Learning To Cut By Looking Ahead: Cutting Plane Selection via Imitation Learning

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Andreas Krause, Laurent Charlin, Chris J. Maddison

Integer Programming

Integer programs are linear programs (LPs) with integral variables:

$$z^{OPT} := \min_x w^\intercal x \quad \text{subject to} \quad Ax \leq b,\ x \in \mathbb{Z} \tag{1}$$

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Solving integer programs (IPs) is hard, but ubiquitous...

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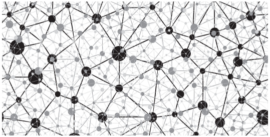
Power Production

Vehicle Routing

Server Load Balancing

Portfolio Optimization



Neural Network Verification

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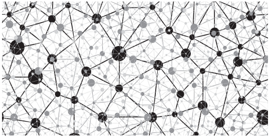

Neural Network Verification

Why ML? $\rightarrow$ IPs are *many* and *similar* in most applications.

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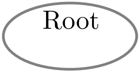

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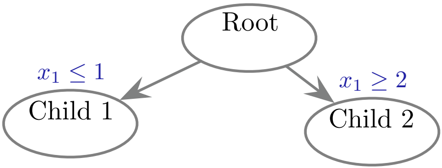

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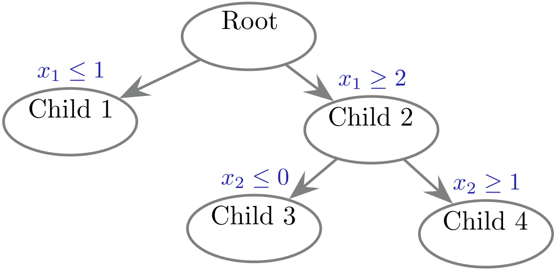

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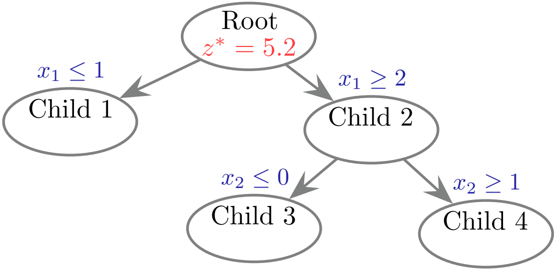

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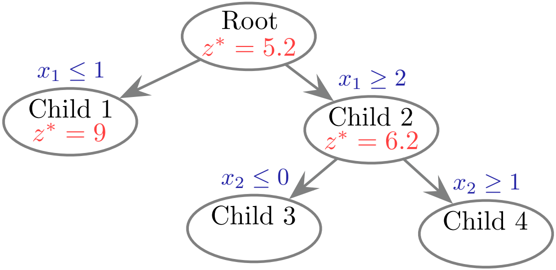

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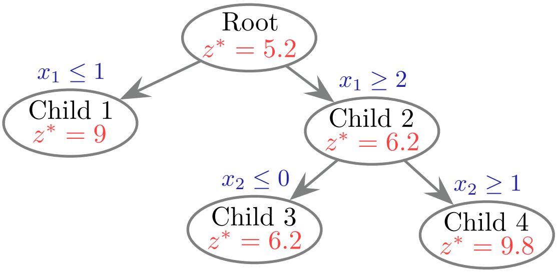

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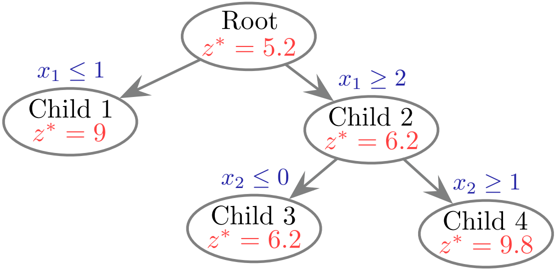

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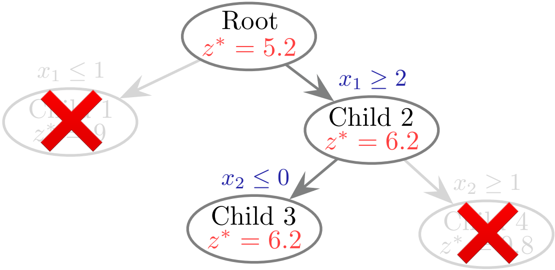

