On Last-Iterate Convergence Beyond Zero-Sum Games

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Main Focus: **No-Regret Learning** Dynamics in **Games**

- What happens if learning agents play against each other in games?
- Traditional no-regret learning fails to converge to a Nash equilibrium
- Recent progress using optimistic variants **(optimistic mirror descent)**

$$z^{(t+1)} = z^{(t)} - 2\eta\nabla_z f(z^{(t)}) + \eta\nabla_z f(z^{(t-1)})$$

Our Questions

- How fast is the convergence to Nash equilibria?
- What if players are using different updates rules?
- Convergence beyond zero-sum games (e.g. nearly zero-sum)?
- Guarantees in terms of the social welfare?

Regret-Based Framework for Last-Iterate Convergence

- So far, the analysis of last-iterate convergence differs from the analysis of the players' regrets
- We show last-iterate guarantees using regret bounds
- We inherit the robustness stemming from a **regret-based analysis**

Implications

- Tight $O(1/\sqrt{T})$ rates for games with **nonnegative sum of regrets**
- Captures MVI property, and the weak MVI property
- Our guarantees apply even if players use different learning rules
- Last-iterate convergence implies optimal $O(1)$ regret for each player
- **Beyond zero-sum**: Games that satisfy an **approximate minimax theorem**

No-Regret Learning Can Outperform the Price of Anarchy

- Variants of optimistic mirror descent **outperform** the (robust) price of anarchy when they do **not** converge to Nash equilibria
- The price of anarchy corresponds to the social welfare of the **worst** Nash equilibrium in the game
- Cycling behavior can improve efficiency in games

Questions?

https://arxiv.org/abs/2203.12056

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