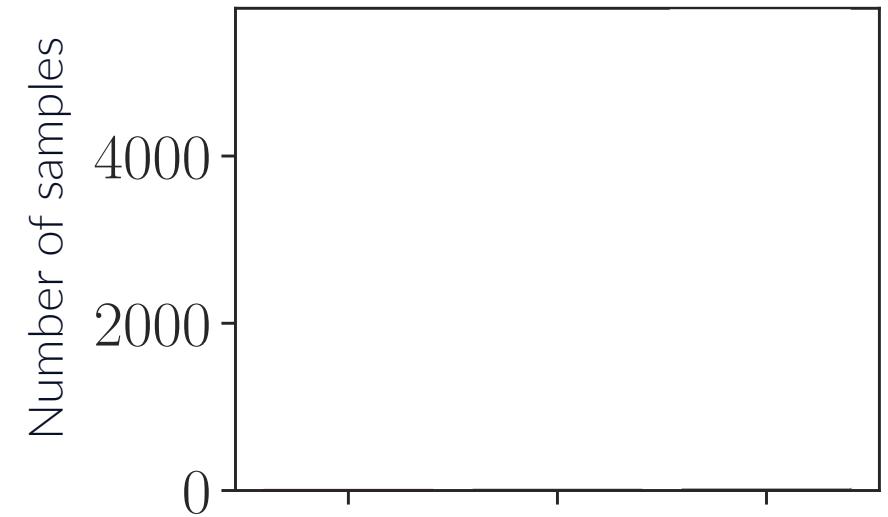


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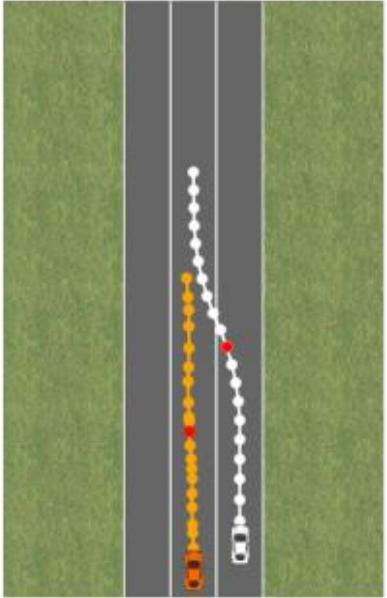
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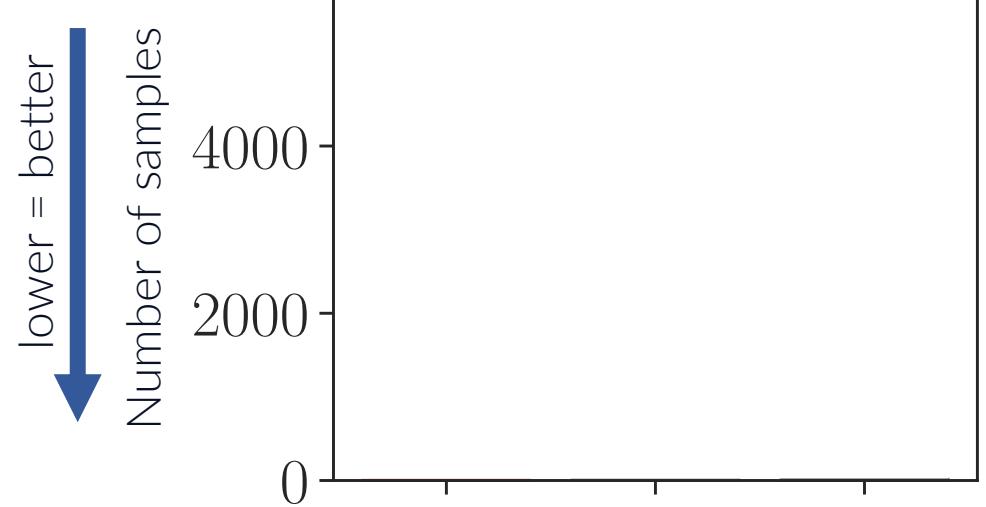
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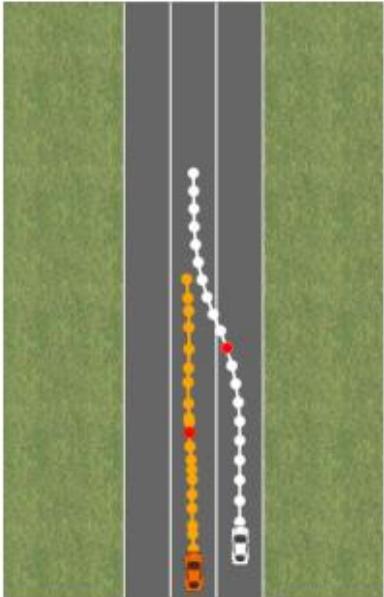
