The Multivariate Community Hawkes Model for Dependent Relational Events in Continuous-time Networks

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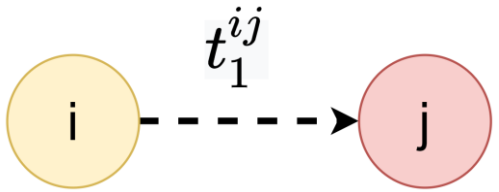
Motivation

- The Multivariate Community Hawkes (**MULCH**):
  - Flexible model for continuous-time networks that introduces dependence between node pairs in a controlled manner.

- **Continuous-time networks:**
  - Events between node pairs (i, j) at time t^{ij}
  - Ex: messages between users on social media.



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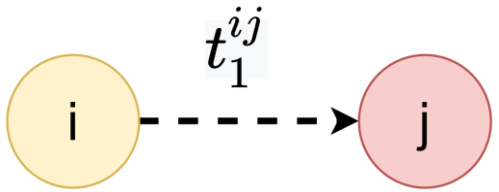

- **Goal:** Analyze interactions between nodes and simulate realistic networks.

Background: Multivariate Hawkes Process

- Multivariate Hawkes Process :
  - **Temporal point process** generates a sequence of timestamps $\{t_1, t_2, \dots,\}$.
  - Self and Mutually-exciting temporal processes.
  - Event in one process excites future events in self and other processes.

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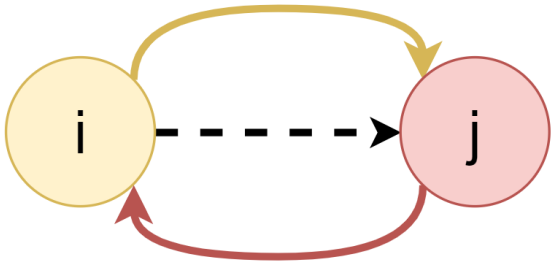
- Network Setup
  - node pairs (i, j) = processes
  - An Event between (i, j) (dashed) →future events in both (i, j) and (j, i) (solid).



$\{t_1^{ij}, t_2^{ij}, \dots, \}$

$\{t_1^{ji}, t_2^{ji}, \dots, \}$

Background: Multivariate Hawkes Process

- **Intensity function:** instantaneous rate of events at time t

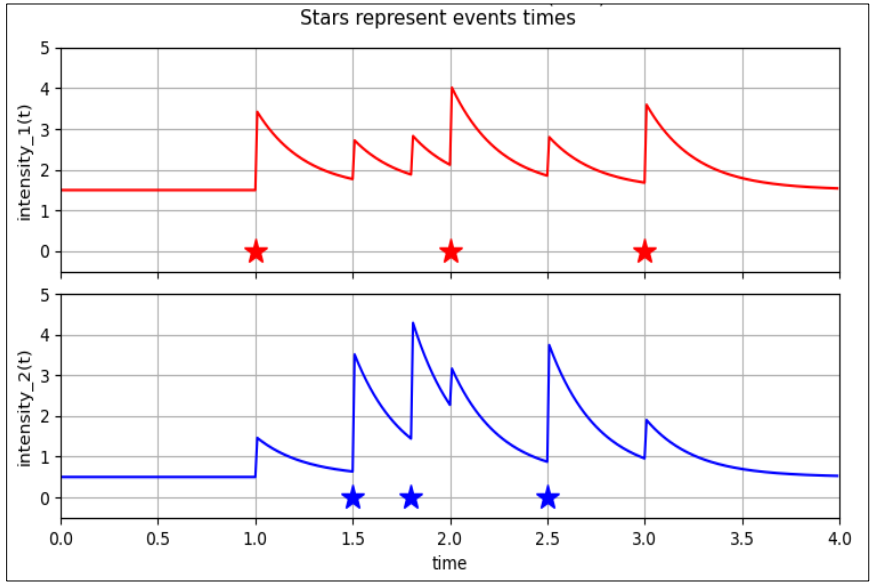
- Intensity of a node pair (i, j) – exponential kernel:

$$\lambda_{ij}(t) = \mu_{ij} + \sum_{(x,y)} \alpha^{xy \to ij} \sum_{s:\, t_s^{xy} < t} \beta e^{\beta(t - t_s^{xy})}$$

μ_{ij} : baseline intensity

$\alpha^{xy \to ij}$: excitation from (x, y) to (i, j)

