Partial Counterfactual Identification from Observational and Experimental Data

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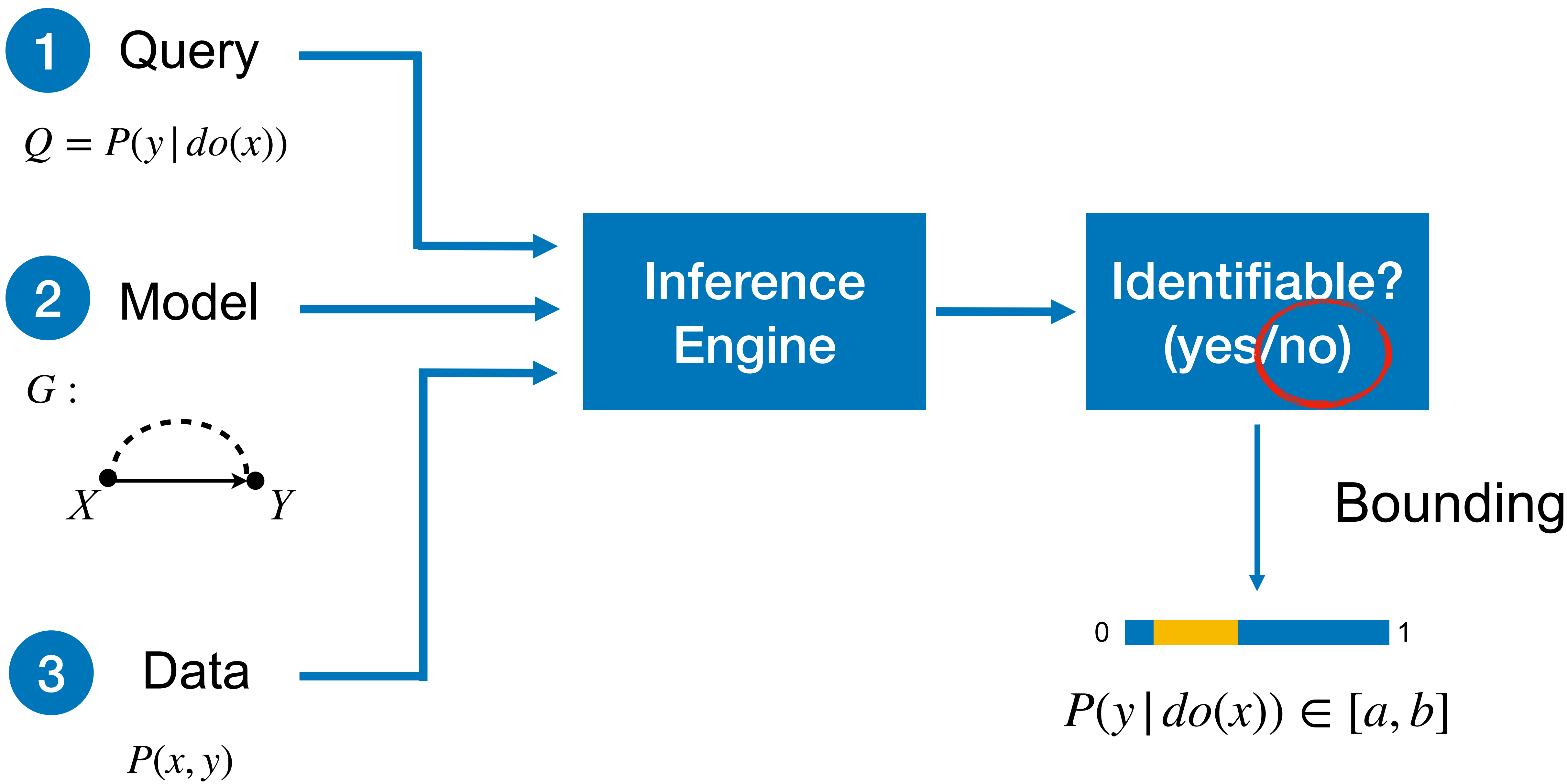
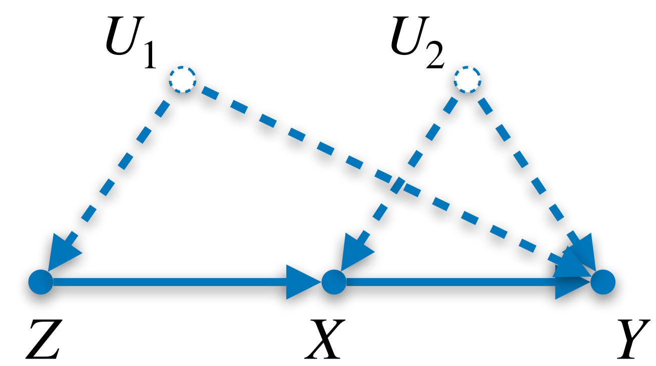
The Partial Identification Problem