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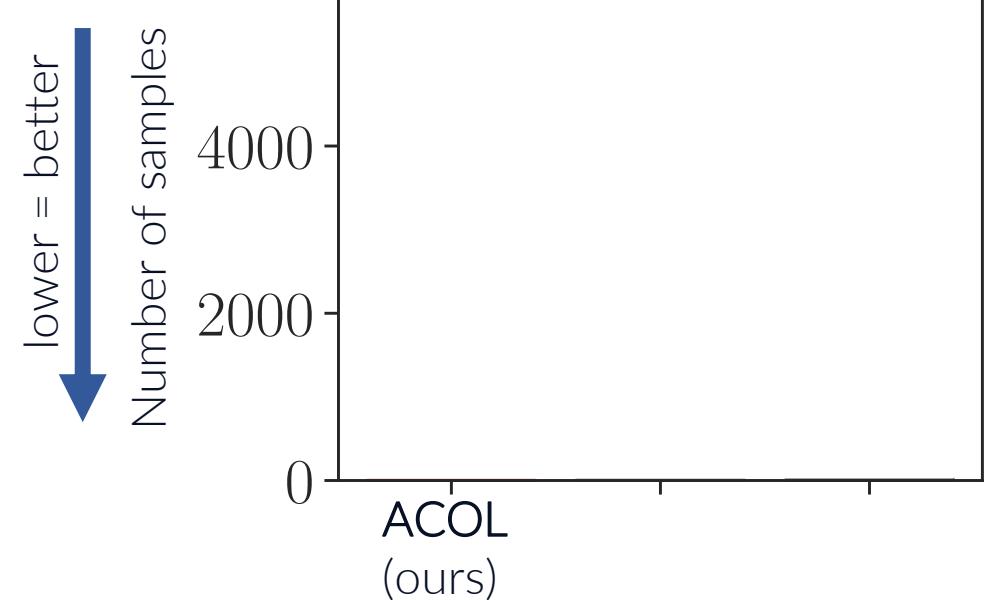
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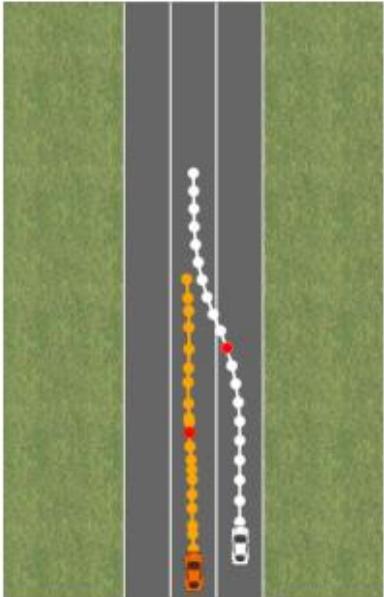
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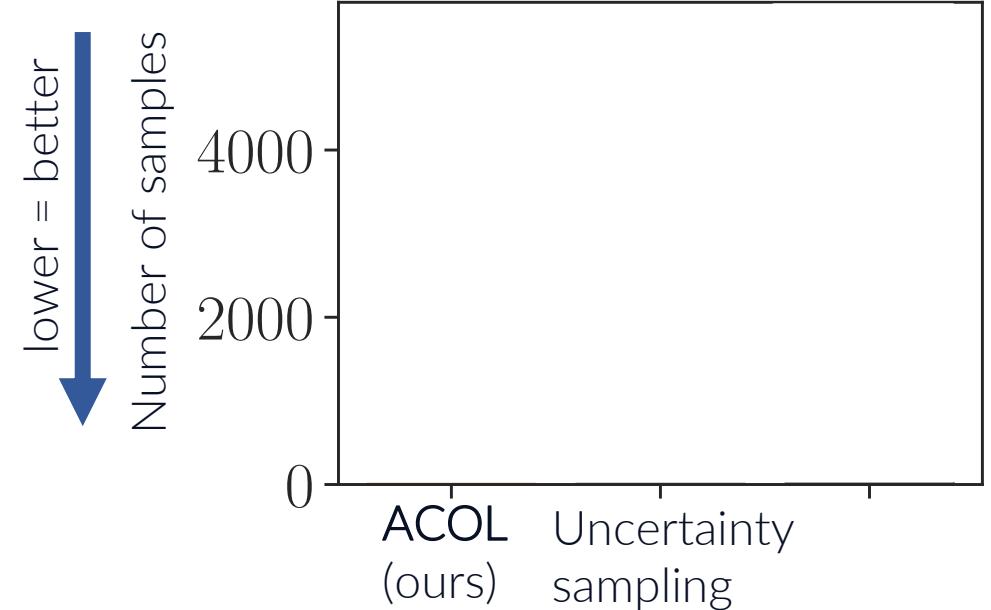
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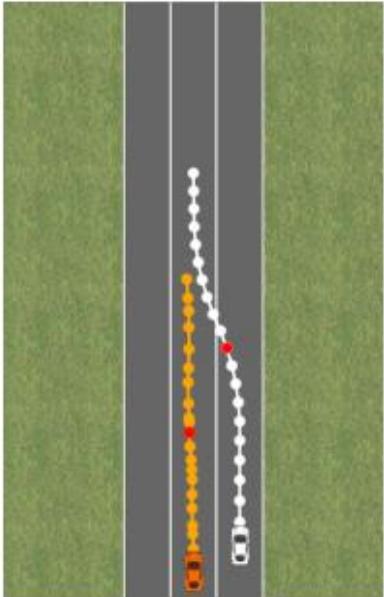