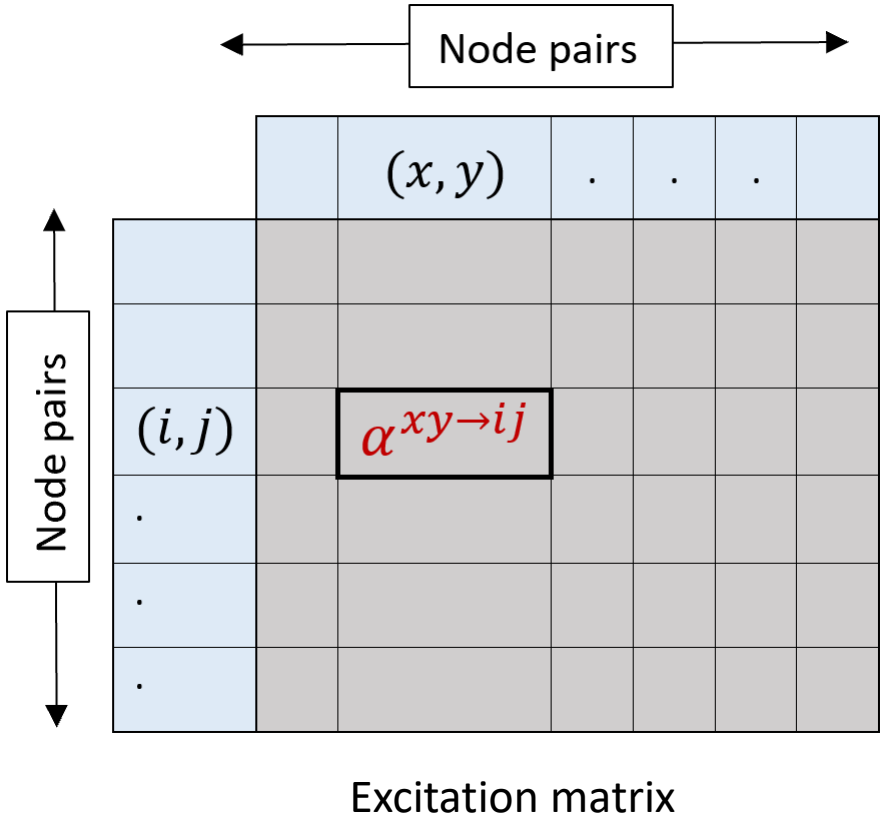
β : exponential decay rate



Problem!

$$\lambda_{ij}(t) = \mu_{ij} + \sum_{(x,y)} \boldsymbol{\alpha^{xy \to ij}} \sum_{s:\, t_s^{xy} < t} \beta e^{\beta(t - t_s^{xy})}$$

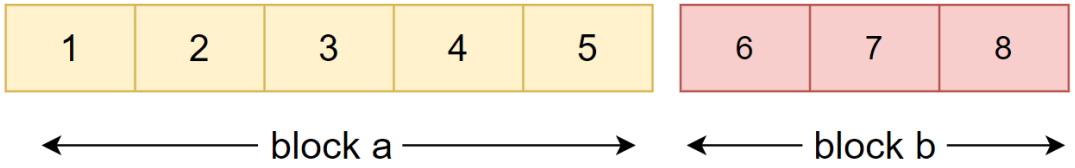
- For a directed network with n nodes:
  - Number node pairs $= n(n-1) \approx n^2$
  - **Excitation matrix $\boldsymbol{\alpha} \approx n^2 \times n^2$**
  - Intractable even for $n = 100$ nodes!



Excitation matrix

MULCH Assumptions

1. Adapted from Stochastic Block Model (SBM), assume K blocks/communities:
   - Each node $i \in$ block a
   - Each node pair $(i, j) \in$ block pair (a, b)