Partial Identification of Causal Effects

Task. Given the observational distribution $P(\mathbf{v})$ in an arbitrary causal diagram G, bound $P(\mathbf{y} \mid do(\mathbf{x}))$ for any $\mathbf{X}, \mathbf{Y} \subseteq \mathbf{V}$.



- We assume that the domain of $\mathbf{V}$ is discrete and finite.
- Let $\mathscr{M}$ denote the set of all possible SCMs compatible with G.
- Given $P(\mathbf{v})$, $P(\mathbf{y} \mid do(\mathbf{x}))$ is bounded in $[a, b]$ where:

$$a = \min P_M(\mathbf{y} \mid do(\mathbf{x})), \qquad \forall M \in \mathscr{M},$$
$$\text{s.t.}$$
$$b = \max P_M(\mathbf{y} \mid do(\mathbf{x}))\,. \qquad P_M(\mathbf{v}) = P(\mathbf{v})\,.$$

Solving this optimization is difficult since parametric form of $\mathscr{F}, P(U)$ are not provided.

Canonical Causal Models

Definition. A canonical SCM is a SCM $M = \langle \mathbf{V}, \mathbf{U}, \mathscr{F}, P(\mathbf{U}) \rangle$ where

- Every $V \in \mathbf{V}$ is decided by a function $v \leftarrow f_V(\mathrm{pa}_V, u_V)$ taking values in a discrete and finite domain Ω_V.

- Every $U \in \mathbf{U}$ are drawn from a discrete domain Ω_U with cardinality

$$|\Omega_U| = \prod_{V \in \mathbf{C}(U)} |\Omega_{\mathrm{Pa}_V}| \times |\Omega_V|$$

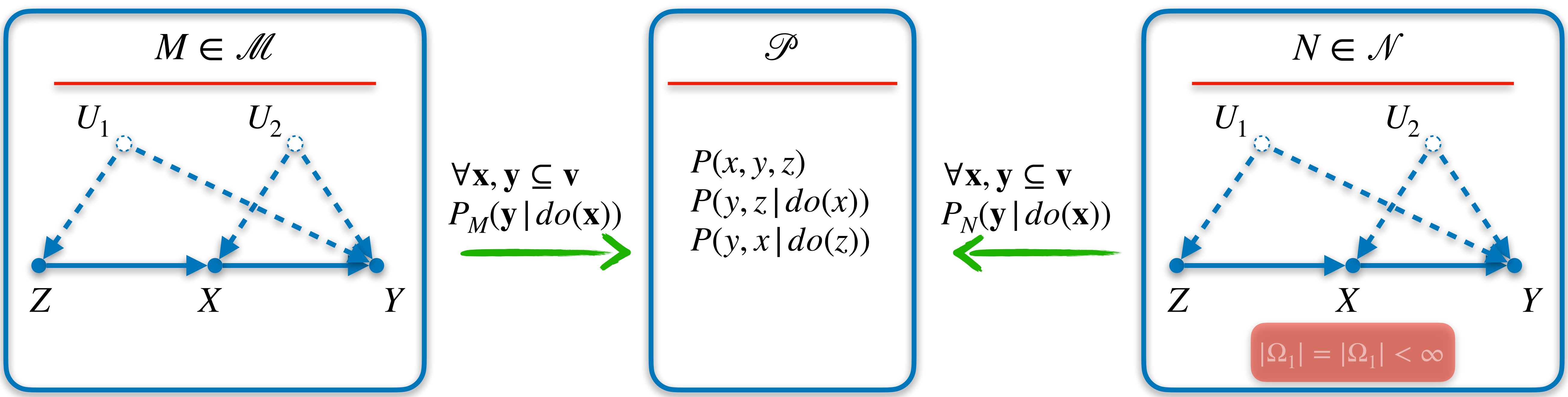
where $\mathbf{C}(U)$ is the c-component in G that covers U.