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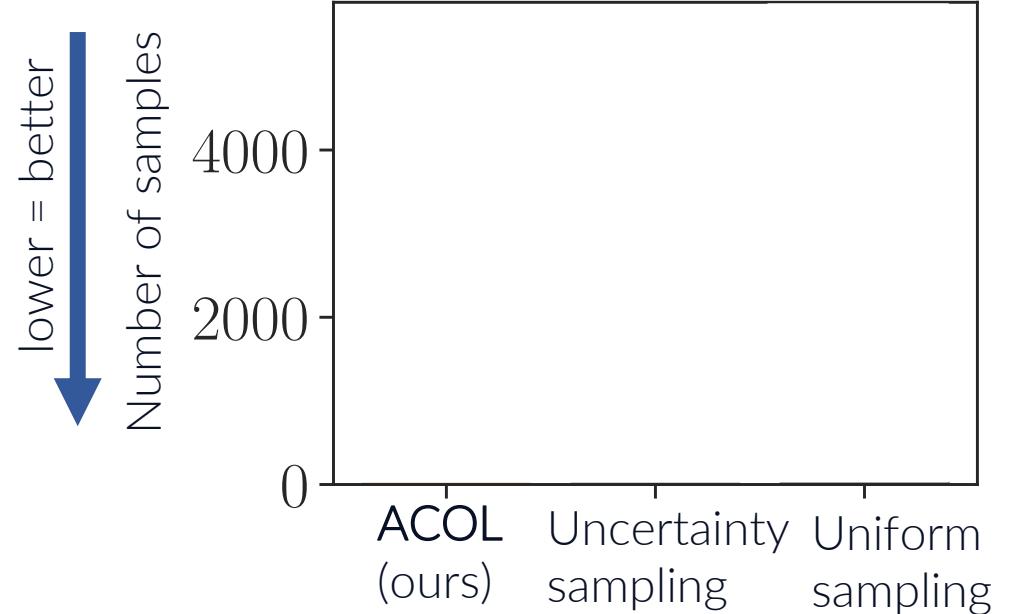
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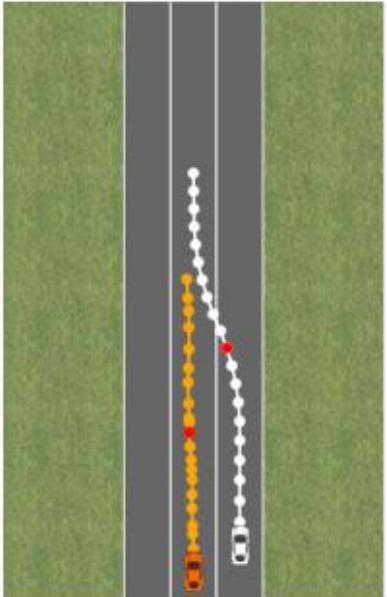
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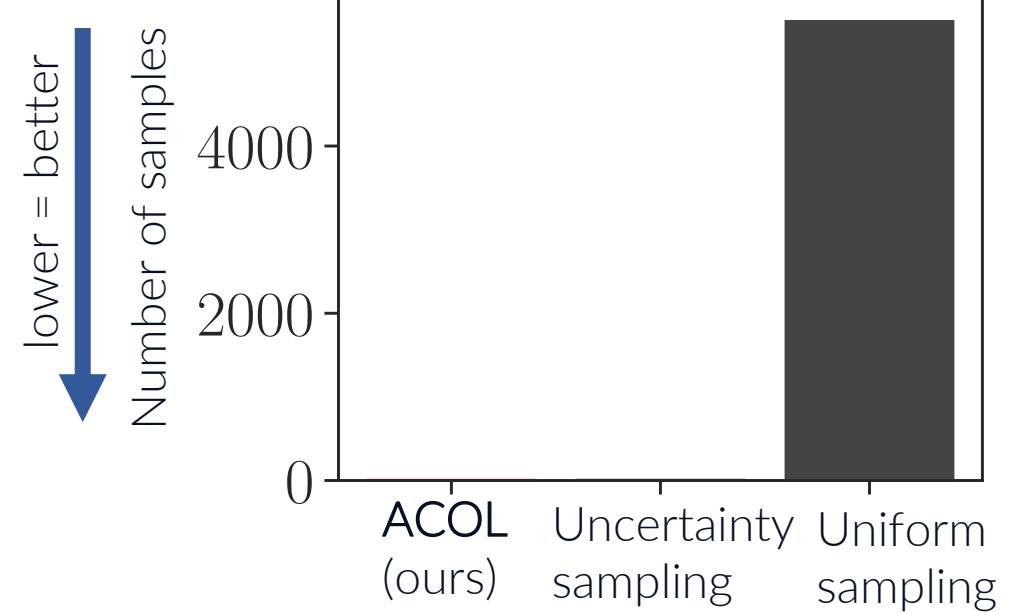
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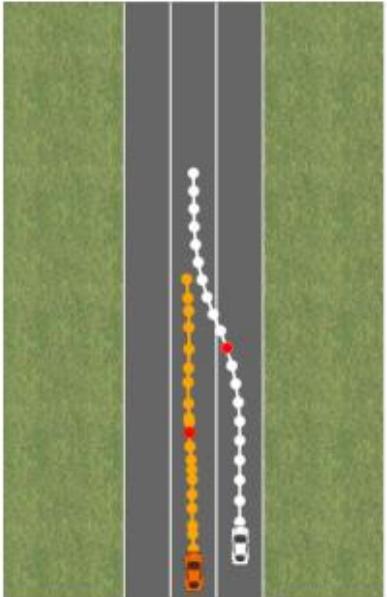
