Volatility Based Kernels and Moving Average Means For Accurate Forecasting with Gaussian Processes

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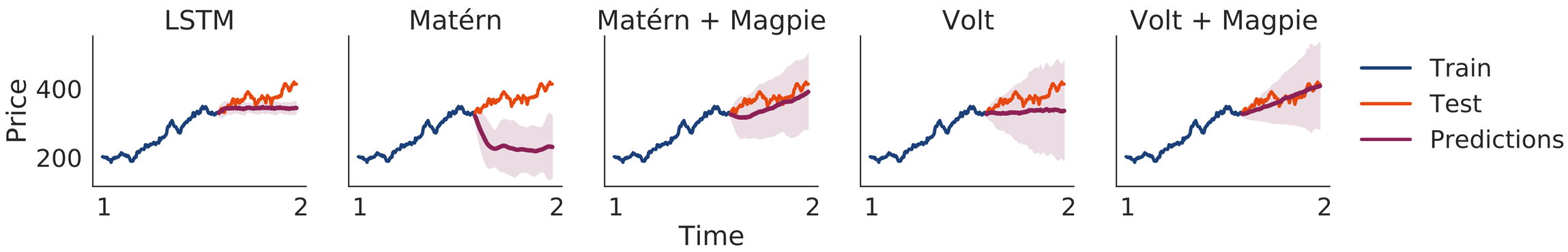
Volt and *Magpie*

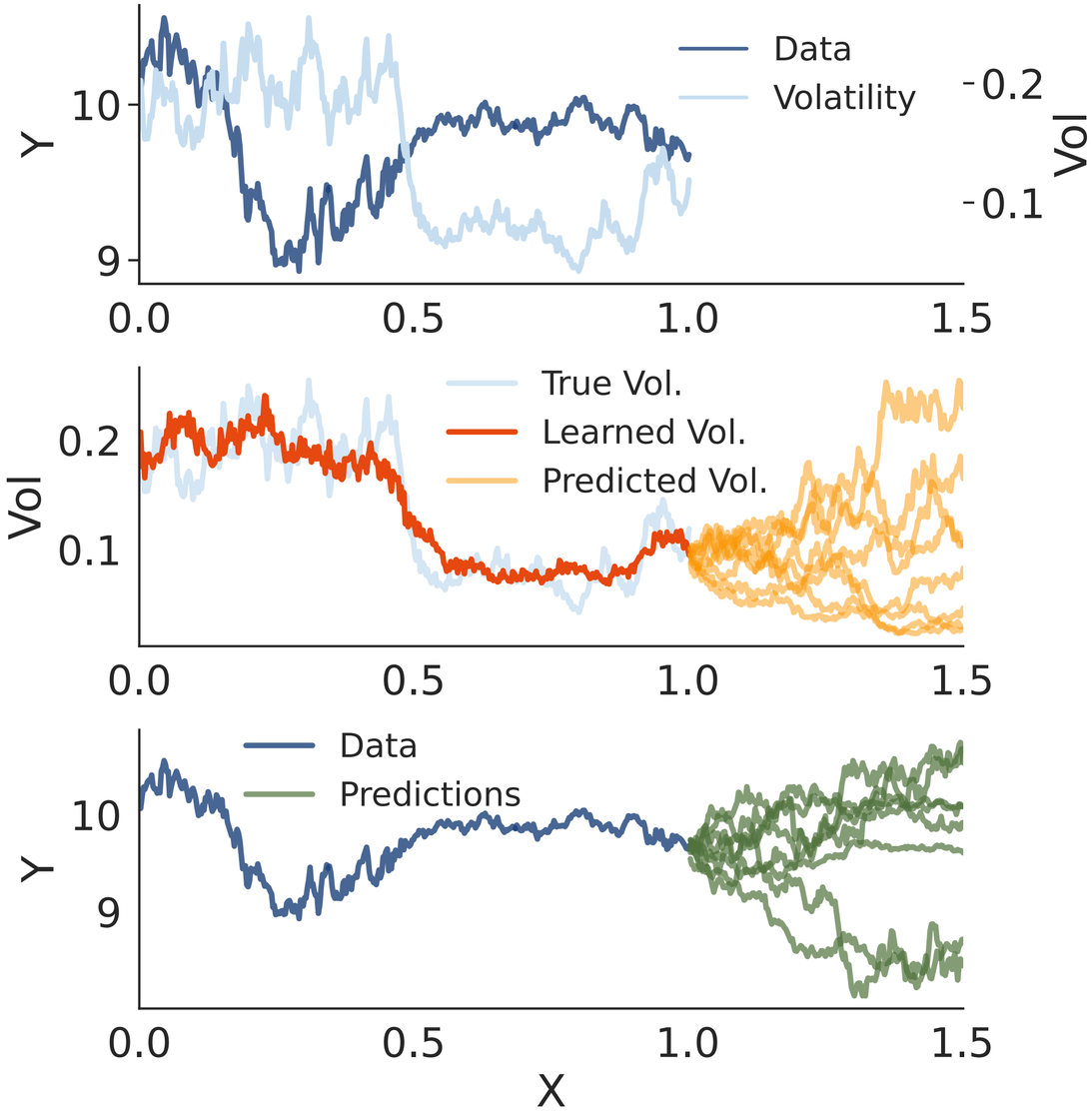
- Typical Gaussian process modeling involves selecting a standard mean function and a flexible kernel
- We derive a new class of mean and covariance functions, *Volt* and *Magpie*, taking direct inspiration from domain specific models
- Volt and Magpie give accurate forecasts when the data are stochastic and we don’t expect to be able to make precise predictions at long time horizons