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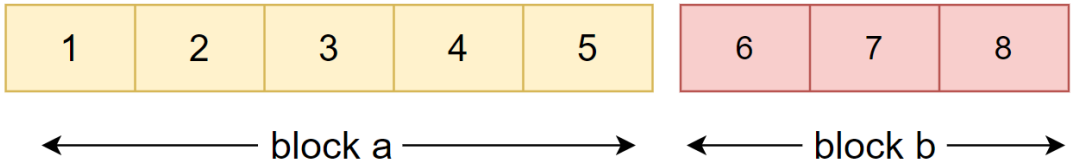
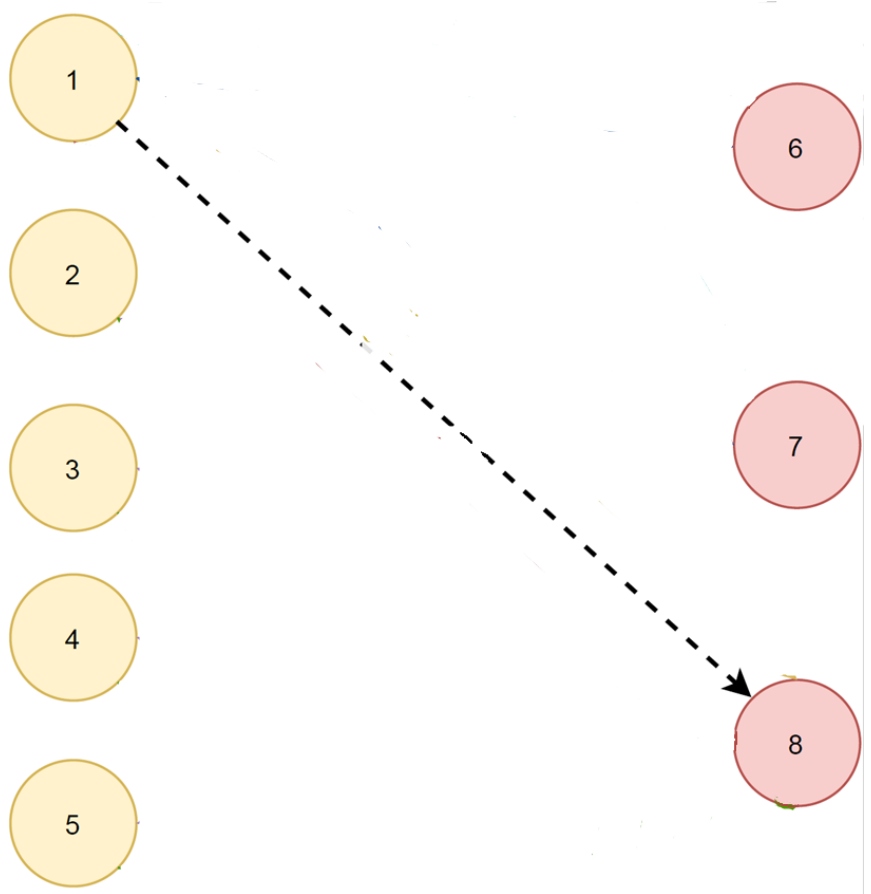


2. A node pair $(i, j) \in$ block pair (a, b) can only be excited by node pairs in block pair (a, b) & block pair (b, a)

MULCH Assumptions

3. Inspired by research in social sciences, we model **6 types of excitation:**

- Example:
  - Nodes 1-5 in block a
  - Nodes 6-8 in block b
  - Message between (1,8)



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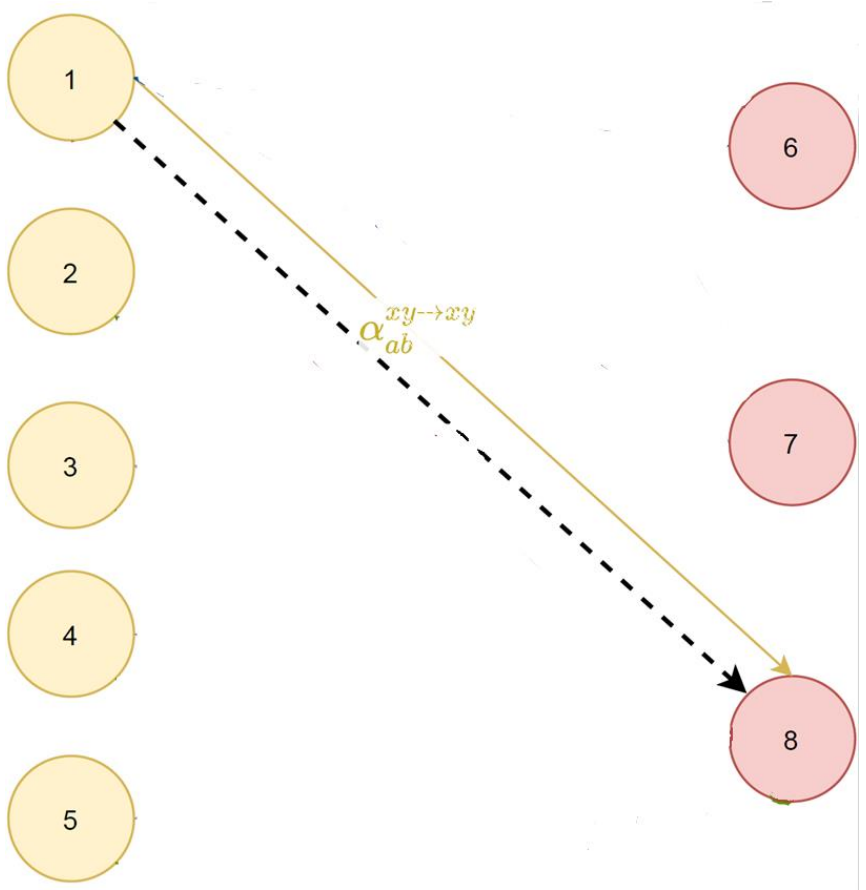

