Continuous-Time Modeling of Counterfactual Outcomes Using Neural Controlled Differential Equations

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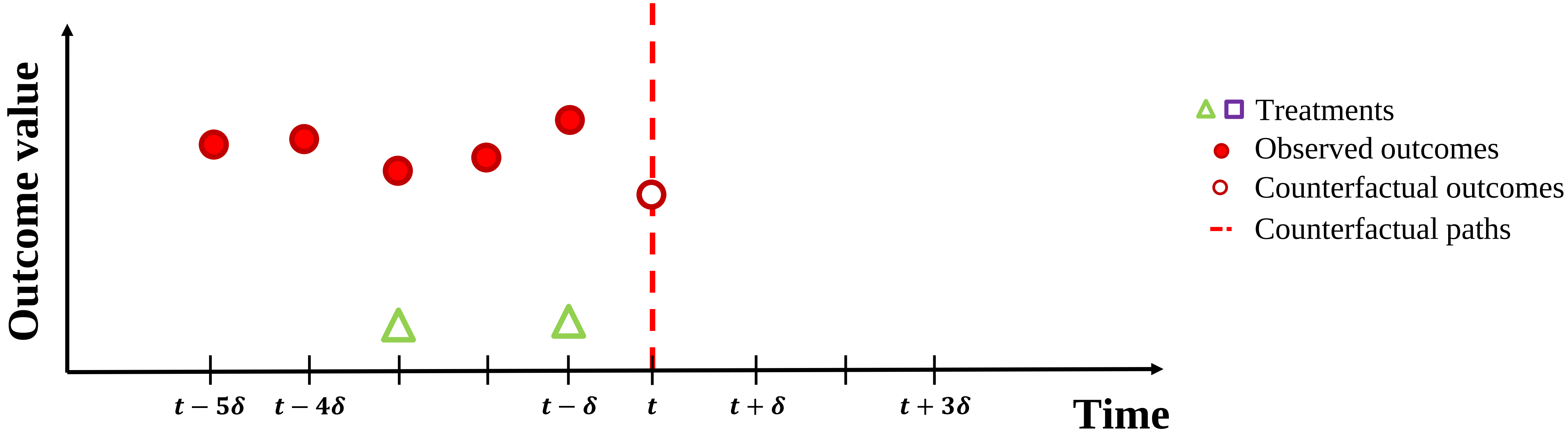
Treatment effects over time

- **Aim?** Estimate **counterfactual outcomes** for an individual given their history under a future sequence of treatments



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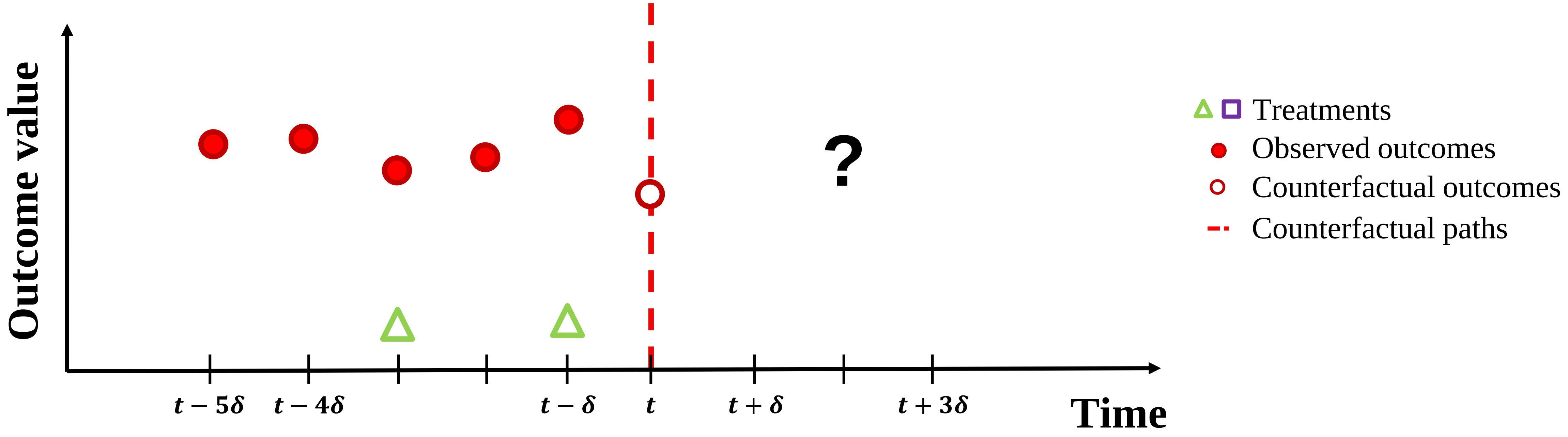

