FedNL:
Making Newton-Type Methods Applicable to Federated Learning

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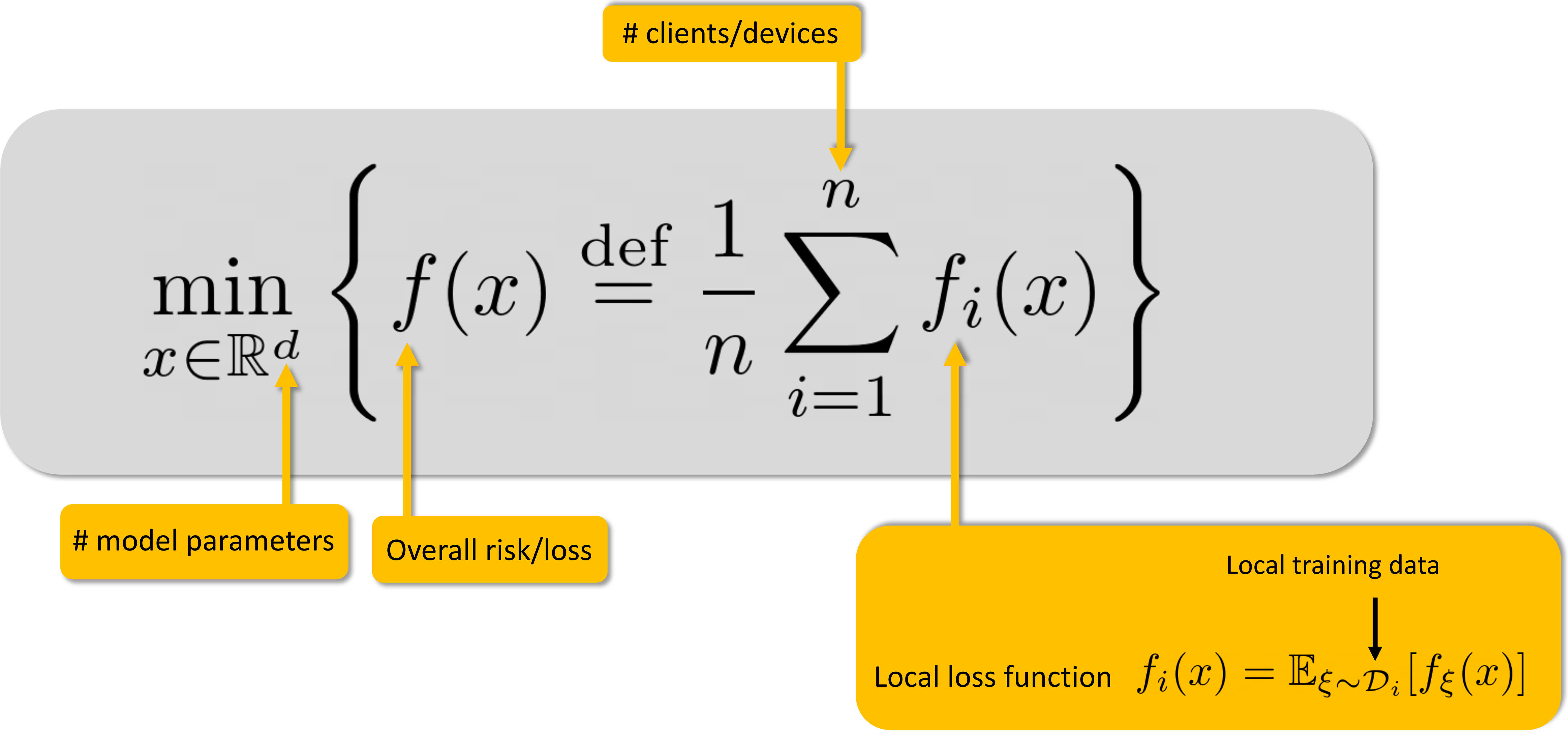
The Problem



clients/devices

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) \overset{\text{def}}{=} \frac{1}{n} \sum_{i=1}^{n} f_i(x) \right\}$$

model parameters

Overall risk/loss

Local training data

Local loss function $f_i(x) = \mathbb{E}_{\xi \sim \mathcal{D}_i}[f_\xi(x)]$

The Problem

Hessians are Lipschitz continuous

$$\|\nabla^2 f_i(x) - \nabla^2 f_i(y)\| \leq L\|x-y\|$$

2nd order smooth

non-convex

$$\min_{x\in\mathbb{R}^d}\left\{ f(x) \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} f_i(x) \right\}$$