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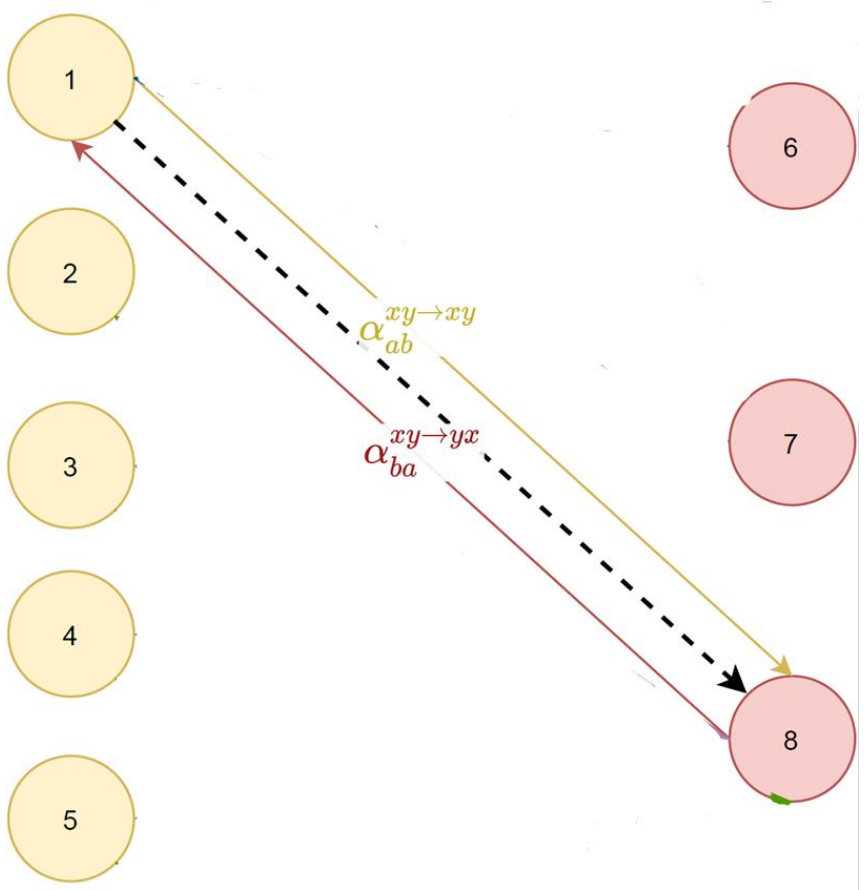

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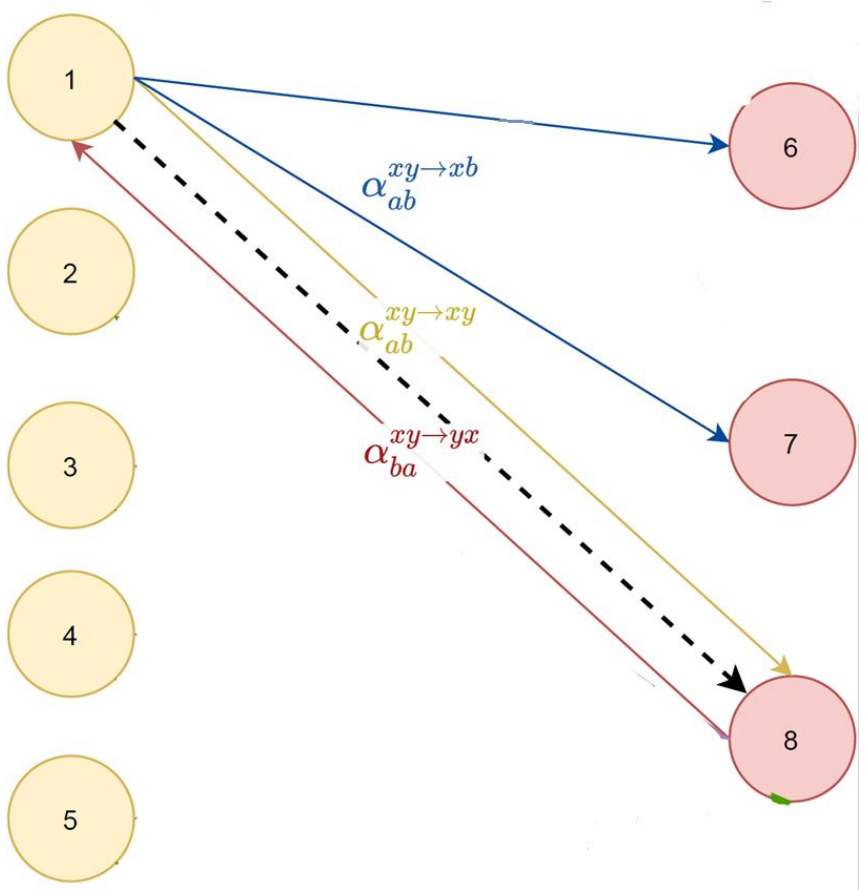