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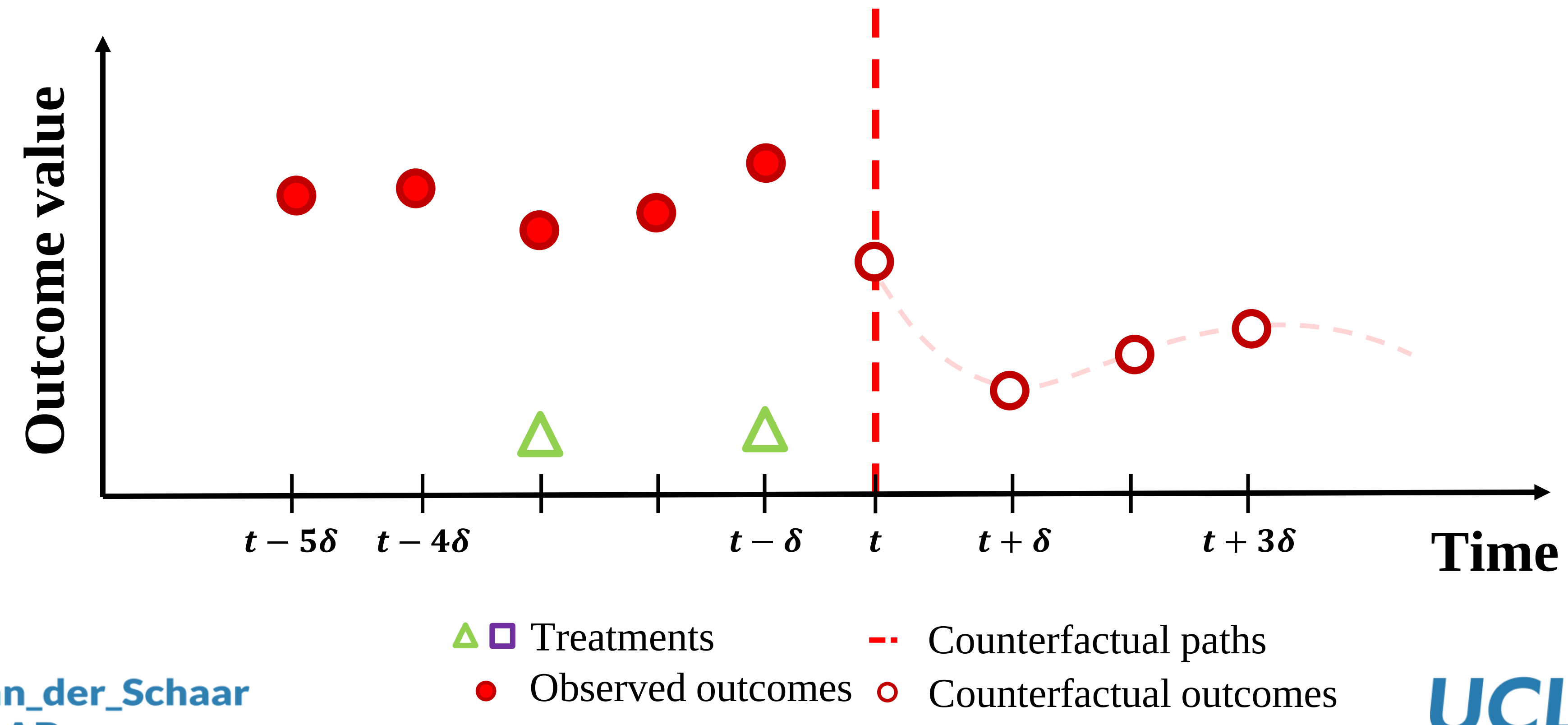