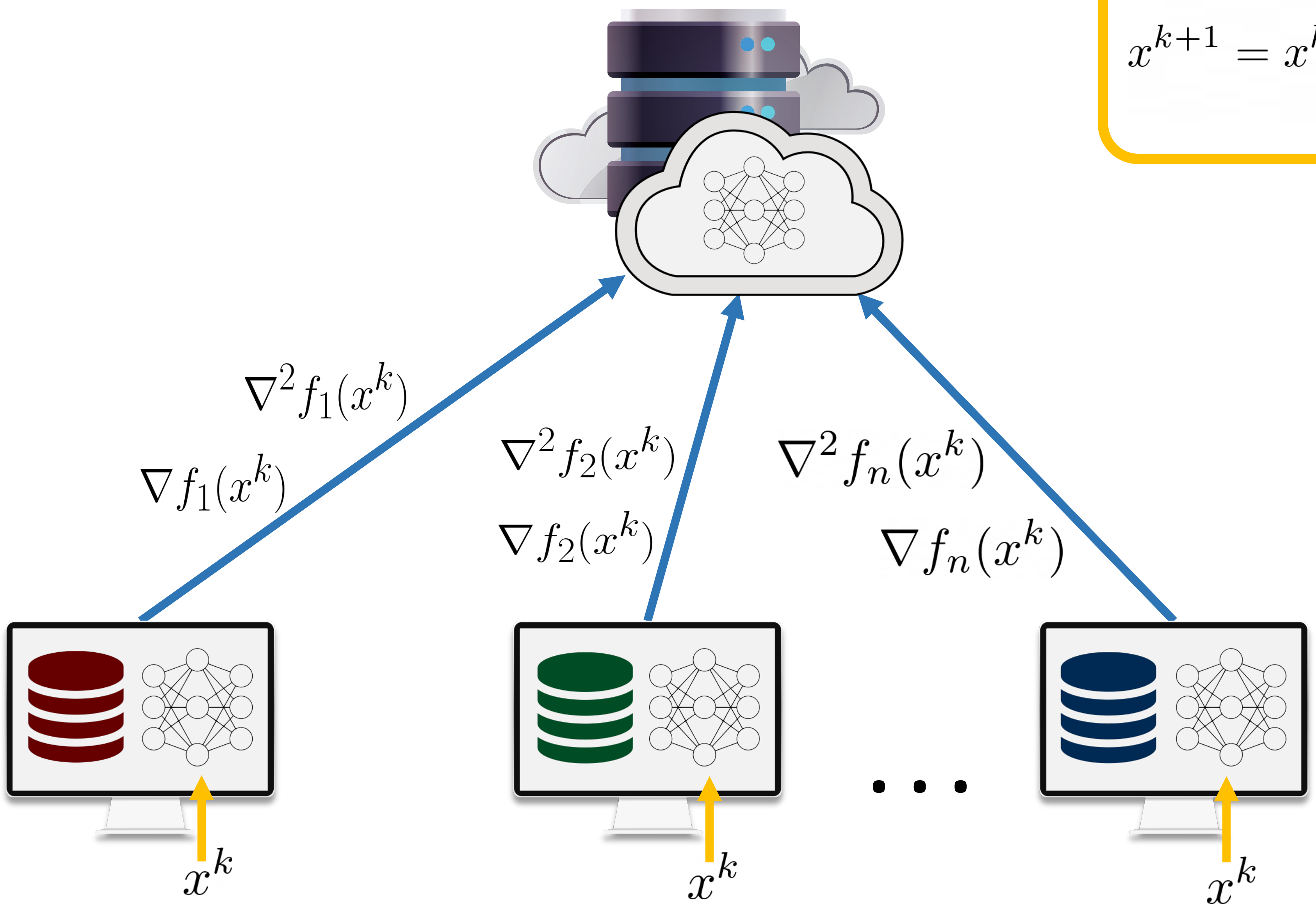
Strongly convex

for some $\mu > 0$ and for any $x, y \in \mathbb{R}^d$

$$f(x) \geq f(y) + \langle \nabla f(x), x-y\rangle + \tfrac{\mu}{2}\|x-y\|^2$$

Distributed Implementation of Newton's method

