



Optimally Controllable Perceptual Lossy Compression

Zeyu Yan, Fei Wen, Peilin Liu

Department of Electronic Engineering /
Brain-inspired Application Technology Center,
Shanghai Jiao Tong University, China

D-P tradeoff in lossy compression

low bit-rate

tradeoff

low distortion



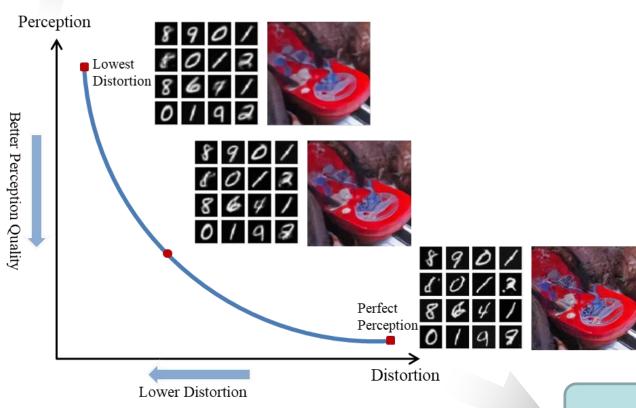
lower distortion blurred details



higher distortion clear details

D-P tradeoff in lossy compression

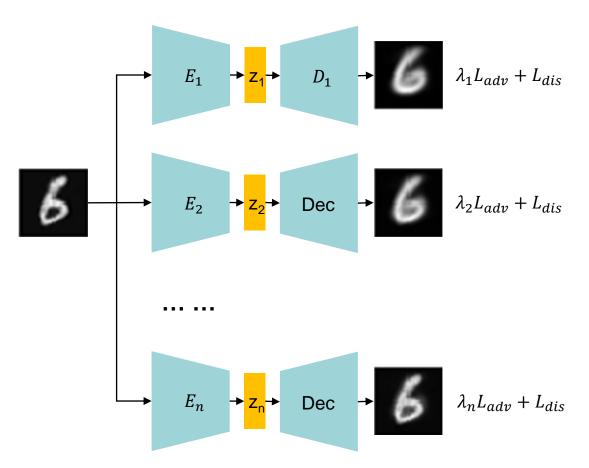
Distribution divergence



Sample distance

How to achieve optimal distortion-perception tradeoff?

Distortion-plus-adversarial loss (DAL) $L = \lambda L_{adv} + L_{dis}$

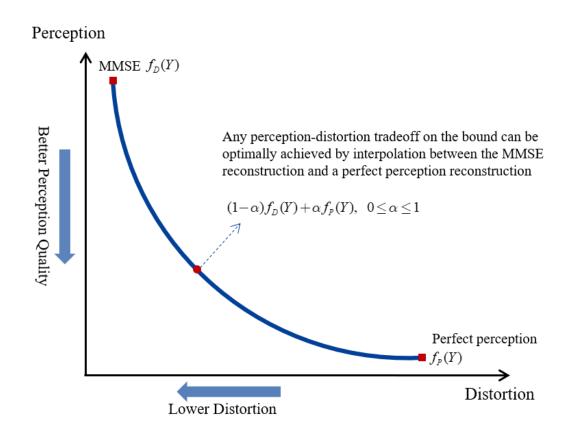


Hard to quantitatively control D-P tradeoff

Infinite number of encoder-decoder pairs are needed to fit D-P tradeoff

Contribution 1 (a nontrivial theoretical finding):

- one encoder and two decoders are enough for optimally achieving arbitrary D-P tradeoff in certain condition,
- the perceptual quality (Wasserstein-2 distance) and distortion (MSE) can be quantitatively controlled by interpolating outputs of two decoders,



Distortion-perception function can be expressed as

$$D(P) := \min_{E \in \Omega, G} \mathbb{E} ||X - G(E(X))||^{2}$$
s.t. $W_{2}^{2}(p_{X}, p_{G(E(X))}) \leq P$,

 Ω : the set of encoders with a given bit-rate R

 W_2^2 : squared Wasserstein-2 distance

Theorem 1. Let (E_d, G_d) be an optimal encoder-decoder pair to $D(+\infty)$, and G_p be an optimal decoder to D(0) for a fixed encoder E_d . Denote $Z_d := E_d(X)$ and $P_d := W_2^2(p_X, p_{G_d(Z_d)})$. Then, these hold:

i) E_d is an optimal encoder for any P > 0.

ii) Let
$$\alpha = \min\left(\sqrt{\frac{P}{P_d}}, 1\right) \in [0,1]$$
, define
$$G_{\alpha}^*(Z_d) \coloneqq \alpha G_d(Z_d) + (1-\alpha)G_p(Z_d)$$
then (E_d, G_{α}^*) is an optimal encoder-decoder pair to $D(P)$.