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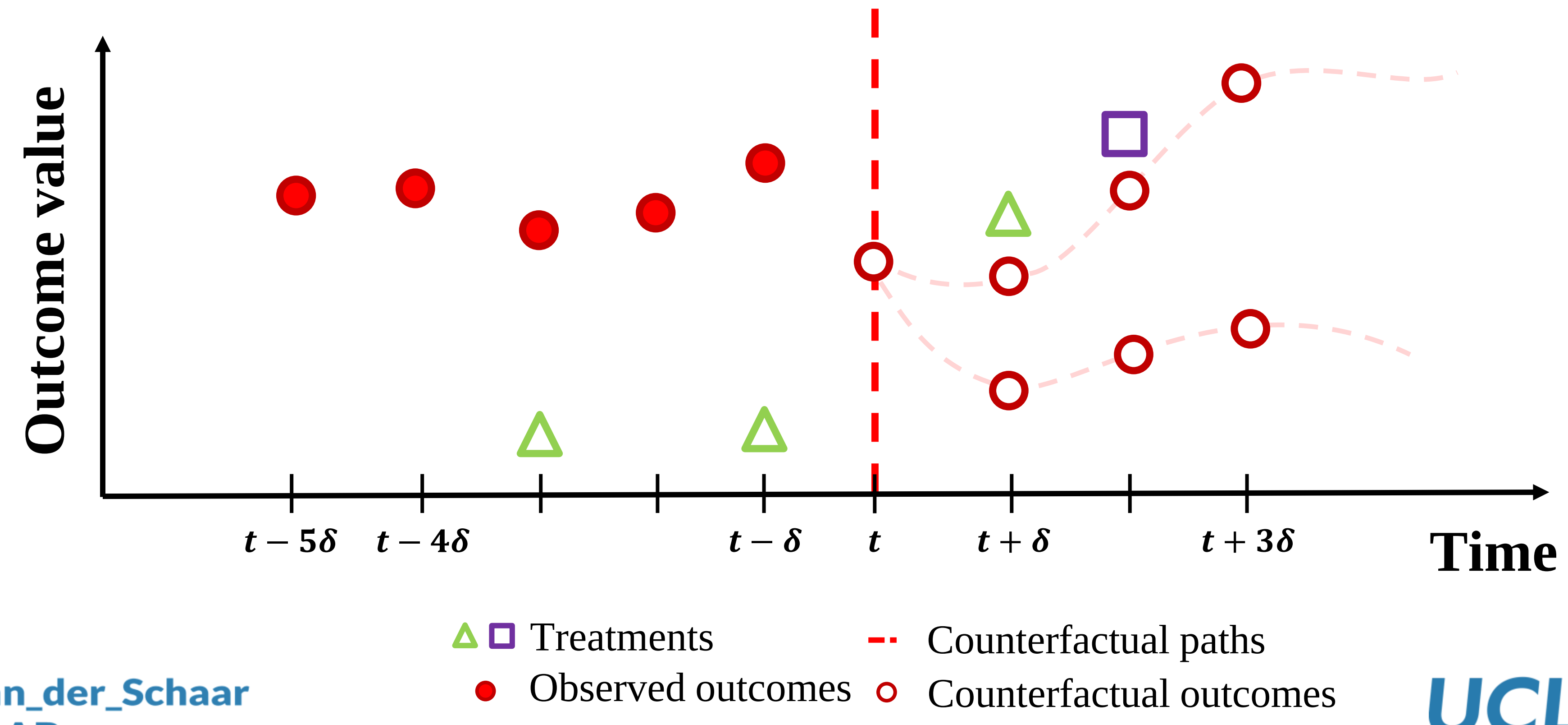

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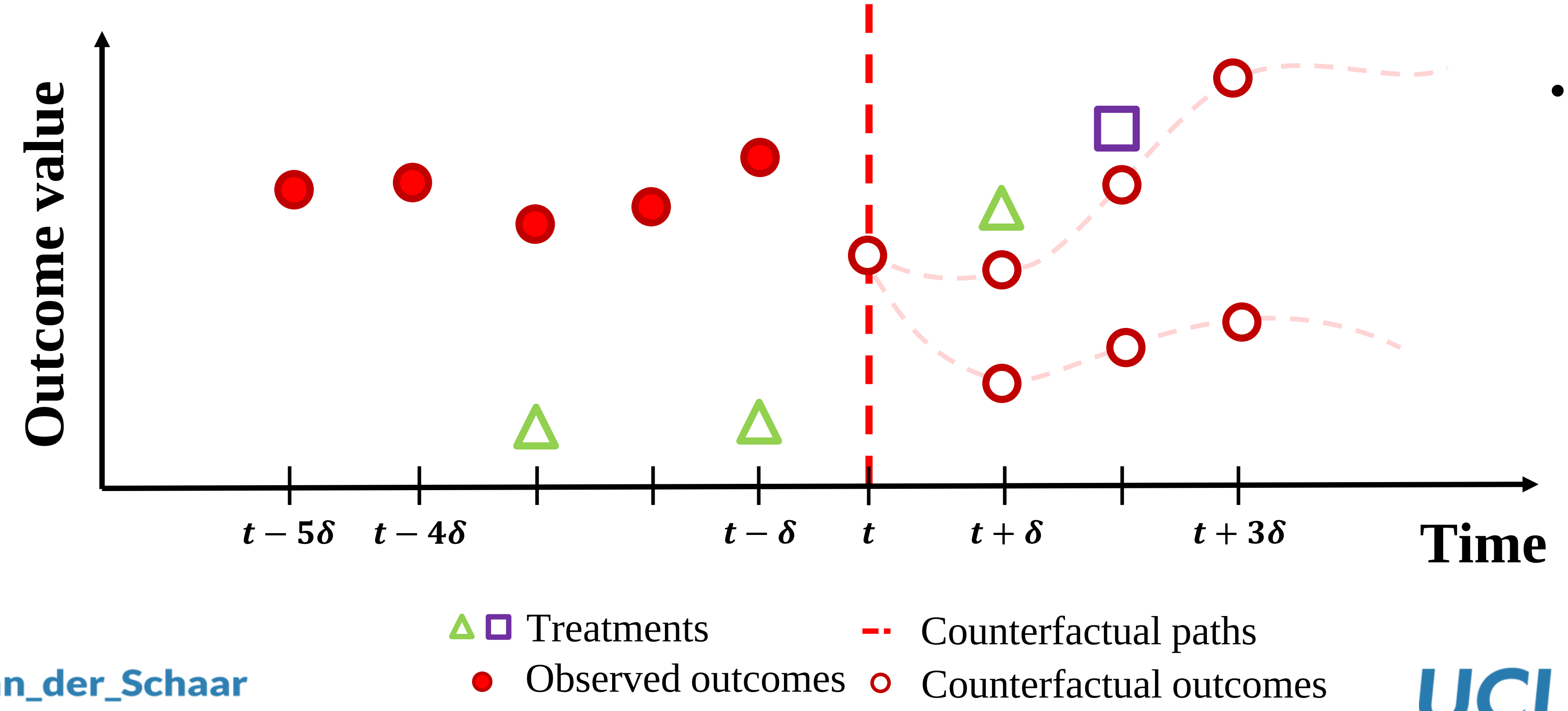