Two endogenous variables are in the same c-component if and only if they are connected by a bi-directed path.

Canonical SCMs

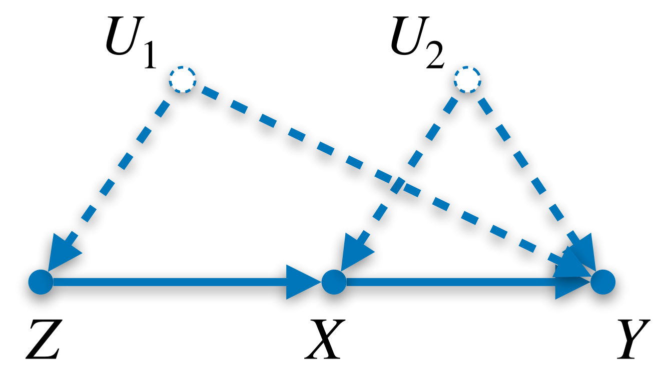
Theorem. For any SCM M, there exists a canonical SCM N s.t.

1. M and N are compatible with the same causal diagram G;
2. For any subsets $\mathbf{X}, \mathbf{Y} \subseteq \mathbf{V}$, $P_M(\mathbf{y} \mid do(\mathbf{x})) = P_N(\mathbf{y} \mid do(\mathbf{x}))$.



Partial Identification of Causal Effects: Revisit

Task. Given the observational distribution $P(\mathbf{v})$ in an arbitrary causal diagram G, bound $P(\mathbf{y} \mid do(\mathbf{x}))$ for any $\mathbf{X}, \mathbf{Y} \subseteq \mathbf{V}$.

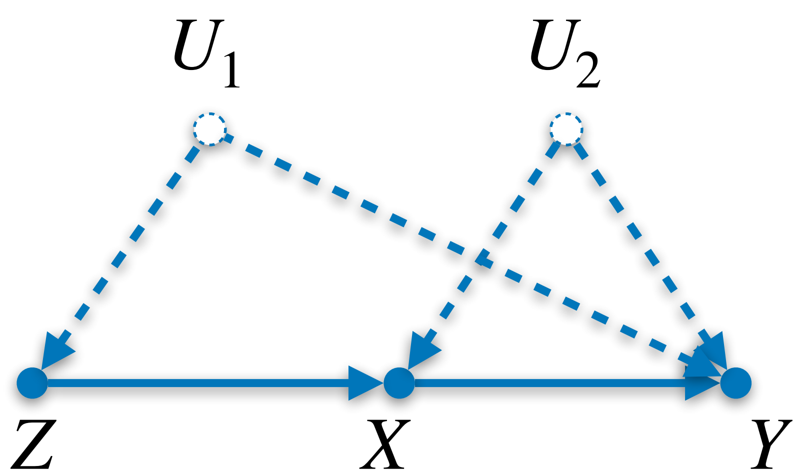


- We assume that the domain of $\mathbf{V}$ is discrete and finite.
- Let $\mathcal{N}$ denote the set of all canonical SCMs compatible with G.
- Given $P(\mathbf{v})$, $P(\mathbf{y} \mid do(\mathbf{x}))$ is bounded in $[a, b]$ where:

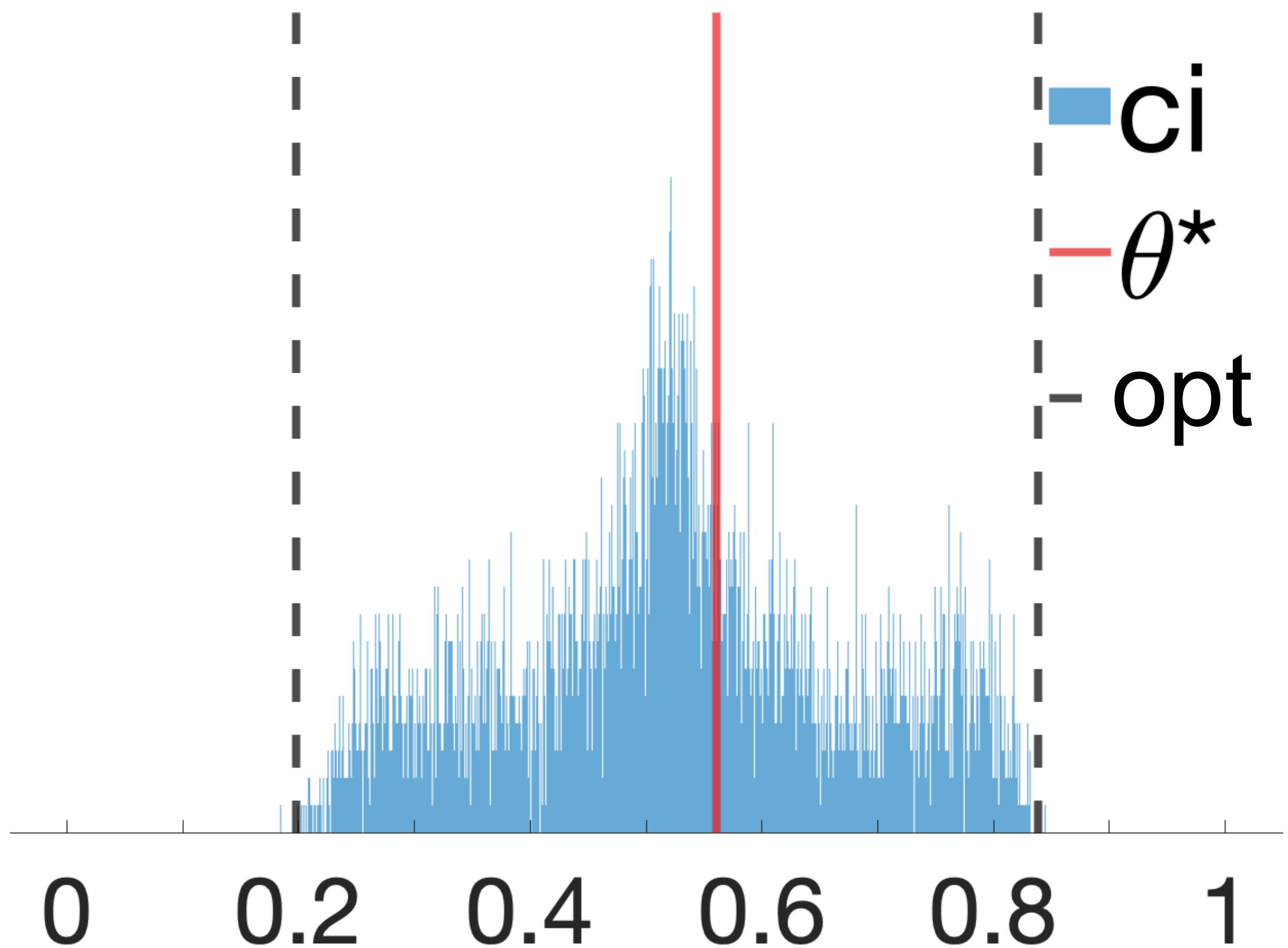
$$\begin{aligned} a &= \min P_N(\mathbf{y} \mid do(\mathbf{x})), \\ b &= \max P_N(\mathbf{y} \mid do(\mathbf{x})) . \end{aligned} \quad \text{s.t.} \quad \begin{aligned} &\forall N \in \mathcal{N}, \\ &P_N(\mathbf{v}) = P(\mathbf{v}) . \end{aligned}$$

This problem is reducible to an equivalent polynomial optimization program

Example: Non-IV



- $X, Y, Z \in \{0,1\}$
- $U_1, U_2 \in \mathbb{R}$
- Data - $P(x, y, z)$
- Query - $P(y \mid do(x))$



$N = 1000$

Conclusions

- We introduce canonical causal models that could represent all interventional distributions in an arbitrary causal diagram.
- It reduces partial causal identification to equivalent polynomial programs.
- What is in the paper (Contributions):
  - Generalized canonical SCMs that could represent all counterfactual distributions in a causal digram.
  - Effective posterior sampling methods to approximate optimal bounds over unknown counterfactual probabilities from observational and experimental data.