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^{n} \nabla^2 f_i(x^k)\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} \nabla f_i(x^k)\right)$$



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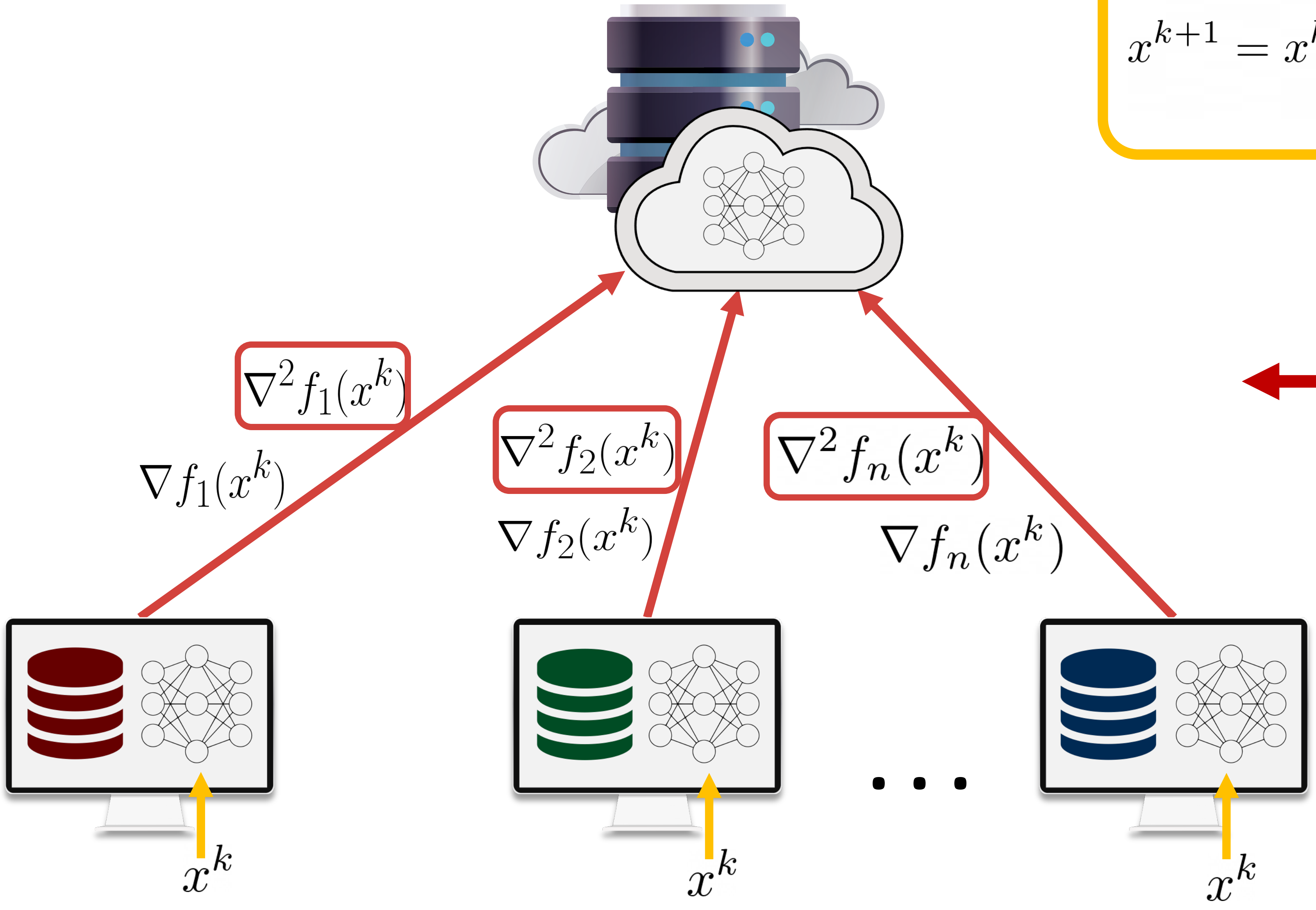