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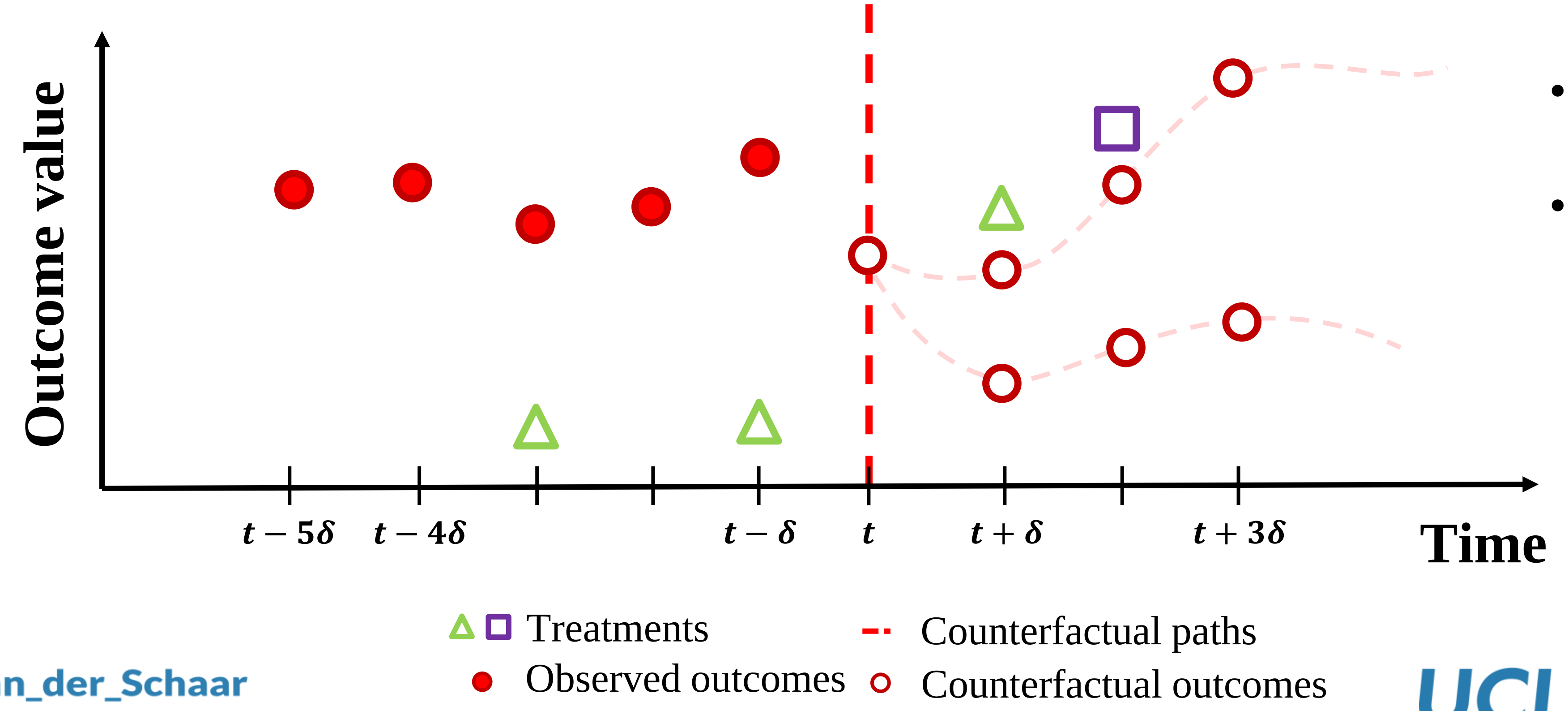