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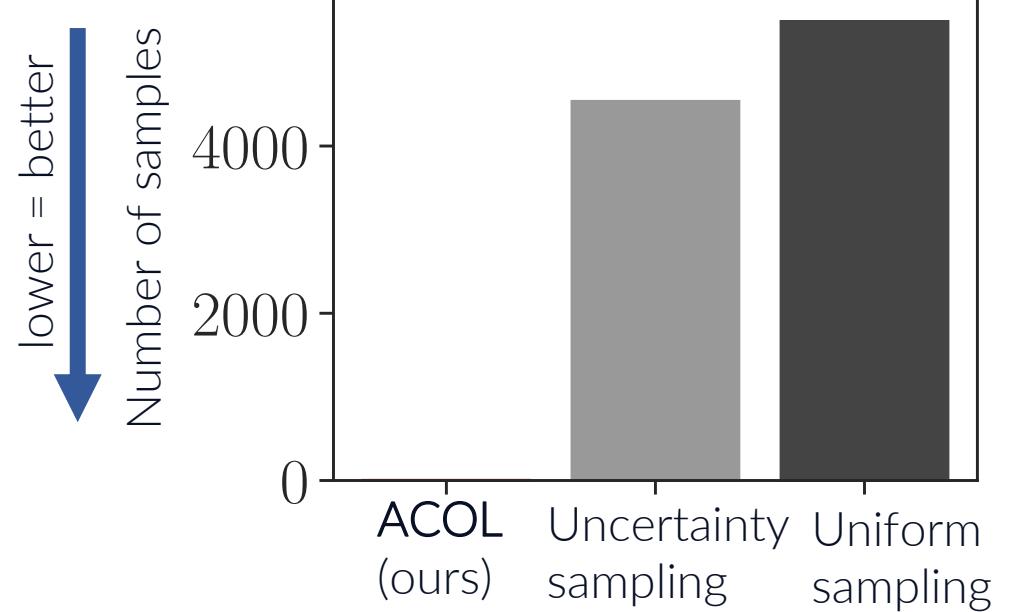
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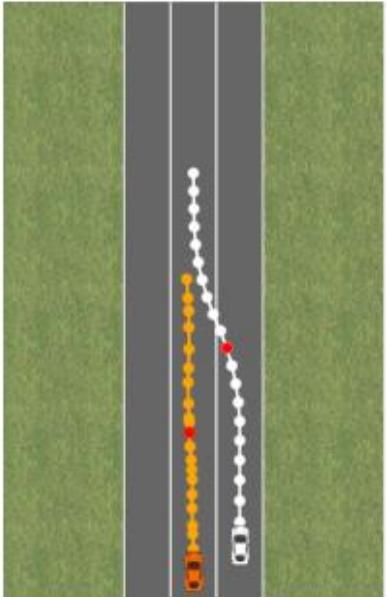
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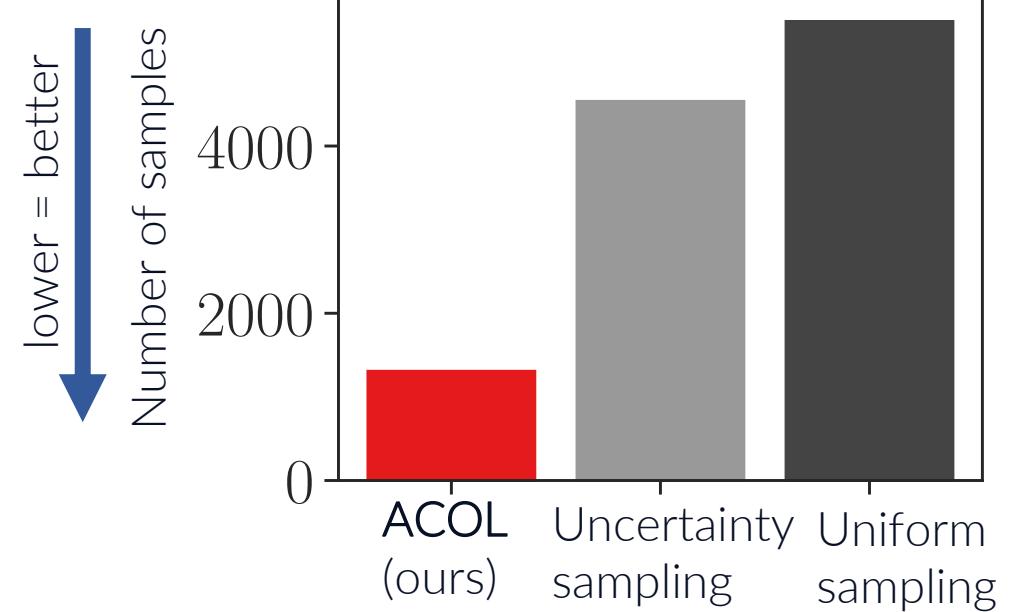