Communication issue

$\nabla^2 f_i(x^k)$ — d^2 floats

Newton’s Method

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Can be computed locally

Expensive to communicate: $\mathcal{O}(d^2)$

Can be computed locally

Easy to communicate: $\mathcal{O}(d)$

- ✗ $\mathcal{O}(d)$ communication cost per round
- ✓ Implementability in practice
- ✓ Local quadratic convergence rate independent of the condition number

Hessian Lipschitz constant

$$\|x^{k+1} - x^*\| \leq \frac{L}{2\mu}\|x^k - x^*\|^2$$

Strong convexity constant

Local quadratic rate

Newton Star Method

The unique minimizer of $f(x)$

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Rustem Islamov, Xun Qian and Peter Richtárik
Distributed second order methods with fast rates and compressed communication,
ICML 2021.

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Can NOT be computed locally

Single communication of $\mathcal{O}(d^2)$

Can be computed locally

Easy to communicate: $\mathcal{O}(d)$

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Learning the Optimal Hessian Matrices

Newton Star

$$x^{k+1} = x^k - \left(\frac{1}{n} \sum_{i=1}^{n} \boxed{\nabla^2 f_i(x^*)} \right)^{-1} \nabla f(x^k)$$

Learning the Optimal Hessian Matrices

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Idea! Learn the optimal Hessians $\boxed{\nabla^2 f_i(x^*)}$ in communication efficient manner:

$\boxed{(i)}\, \mathbf{H}_i^k \to \nabla^2 f_i(x^*)$ as $k \to \infty$ $\quad \boxed{(ii)}\, \mathbf{H}_i^{k+1} - \mathbf{H}_i^k$ is compressed

$$x^{k+1} = x^k - \left(\frac{1}{n} \sum_{i=1}^{n} \boxed{\mathrm{H}_i^k} \right)^{\boxed{-1}} \nabla f(x^k)$$

$$= x^k - \left(\boxed{\mathrm{H}^k}\right)^{\boxed{-1}} \nabla f(x^k)$$

FedNL: Two Options for Updating the Global Model

Option 1

$$x^{k+1} = x^k - \left(\left[\mathbf{H}^k \right]_{\mu} \right)^{-1} \nabla f(x^k)$$

Projection onto the cone of positive definite matrices

Option 2

$$x^{k+1} = x^k - \left(\mathbf{H}^k + l^k \mathbf{I} \right)^{-1} \nabla f(x^k)$$

$$l^k = \frac{1}{n} \sum_{i=1}^{n} \|\mathbf{H}_i^k - \nabla^2 f_i(x^k)\|_{\mathrm{F}}$$

FedNL: Hessian Learning Rate Options

$$\mathbf{H}_i^{k+1} = \mathbf{H}_i^k + \alpha \mathcal{C}_i^k(\nabla^2 f_i(x^k) - \mathbf{H}_i^k)$$

Stepsize depends only on the compression, e.g.,

$$\alpha = 1$$

$$\alpha = 1 - \sqrt{1 - \delta}$$

$$\alpha = \frac{1}{\omega + 1}$$

FedNL: New Hessian Learning Technique

$$\mathbf{H}_i^{k+1} = \mathbf{H}_i^k + \alpha \boxed{\mathcal{C}_i^k} (\nabla^2 f_i(x^k) - \mathbf{H}_i^k)$$