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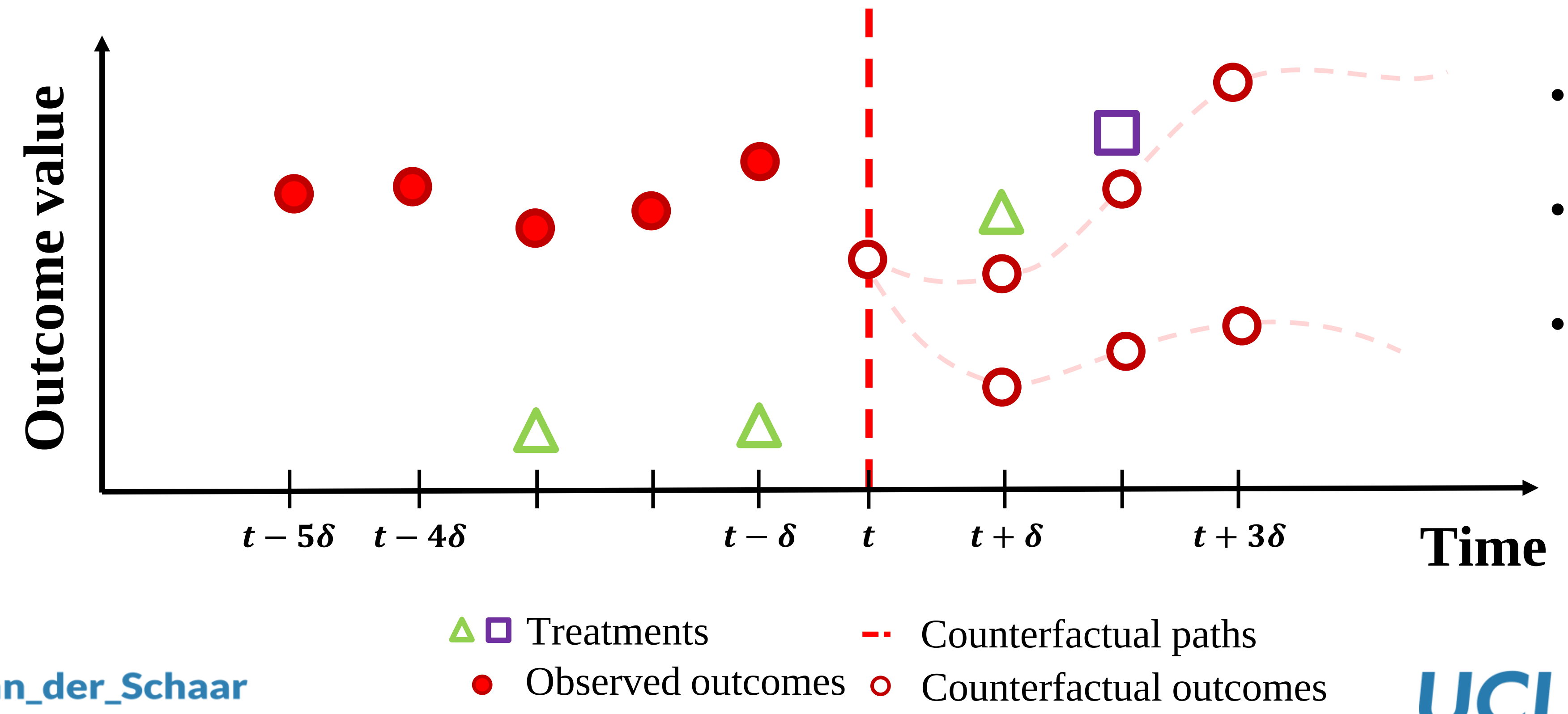

- **Whether to treat?**
- **How to treat?**
- **When to treat?**

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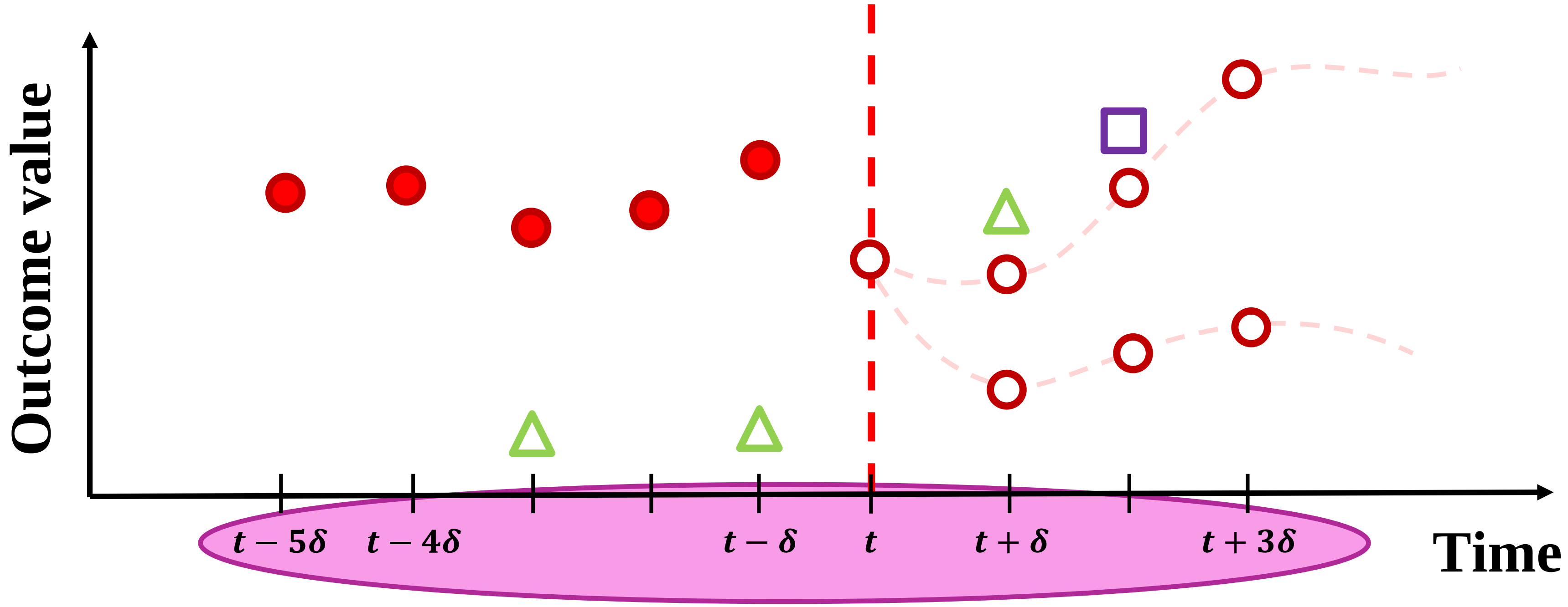
Treatment effects over time