Stochastic Volatility Kernels

The *Volt* model

- Taking motivation from stochastic volatility models we introduce *Volt*
- Volt works in three steps:
  - Infer a latent volatility path
  - Use a latent GP to forecast volatilities into the future
  - Forecast in data space conditioned on the volatility forecasts



Volt

A Stochastic Volatility GP Kernel

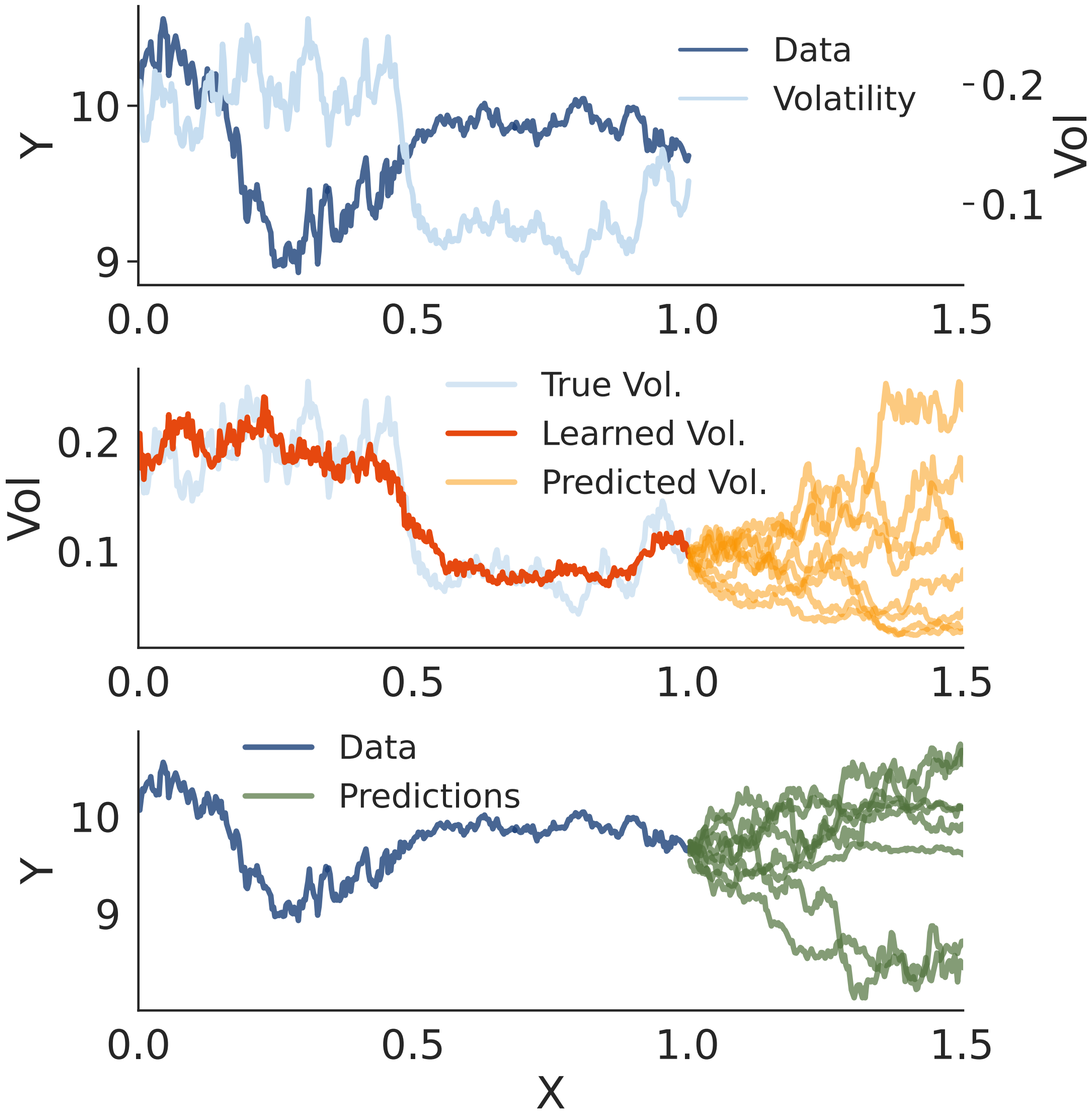
- With a data $S(t)$ and volatility $V(t)$ we assume the following joint SDE:

$$v(t) = \log V(t) \qquad dv(t) = -\frac{\sigma^2}{2}dt + \sigma dZ(t)$$
$$s(t) = \log S(t) \qquad ds(t) = \mu_s dt + V(t)dW(t)$$

- The corresponding kernel functions are then:

$$K_v(t, t') = \sigma^2 \min\{t, t'\}$$

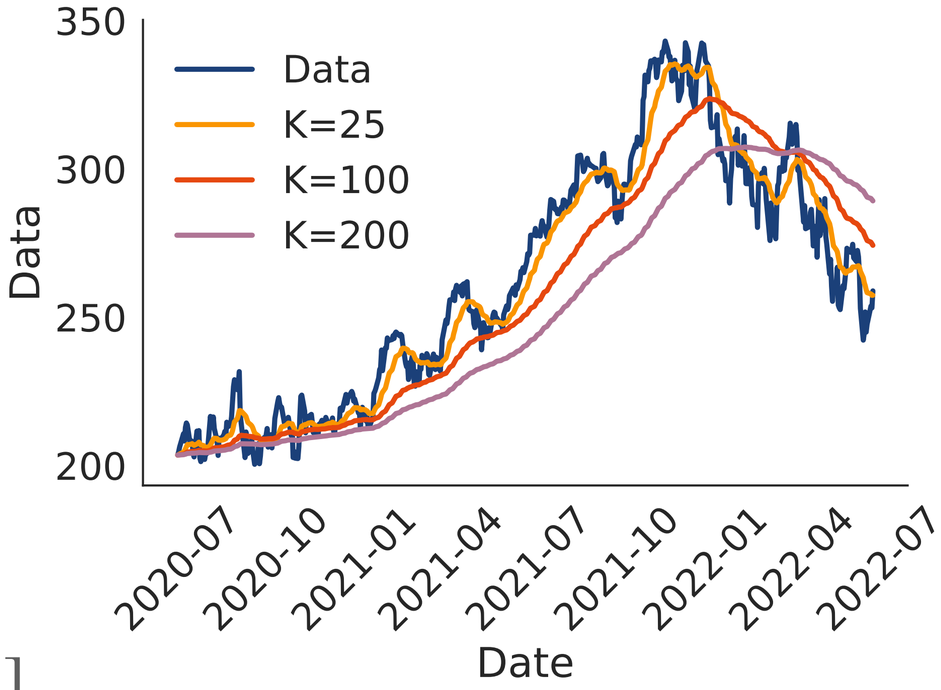
$$K_s(t, t'; V(t)) = \int_0^{\min\{(t,t')\}} V(t)^2 dt$$



Magpie

Moving Average Gaussian Processes

- Volt gives well calibrated uncertainties but ignores any mean trends in the data
- Moving average models have been widely used in climatology and quantitative finance
- We introduce *Magpie,* using exponential moving averages as a mean function in GPs