Compression operator

Contractive compressor $\mathbb{C}(\delta),\ \delta \in [0,1)$

$$\|\mathcal{C}(\mathbf{M})\|_{\mathrm{F}} \leq \|\mathbf{M}\|_{\mathrm{F}}$$

$$\|\mathcal{C}(\mathbf{M}) - \mathbf{M}\|_{\mathrm{F}}^2 \leq (1-\delta)\|\mathbf{M}\|_{\mathrm{F}}^2 \quad \forall\, \mathbf{M} \in \mathbb{R}^{d\times d}$$

Unbiased compressor $\mathbb{B}(\omega),\ \omega \geq 0$

$$\mathbb{E}[\mathcal{C}(\mathbf{M})] = \mathbf{M}$$

$$\mathbb{E}\left[\|\mathcal{C}(\mathbf{M}) - \mathbf{M}\|_{\mathrm{F}}^2\right] \leq \omega\|\mathbf{M}\|_{\mathrm{F}}^2 \quad \forall\, \mathbf{M} \in \mathbb{R}^{d\times d}$$

Greedy sparsification (Top-K)

$$\begin{bmatrix} -0.4 & 12.1 & 0.76 \\ 2.8 & -9.7 & -1.1 \\ 0.24 & 4.5 & 0.9 \end{bmatrix} \xrightarrow{K=2} \begin{bmatrix} 0 & 12.1 & 0 \\ 0 & -9.7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Random sparsification (Rand-K)

$$\begin{bmatrix} -0.4 & 12.1 & 0.76 \\ 2.8 & -9.7 & -1.1 \\ 0.24 & 4.5 & 0.9 \end{bmatrix} \xrightarrow{K=2} \frac{9}{2} \begin{bmatrix} -0.4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 4.5 & 0 \end{bmatrix}$$

factor preserving unbiasedness

FedNL: Local Convergence Theory

$$\|x^k - x^*\|^2 \boxed{\leq} \frac{1}{2^k}\|x^0 - x^*\|^2$$

Local (fixed) linear rate

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Local (fixed) linear rate

Lyapunov function

$$\Phi^k := 6\boxed{B}H^2\|x^k - x^*\|^2 + \frac{1}{n}\sum_{i=1}^{n}\|\mathbf{H}_i^k - \nabla^2 f_i(x^*)\|_{\rm F}^2$$

$$\mathbb{E}[\boxed{\Phi^k}] \boxed{\leq} \left(1 - \min\left\{\boxed{A}, \frac{1}{3}\right\}\right)^k \boxed{\Phi^0}$$

Local linear rates

$x^k \to x^*$

$\mathbf{H}_i^k \to \nabla^2 f_i(x^*)$ for all $i \in [n]$

Constant depending on the choice of the compressor and stepsize

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Local linear rates

$x^k \to x^*$

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Constant depending on the choice of the compressor and stepsize

$$\mathbb{E}\left[\frac{\|x^{k+1} - x^*\|^2}{\|x^k - x^*\|^2}\right] \boxed{\leq} \left(1 - \min\left\{\boxed{A}, \frac{1}{3}\right\}\right)^k \left(C + \frac{D}{12BL_{\mathrm{F}}^2}\right)\frac{\Phi^0}{\mu^2}$$