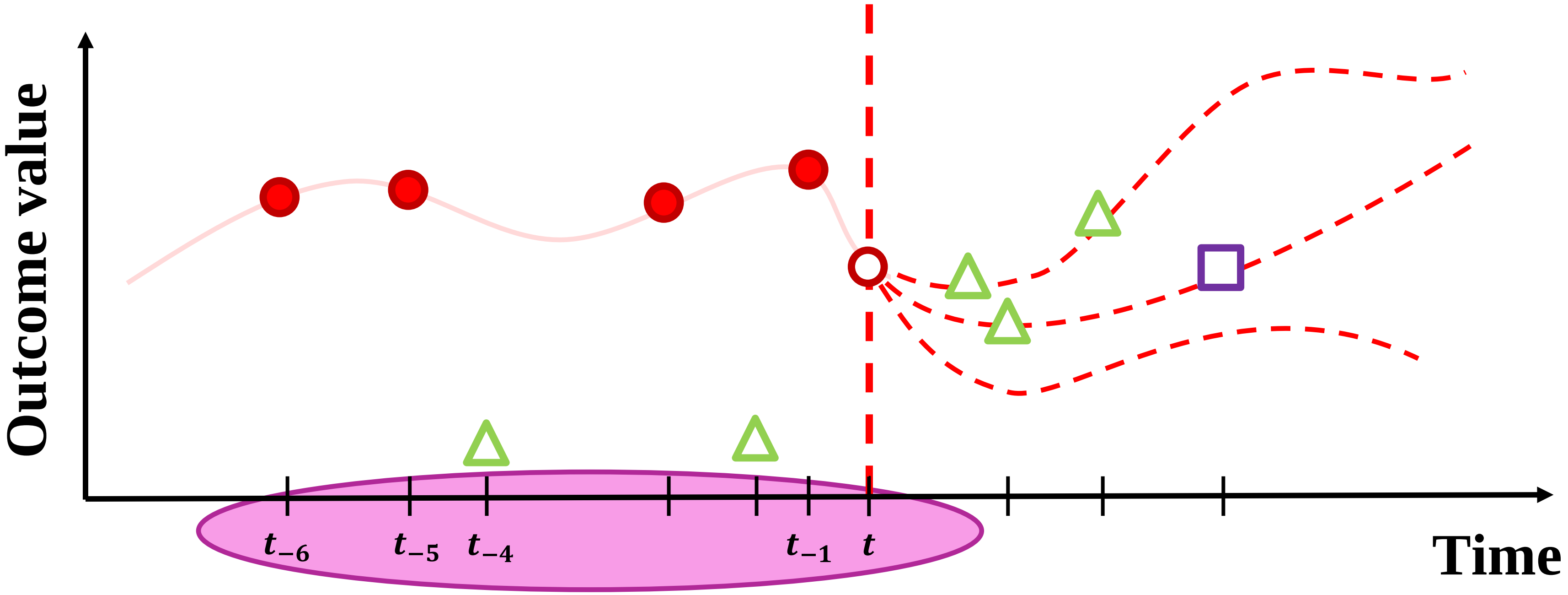
- **Existing work -** Limited to **fixed, regular** time intervals between observations