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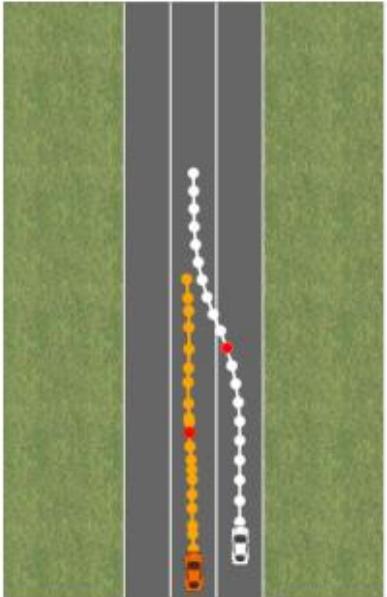
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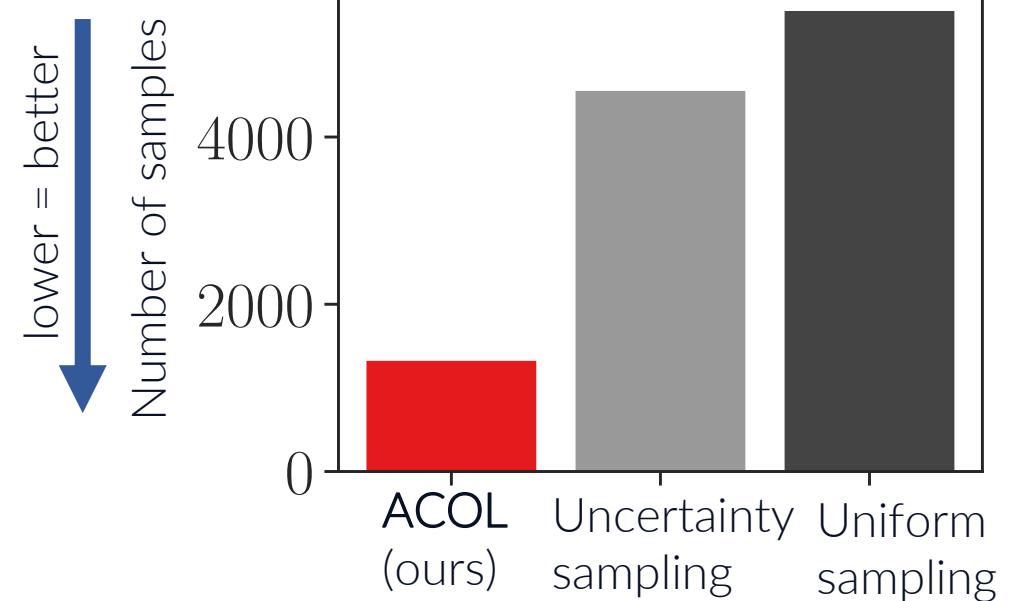
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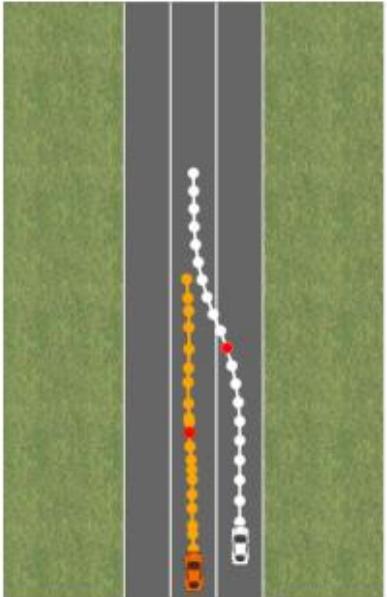


- Driving simulation with known reward but unknown constraints
- We obtain binary observations whether a trajectory is feasible