Local superlinear rate

Experiments: Regularized Logistic Regression

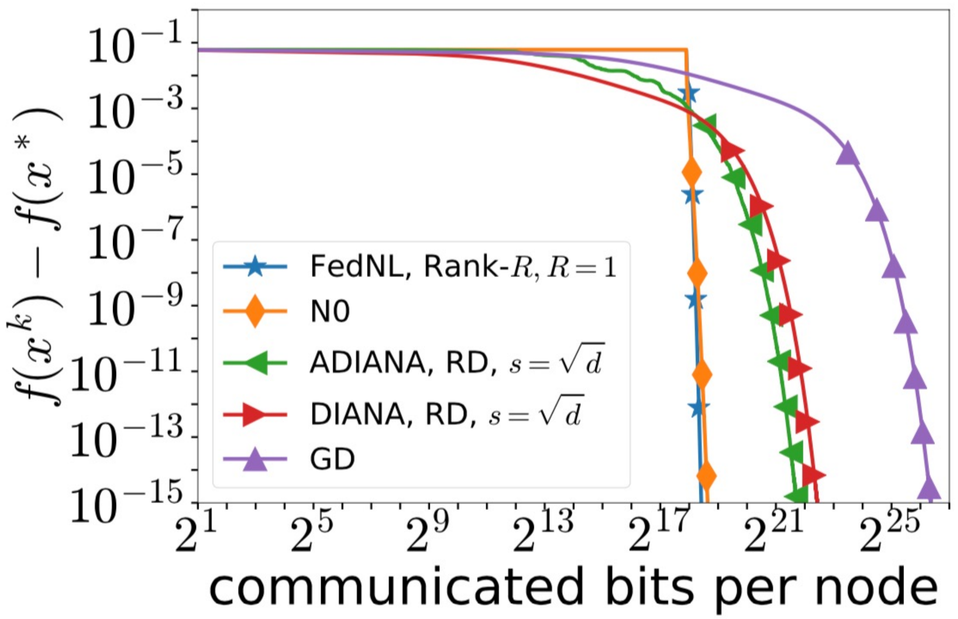
Regularization parameter

Training data points

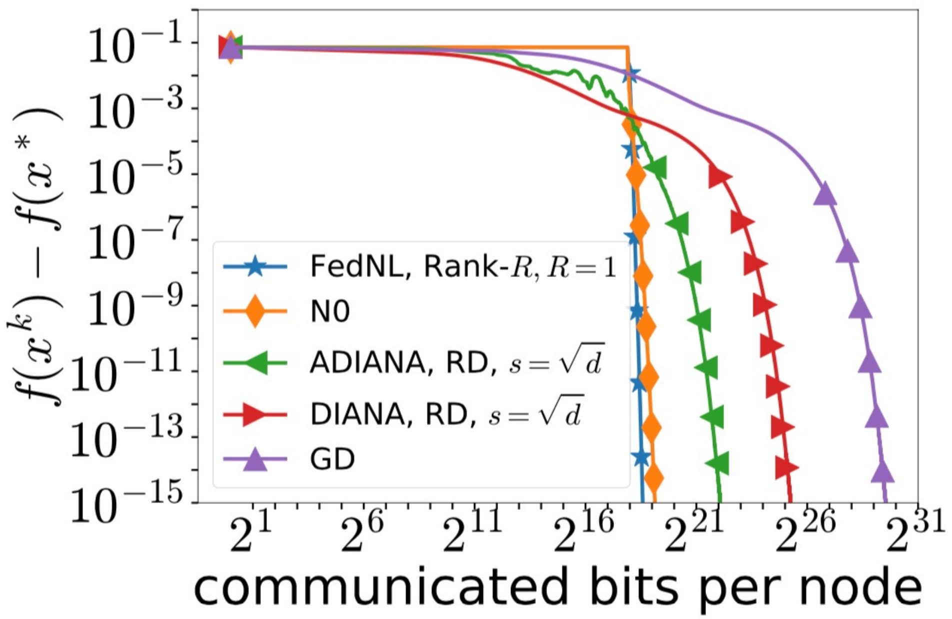
$$\min_{x\in\mathbb{R}^d}\left\{f(x) := \frac{1}{n}\sum_{i=1}^{n} f_i(x) + \overset{\downarrow}{\frac{\lambda}{2}}\|x\|^2\right\}, \qquad f_i(x) = \frac{1}{m}\sum_{j=1}^{m}\log\Big(1+\exp(-b_{ij}\overset{\downarrow}{a_{ij}^\top} x)\Big),$$

where $\{a_{ij}, b_{ij}\}_{j\in[m]}$ are data points at the i-th device. The datasets were taken from LibSVM library [Chang and Lin, 2011]: `a1a`, `a9a`, `w7a`, `w8a`, and `phishing`.