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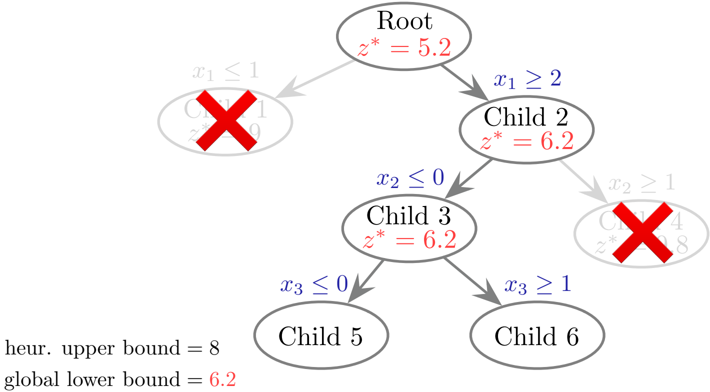

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Branch and Cut Algorithm (B&C)

B&C tightens the lower bound by adding a cut $\boldsymbol{C = (c, c_0)}$,

$$z_C \coloneqq \min_x w^\intercal x \quad \text{subject to} \quad Ax \le b,\ \boldsymbol{c^\intercal x \le c_0}. \tag{2}$$

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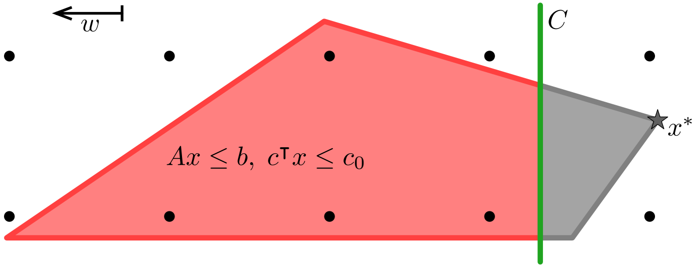
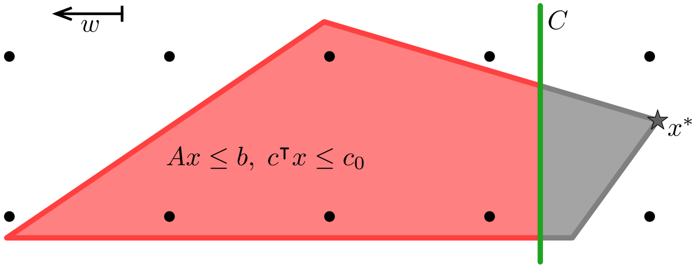
A **cut** $\boldsymbol{C}$ is an extra constraint satisfied by all feasible x of the IP...



..but not by $x^* := \arg\min_x w^\intercal x$ s.t. $\boldsymbol{Ax \leq b}$ (LP relaxation).

Cutting Planes

In practice, many cuts $C \in \mathcal{C}$ can be generated...



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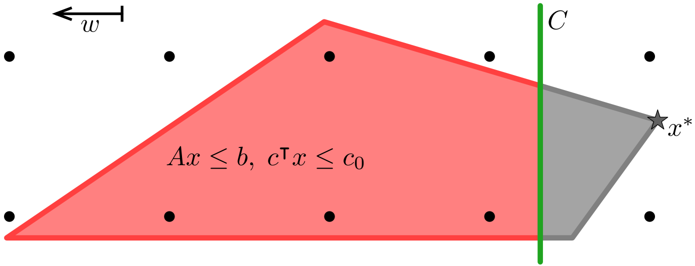

..but only *effective* cuts should be added, because every cut increases the complexity of resolving the linear program (2).

Our Contribution: "Learning To Cut By Looking Ahead"

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To date, modern B&C solvers use *manual heuristics* to select cuts.

Instead, we leverage *machine learning* for cutting plane selection:

- **Looking Ahead:** Propose expensive but effective *lookahead* criterion for cut selection, used as label for supervised training.
- **Learning To Cut:** Propose *NeuralCut*, a graph neural network (GNN) to *imitate* lookahead for cut selection.

Looking Ahead: Lookahead Criterion is strong but expensive

For a cut $C \in \mathcal{C}$, we define the lookahead score s_{LA} as...