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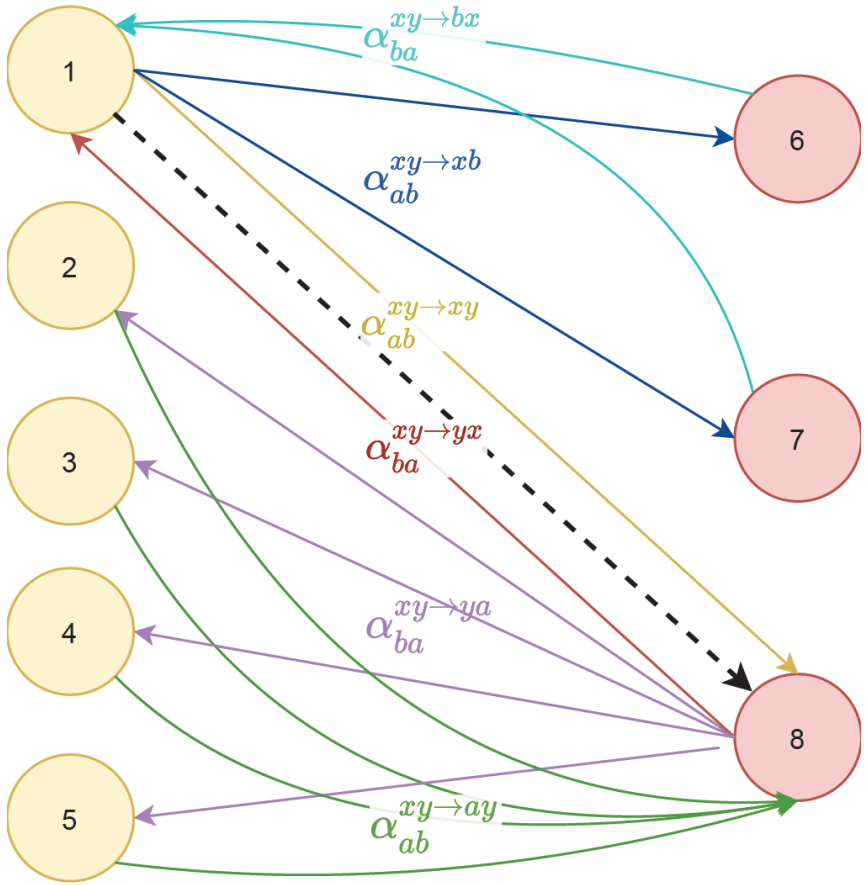
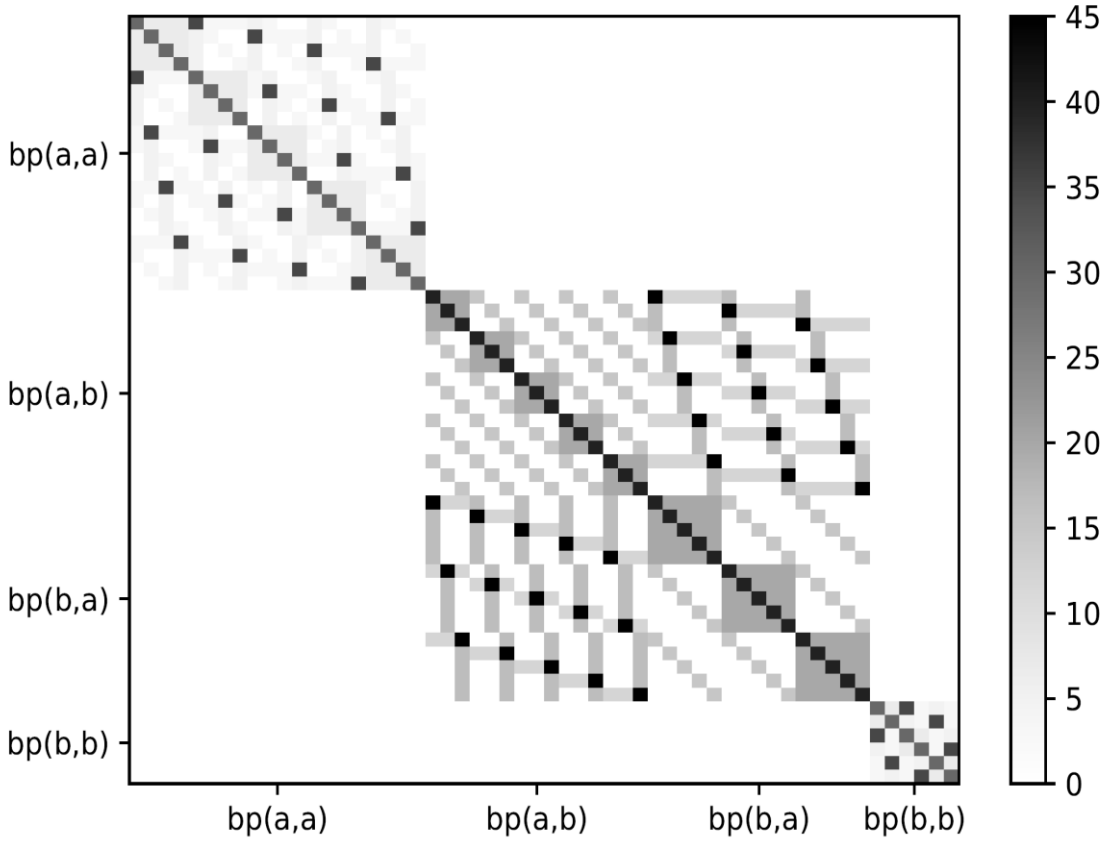
  - ➢ 1 to 6 and 7 (Turn continuation)
  - ➢ Other three (Generalized reciprocity, Allied continuation, Allied reciprocity)