Experiments: FedNL vs Gradient-Type Methods

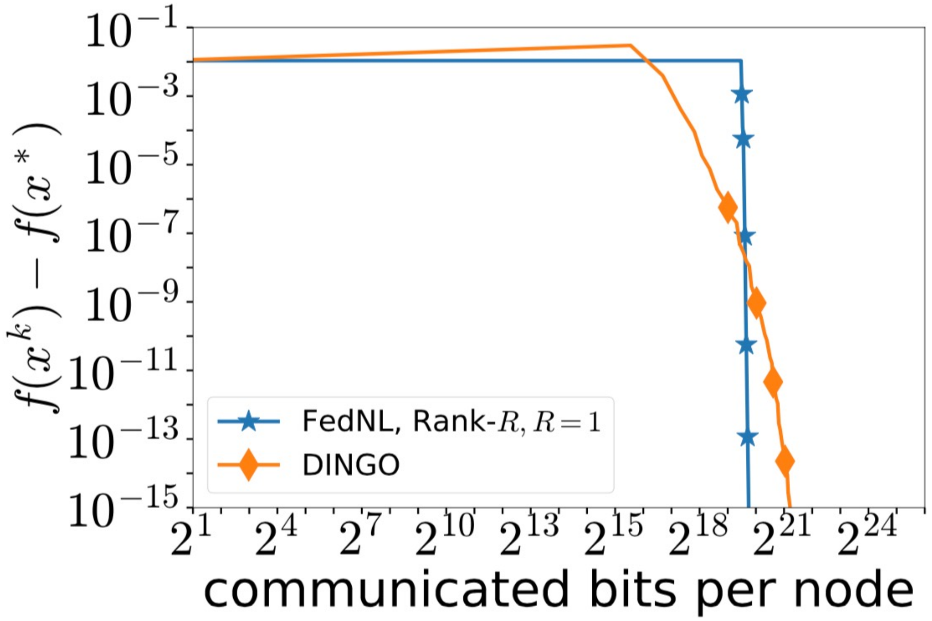


(a) a1a, $\lambda = 10^{-3}$

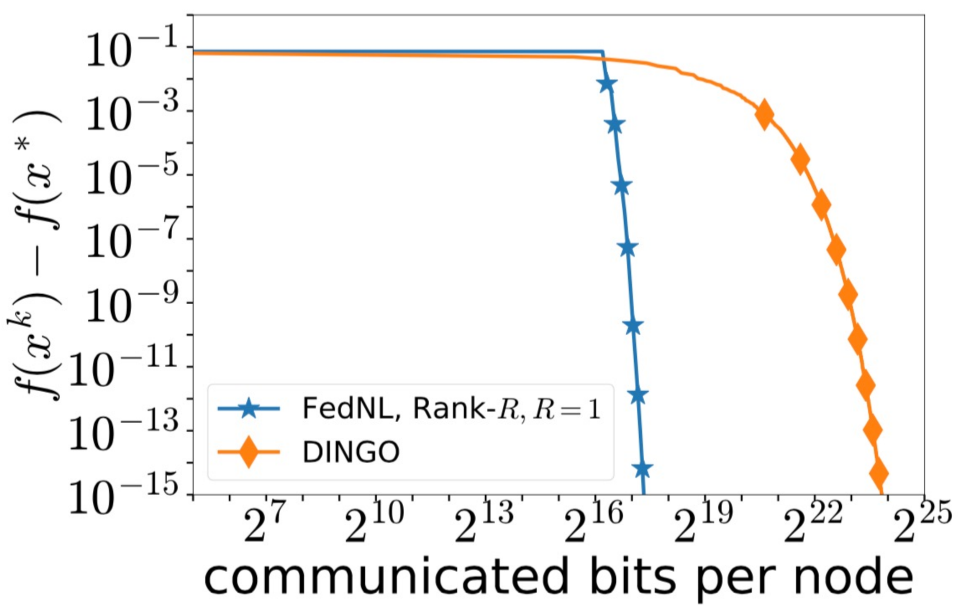


(b) a9a, $\lambda = 10^{-4}$

Experiments: FedNL vs DINGO

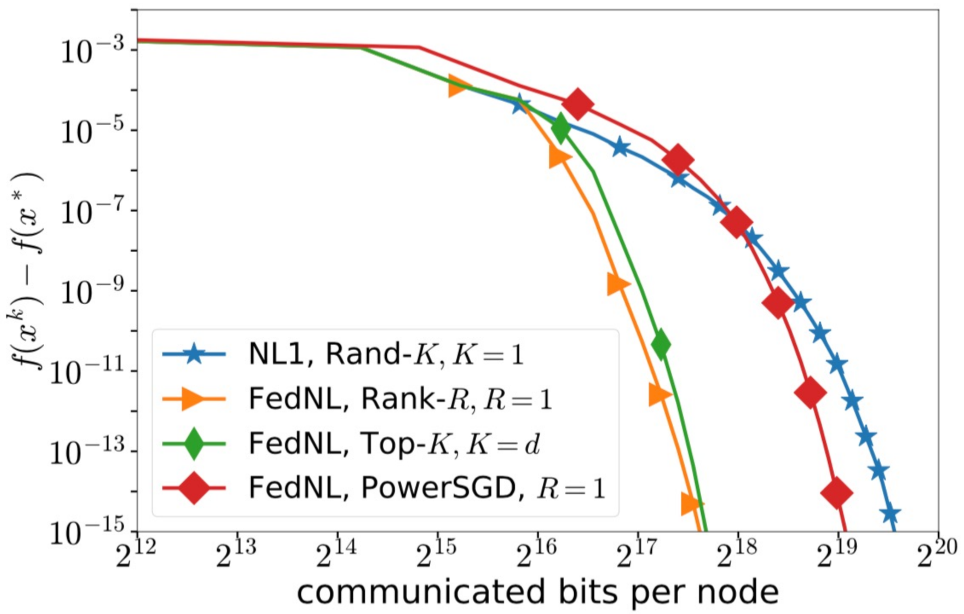