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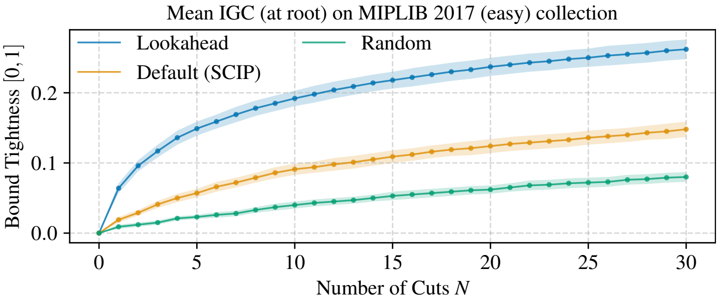

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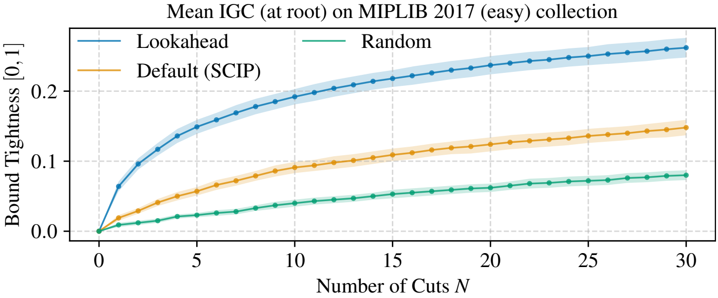

..but computing $s_{LA}(C)$ for all $C \in \mathcal{C}$ is too expensive in practice.

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Hence, we **learn to imitate** lookahead selection with *NeuralCut*.

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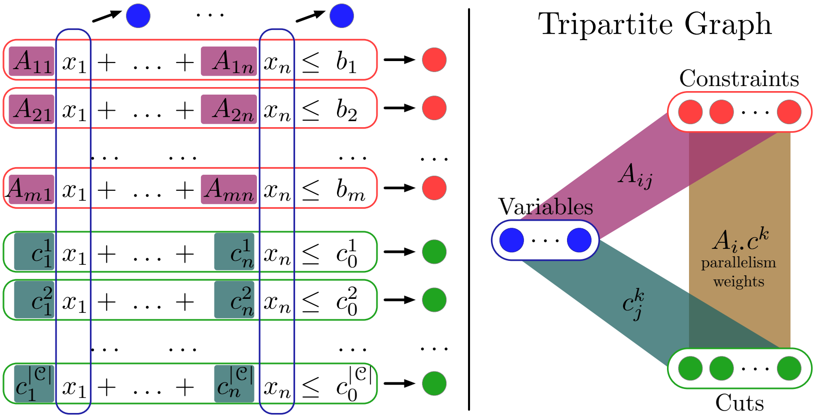
To learn from IPs and cutpools, Neuralcut uses a tripartite graph...



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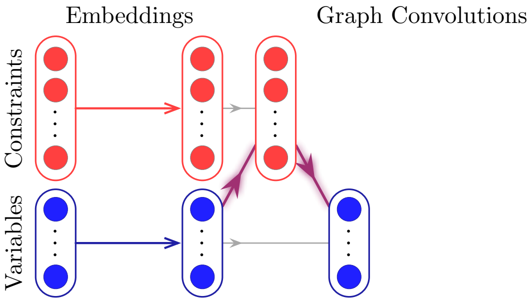

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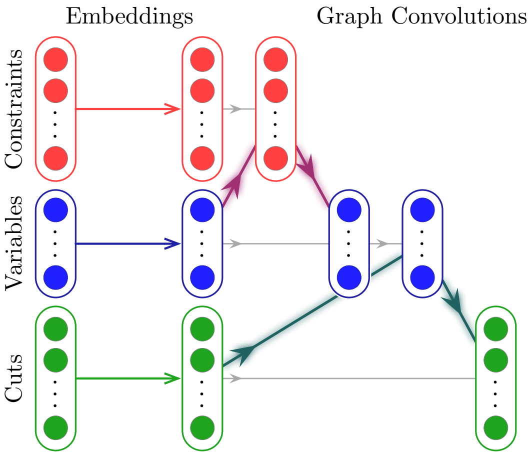

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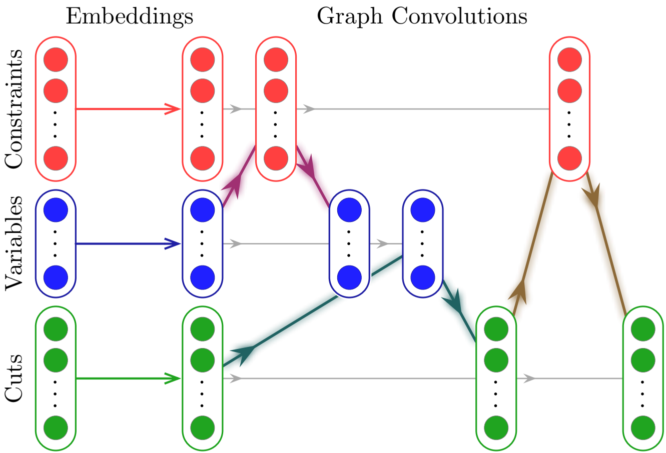