Contribution 2 (perfect perception decoding):

• We propose a training method for perfect perceptual quality decoder G_p .

An augmented training loss without compromising the optimality

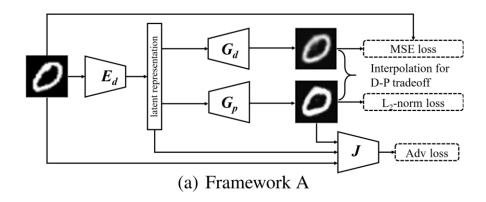
$$\min_{p_{\hat{X}, Z_d}} W_1(p_{\hat{X}, Z_d}, p_{X, Z_d}) \qquad \min_{p_{\hat{X}, X_d}} W_1(p_{\hat{X}, X_d}, p_{X, X_d}) + \lambda \mathbb{E} \|\hat{X} - X_d\|$$

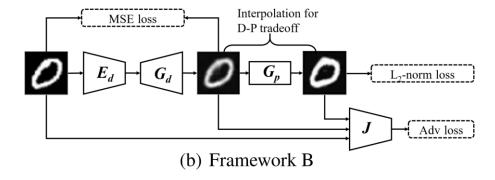
Theorem 2. Let (E_d, G_d) be an MMSE encoder-decoder pair, and $W_1(\cdot, \cdot)$ be the Wasserstein-1 distance. Denote $Z_d = E_d(X)$ and $X_d = G_d(Z_d)$, then these hold:

- i) When $0 \le \lambda < 1$, the optimal solution satisfies $p_{\hat{X},X_d} = p_{X,X_d}$, or equivalently $p_{\hat{X},Z_d} = p_{X,Z_d}$.
- ii) When $\lambda > 1$, the optimal solution satisfies $\hat{X} = X_d$.

Contribution 3 (perfect perception decoding):

 We propose two optimal training frameworks for perfect perceptual decoding, which enables the realization of interpolation based optimal D-P tradeoff.





Results on MNIST

$$\min_{p_{\hat{X},X_d}} W_1(p_{\hat{X},X_d}, p_{X,X_d}) + \lambda \mathbb{E} \|\hat{X} - X_d\|$$

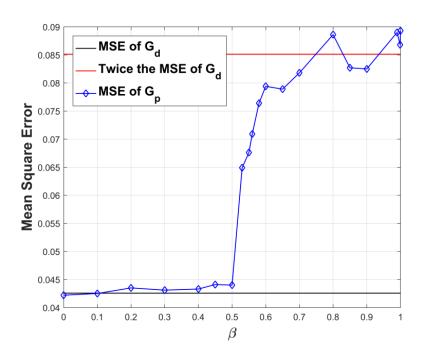
To verify our result in **contribution 2**, we train framework A with loss

$$\max_{\|J\|_{L} \le 1} \mathbb{E}[J(G_{p}(E(X)), E(X))] - \mathbb{E}[J(X, E(X))]$$

$$\min_{G_{p}} (1 - \beta) \mathbb{E}\|G_{p}(X) - X_{d}\| - \beta \mathbb{E}[J(G_{p}(X), E(X))]$$

where
$$\lambda = \frac{1-\beta}{\beta}$$

MSE jumps from $D(+\infty)$ (MSE of G_d) to $2D(+\infty)$ at $\beta = 0.5$ ($\lambda = 1$)