What have we gained?

- **Estimation:** spectral clustering with a likelihood refinement procedure

1. Community/block detection.
2. Excitation matrix has a diagonal structure (sparser).
3. Meaningful estimated parameters.



Results: Predictive Accuracy

- Split dataset into 0.8 train : 0.2 test
- Predictive log-likelihood $\mathcal{L}$ score:

$$\text{test } \mathcal{L} = \frac{(\text{full } \mathcal{L} - \text{train } \mathcal{L})}{\text{number of test events}}$$

Model	Reality	Enron	MID	Facebook
MULCH	**-3.82**	**-5.13**	**-3.53**	**-6.82**
BHM	-5.37	-7.49	-5.33	-14.4
CHIP	-4.83	-5.61	-3.67	-9.46
REM	-6.11	-6.84		
ADM4	-8.52			
DLS	-5.65	-7.57	-4.52	

Results: Generative Accuracy

- Compare counts of **temporal motifs** on Reality Mining

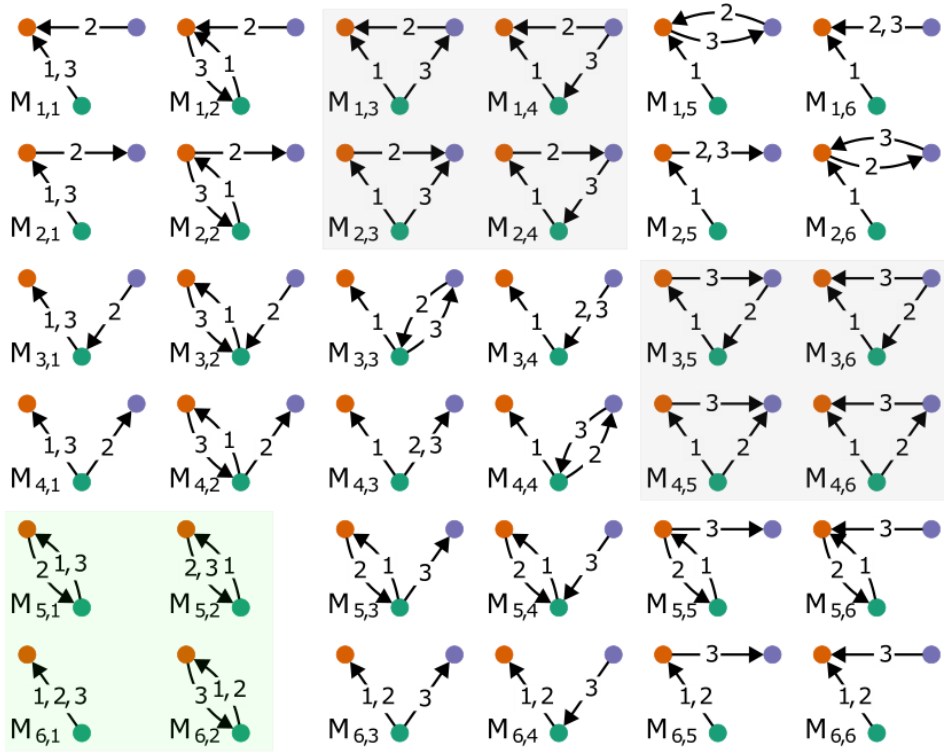
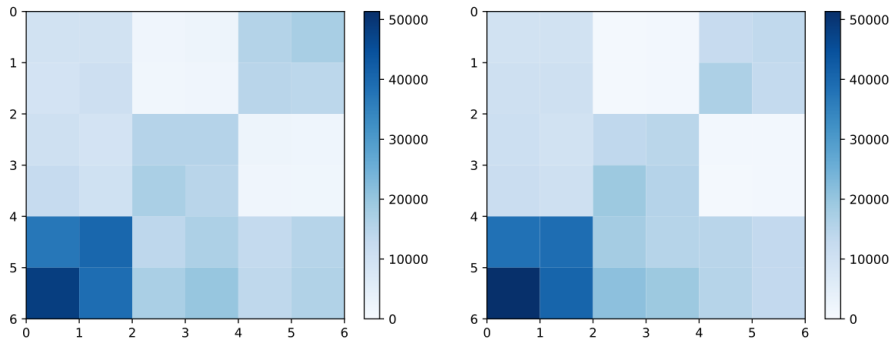
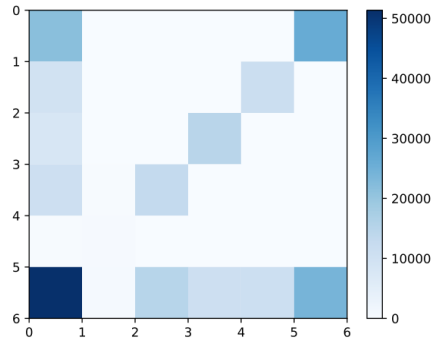


Photo credit: Paranjape, A., Benson, A. R., and Leskovec, J. Motifs in temporal networks. In Proceedings of the 10th ACM International Conference on Web Search and Data Mining, pp. 601–610, 2017.

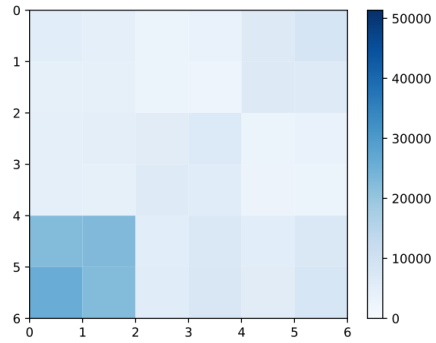


(a) Actual network

(b) MULCH simulated

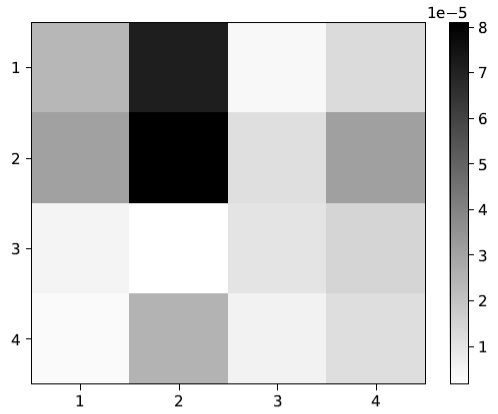


(c) CHIP simulated

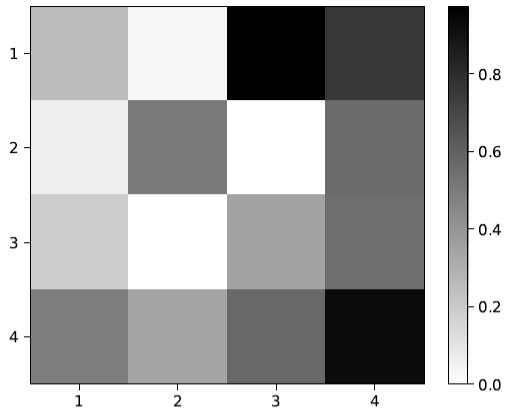


(d) BHM simulated

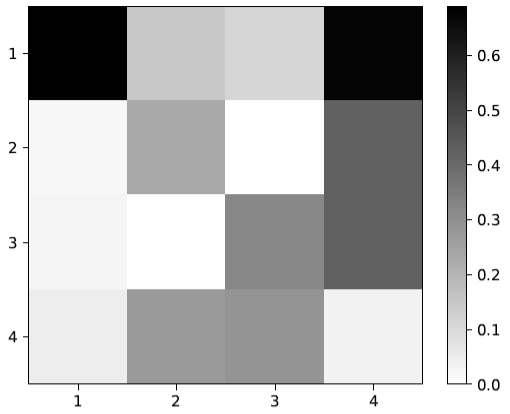
Case Study: Militarized Interstate Disputes dataset