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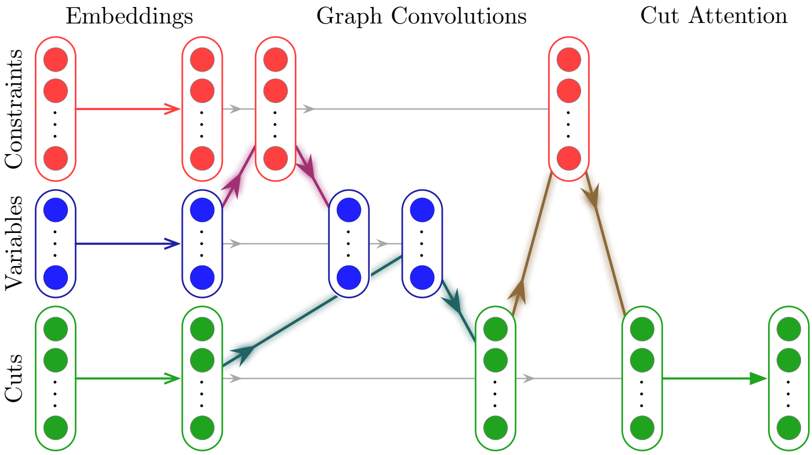

..but adds more convolutions, shortcuts and self-attention on cuts.

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- to minimize a training log-loss **over all cuts**...

$$L(\hat{s}) \coloneqq -\frac{1}{|\mathfrak{C}|} \sum_{C \in \mathfrak{C}} q_C \log \hat{s}_C + (1 - q_C) \log(1 - \hat{s}_C) \quad (4)$$

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- and selects the cut $C^*_{NC} = \arg\max_{C \in \mathfrak{C}} \hat{s}_C$ at test time.

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- ..as a **plug-in of the SCIP[1] B&C solver**...

  ..to select *effective* cuts at root that speed up B&C search

  ($\rightarrow$ small *residual* solving time after processing root).

[1] Gamrath et al., 2020

Experiments: NeuralCut for *pure* cut selection

NeuralCut achieves **tight bounds quickly** on various IP families...

Area above bound tightness curve (↓) on test instances, mean (ste), $N = 30$

		Max. Cut	Packing	Bin. Packing	Planning
	Lookahead	*15.05 (0.09)*	*26.42 (0.07)*	*9.85 (0.33)*	*10.33 (0.04)*
ML	**NeuralCut**	**15.55 (0.09)**	***26.30 (0.08)***	**10.96 (0.33)**	**10.42 (0.04)**

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ML	RL[2]	19.00 (0.09)	27.59 (0.06)	16.06 (0.38)	14.94 (0.08)
Heuristics	Default	16.72 (0.09)	***26.29 (0.08)***	15.42 (0.28)	14.01 (0.06)
Heuristics	Efficacy	17.01 (0.09)	***26.28 (0.08)***	15.19 (0.29)	14.52 (0.06)
Heuristics	Obj. Parallelism	24.01 (0.08)	28.28 (0.05)	22.23 (0.26)	27.71 (0.07)
Heuristics	IntSupport	21.87 (0.07)	28.78 (0.03)	21.14 (0.28)	23.31 (0.08)
Heuristics	Random	21.99 (0.07)	28.73 (0.04)	21.23 (0.28)	23.26 (0.08)

..and **outperforms** heuristic scores and a competing RL approach.

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Experiments: NeuralCut inside a B&C solver

Paired with an ϵ-threshold rule to run inside the SCIP B&C solver on (mixed) integer programs from neural network verification...

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Median performance on test instances

	# cuts	Rel. bound improv.	Residual Time (s)
Default (SCIP B&C)	279	**1.00**	23.65
NeuralCut ($\epsilon = 10^{-5}$)	105	**1.00**	22.35
NeuralCut ($\epsilon = 10^{-4}$)	81	0.99	**20.89**
NeuralCut ($\epsilon = 10^{-3}$)	48	0.98	22.73

...NeuralCut achieves **strong lower bounds at root** with fewer cuts and **speeds up** the remaining B&C search.

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Code: We plan to make code available at `github.com/mbp28`.