Results on MNIST

Samples decoded by conventional framework $L = \lambda L_{adv} + L_{dis}$

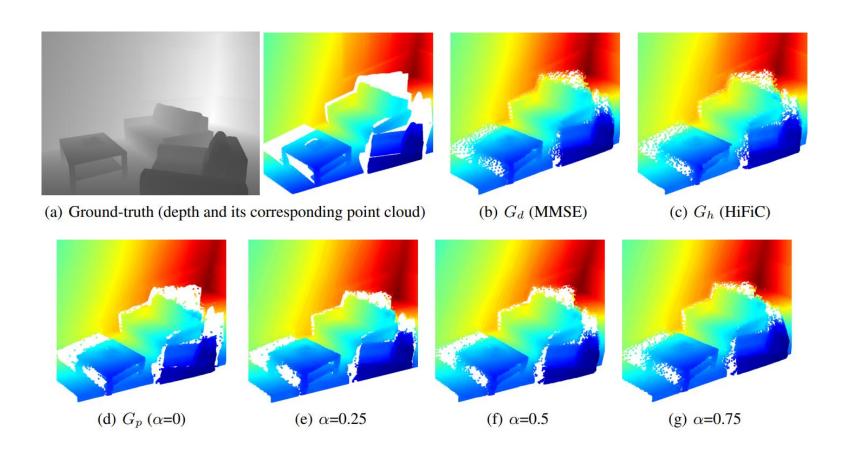
$$L = \lambda L_{adv} + L_{dis}$$

Input	$\lambda=0$ (G_d)	λ=0.1	λ=1	λ=5
MSE: 0	0.043	0.059	0.089	0.114
524	629	977	9 # 9	598
000	6 6 6	0 5 6	Q 3 7	774
789	189	9 8 9	\$ 3 2	499

Samples decoded by our framework

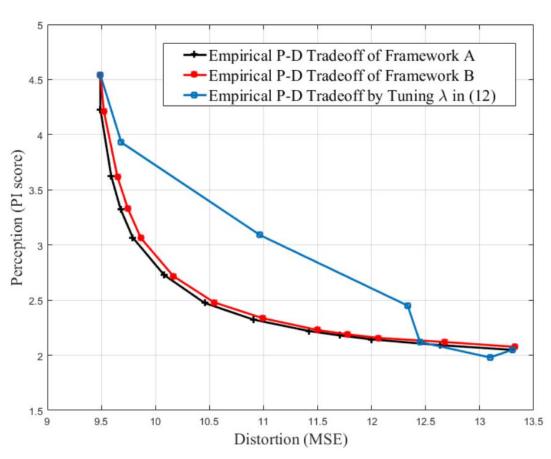
$\alpha=0.8$	$\alpha=0.6$	$\alpha=0.4$	α=0.2	$\alpha=0$ (G_p)
MSE: 0.045	0.049	0.057	0.068	0.082
629	625	625	625	625
189	224	224	224	224

Results on SUNCG dataset

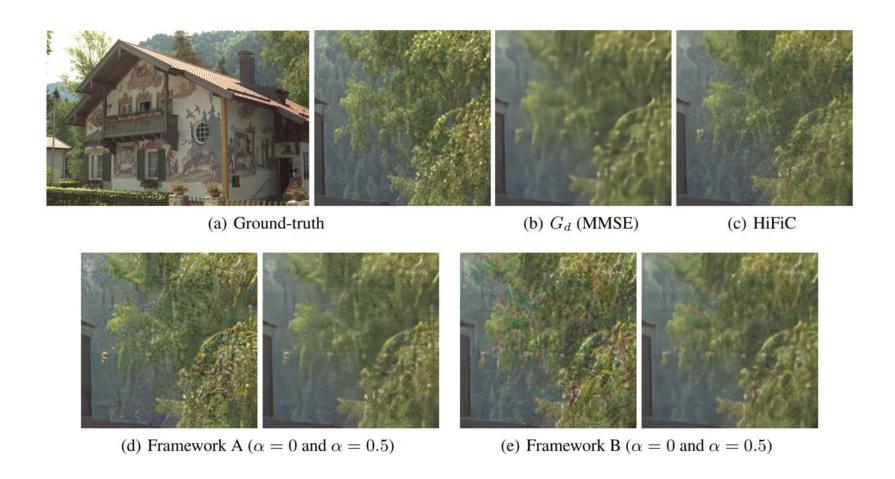


Results on KODAK dataset

Distortion (MSE) vs. perception (PI score)



Results on KODAK dataset



Thank you!