Observations are typically **NOT** at regular intervals

Treatment effects over time

- **Irregular sampling:** Both patient history and future treatment plans



But instead observed **IRREGULARLY**

Treatment effects over time

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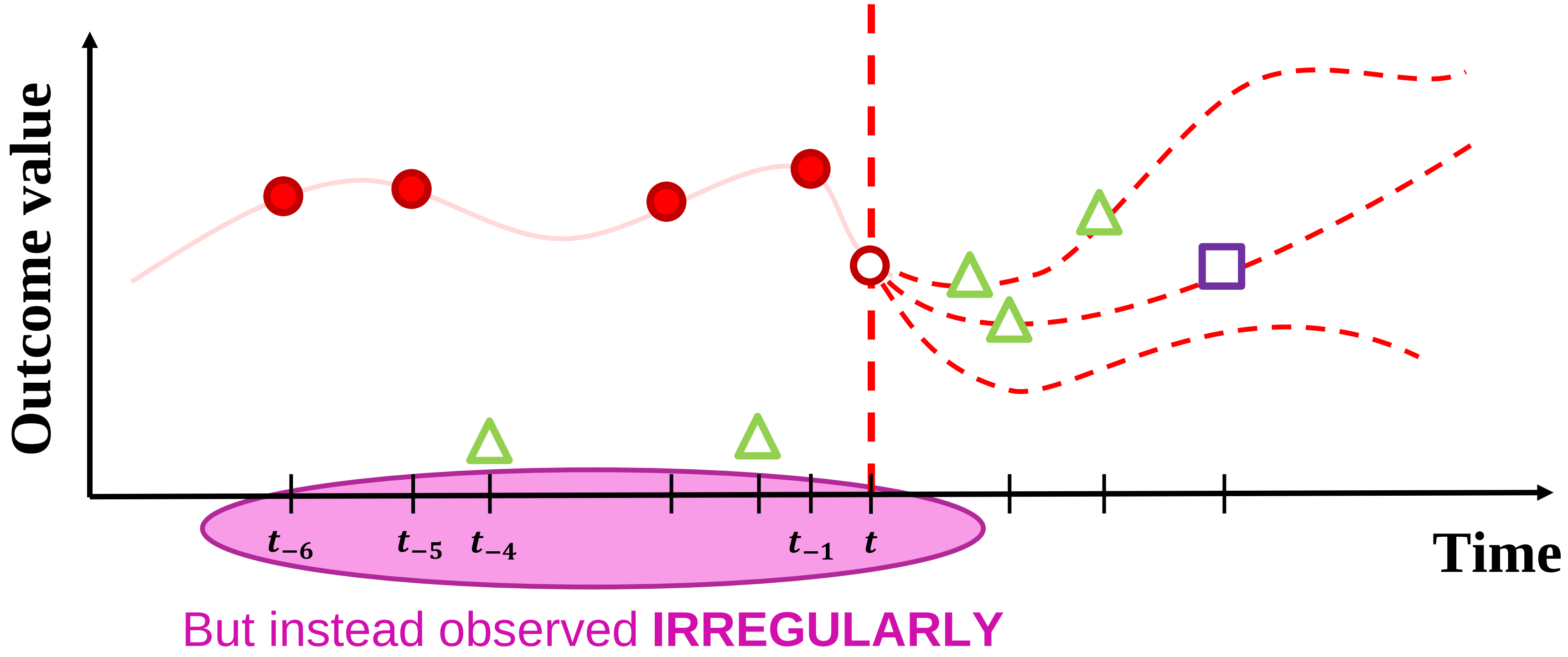
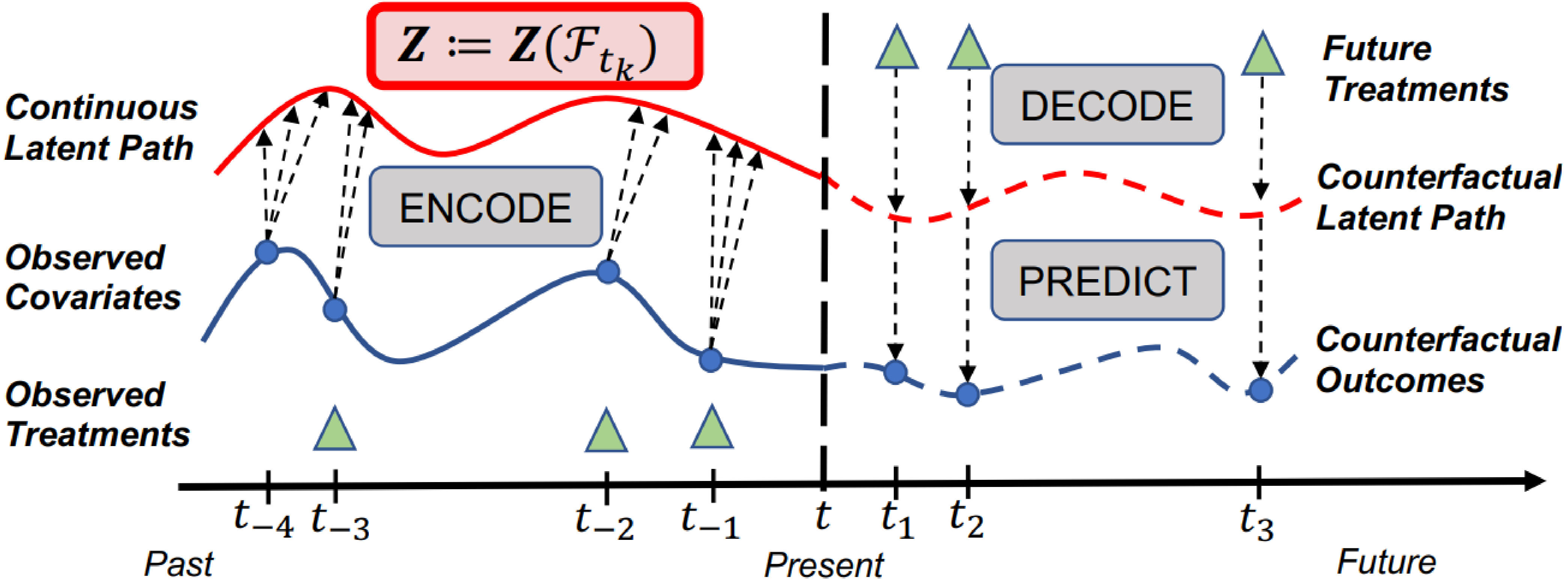


How can we learn from such observation patterns?