(c) w8a, $\lambda = 10^{-3}$



(d) phishing, $\lambda = 10^{-4}$

Experiments: FedNL vs NEWTON-LEARN (NL)



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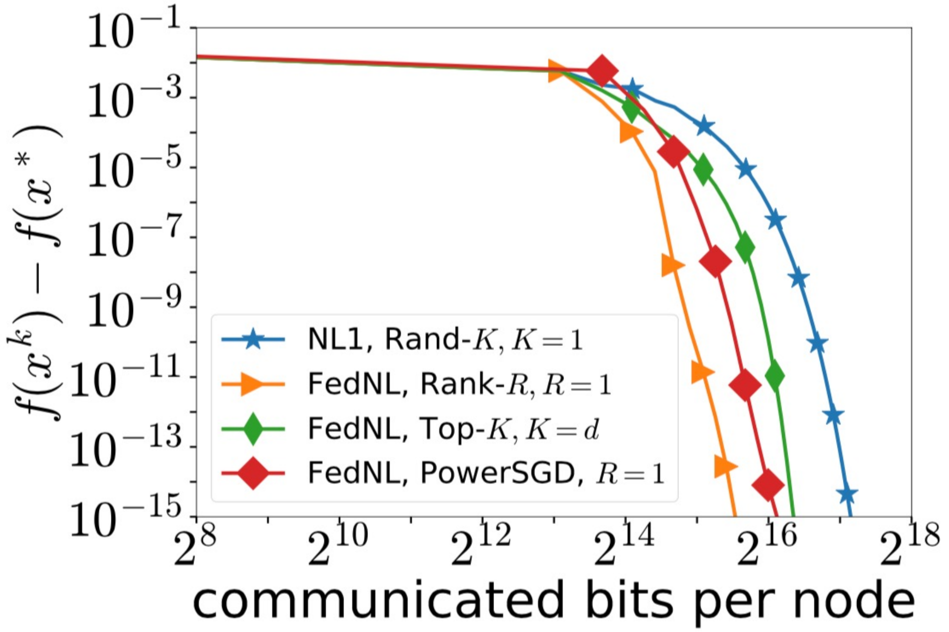

(b) phishing, $\lambda = 10^{-3}$

Thank you