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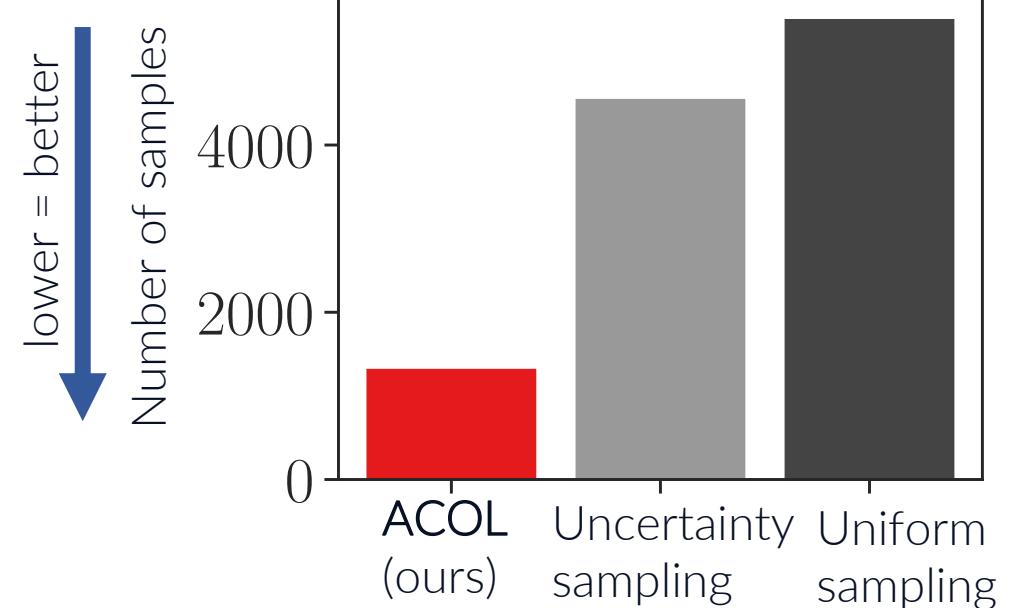


- Constraints are a natural and robust way to represent human preferences

Adaptive constraint learning works well in simulations

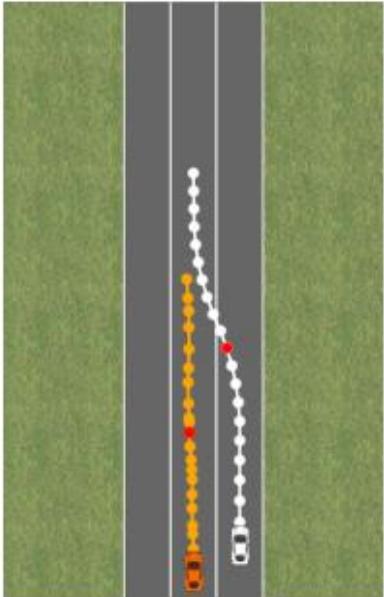


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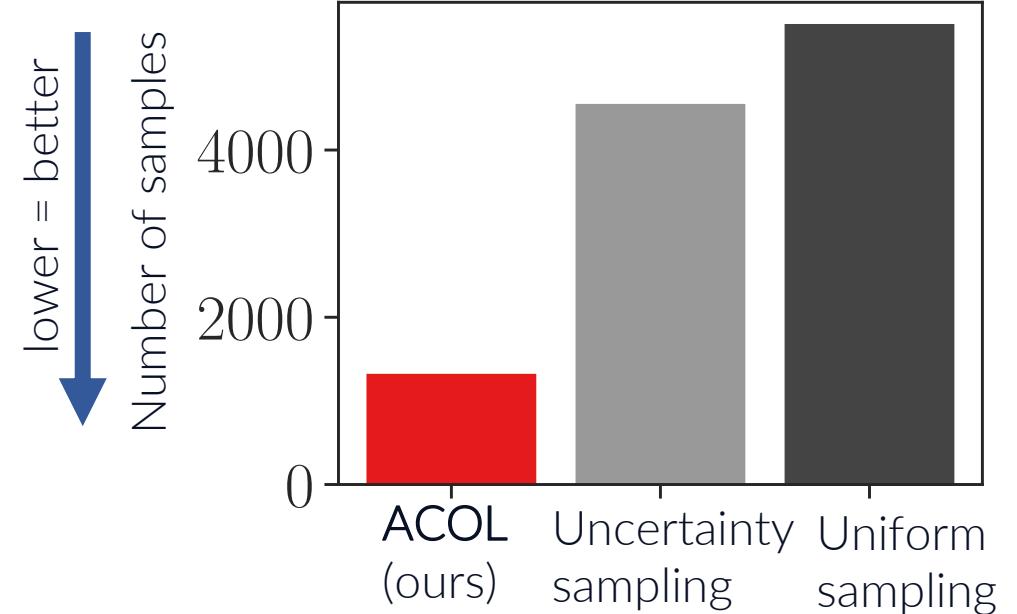


- Constraints are a natural and robust way to represent human preferences
- Adaptive Constraint Learning can efficiently learn constraints in linear bandits

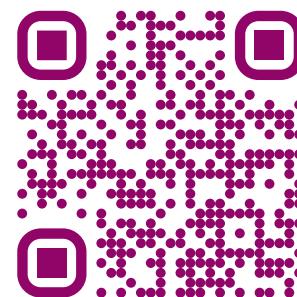
Adaptive constraint learning works well in simulations



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