Proposed Method: TE-CDE

- **Solution:** Treatment Effect Neural Controlled Differential Equation (TE-CDE)
- **Key idea:** Learn a *continuous latent representation* of the patient state as the solution to a CDE
- **Time-dependent confounding:** Address via domain adversarial training (Ganin et al., 2016)



Simulation Environment: Cancer Tumor Growth

Not possible to observe counterfactuals, thus we require a simulation environment.

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- Lung cancer tumor growth model (Geng et al., 2017):

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- Flexible observation process parametrized by a Hawkes process (Hawkes, 1971):

$$\lambda(t, s_t = i) = \lambda_i^o + \sum_{\tau < t_m < t} e^{-2(t - t_m)},$$

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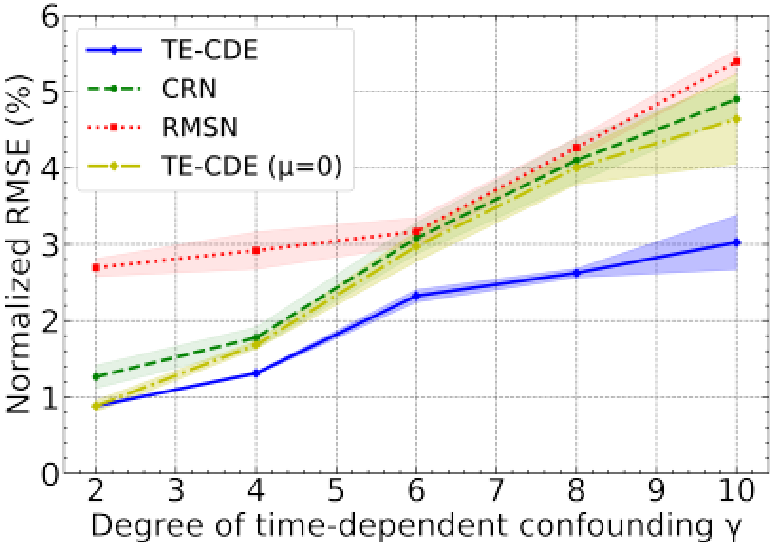
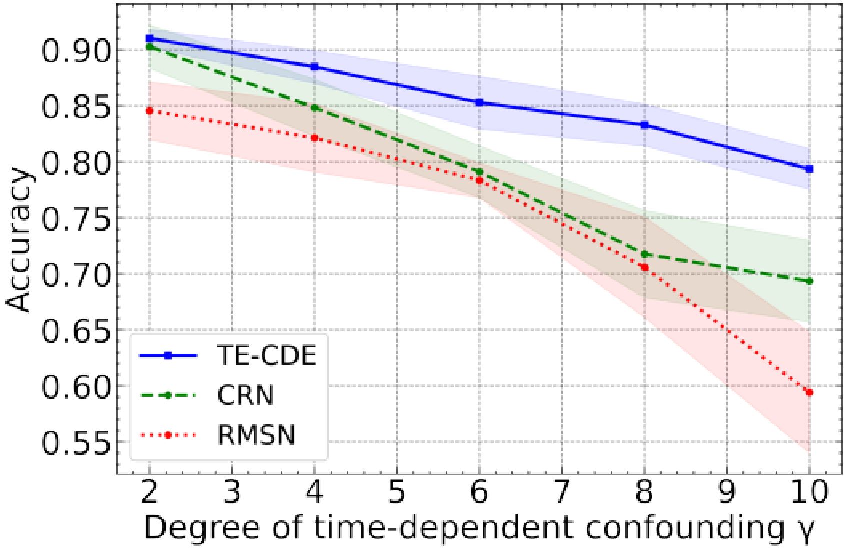
State-dependent
(based on AJCC
cancer stages)

Experiment Results

- Outperform state-of-the-art **discrete-time** baselines CRN (Bica et al., ICLR 2020) and RMSN (Lim et al., NeurIPS 2018)
- Demonstrate importance of **domain adversarial training** ($\mu = 0$)
- Utility for both **counterfactual estimation** and **treatment accuracy**



Counterfactual estimation



Treatment accuracy

Contributions

- Extend counterfactual estimation over time to the **irregularly sampled setting**
- Introduce a **simulation environment** for irregularly sampled observations
- **Propose TE-CDE** as a solution, modeling for the first time a patient's (latent) trajectory as the solution to a CDE
- Demonstrate utility of our approach for both **counterfactual estimation** and **treatment accuracy**

Resources & Acknowledgements

Paper

Code

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ONR
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Cystic Fibrosis Trust