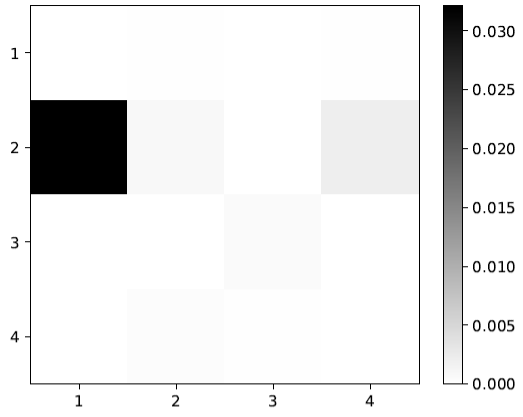
(a) Base intensity μ_{ab}



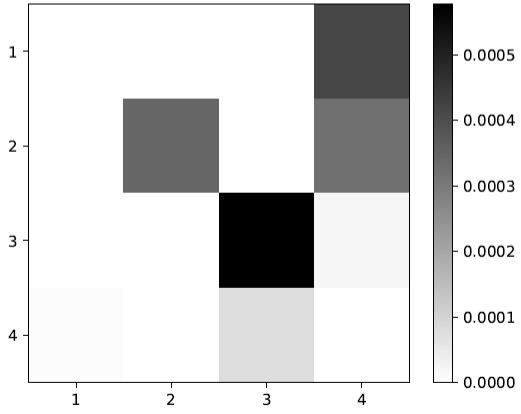
(b) Self excitation $\alpha_{ab}^{xy \to xy}$



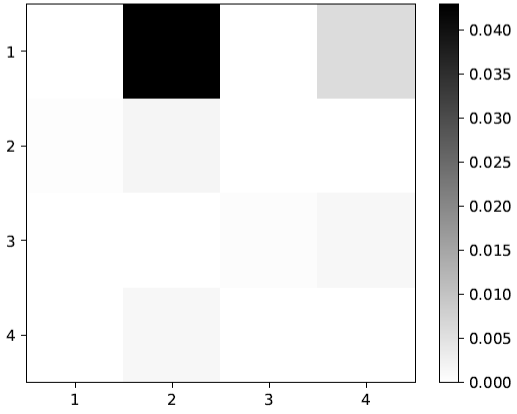
(c) Reciprocal excitation $\alpha_{ab}^{xy \to yx}$



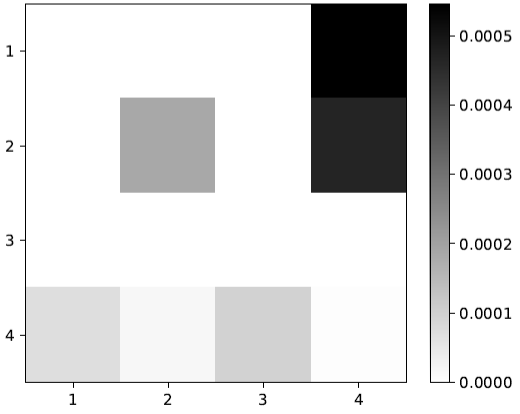
(d) Turn continuation $\alpha_{ab}^{xy \to xb}$



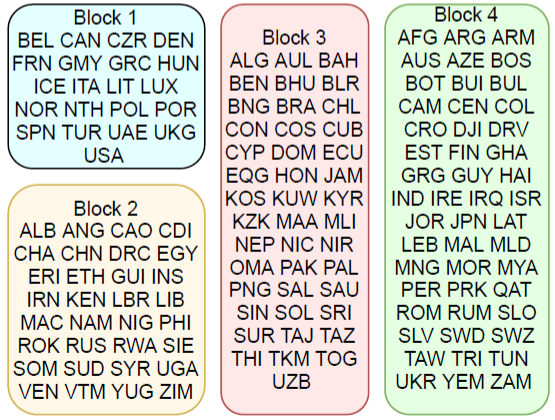
(e) Generalized recip. $\alpha_{ab}^{xy \to ya}$



(f) Allied continuation $\alpha_{ab}^{xy \to ay}$



(g) Allied reciprocity $\alpha_{ab}^{xy \to bx}$



(h) Node memberships

Thank you!

- Code: https://github.com/IdeasLabUT/Multivariate-Community-Hawkes

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