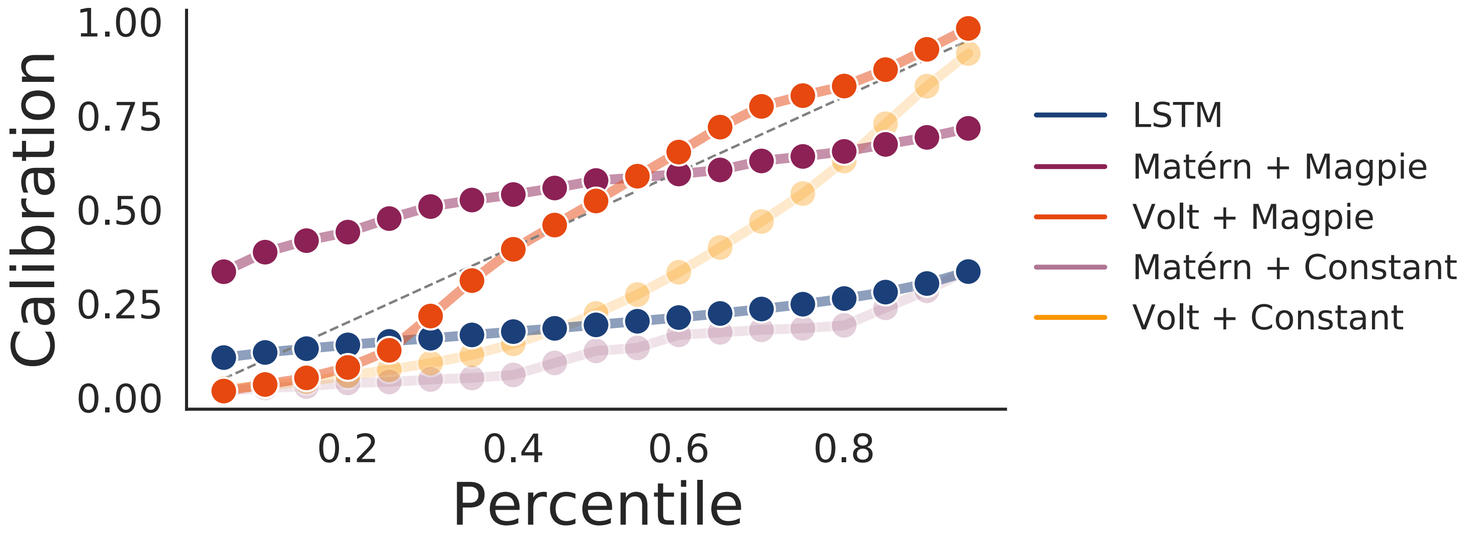


$$EMA(\mathbf{s})_{i+1} = \alpha[s_i + (1-\alpha)s_{i-1} + \cdots + (1-\alpha)^{k-1}s_{i-k-1)}]$$

$$\alpha = 2/(k+1)$$

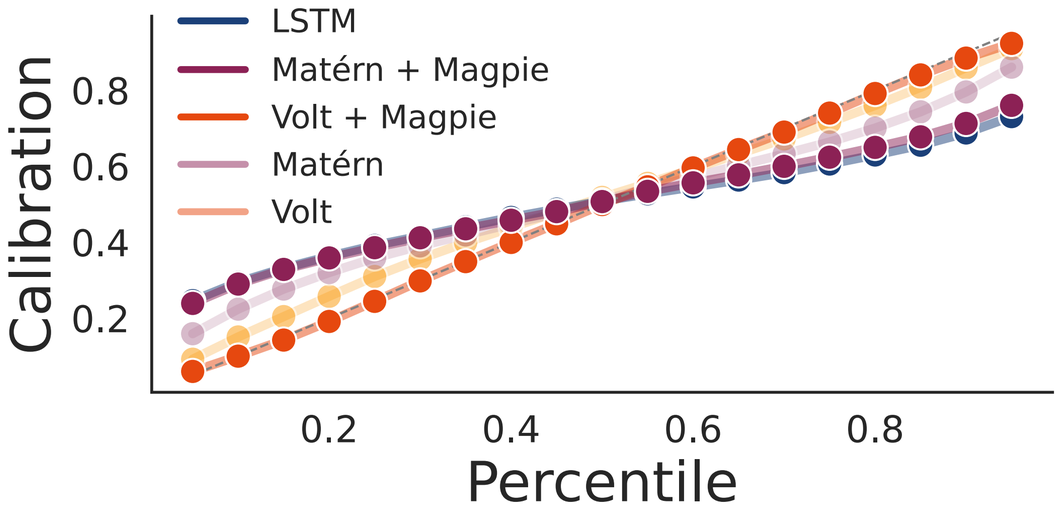
Calibration: Finance

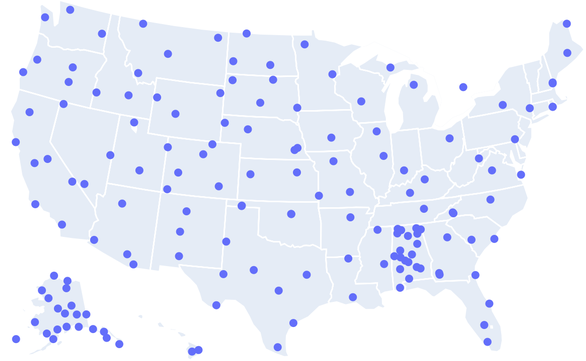
- Forecast 75-100 days into the future for the last 2 years of the NASDAQ 100
- Compare Volt and Magpie over tens of thousands of individual predictions
  - Together Volt and Magpie produce calibrated forecasts at all quantiles giving a reliable tool for predicting distributions

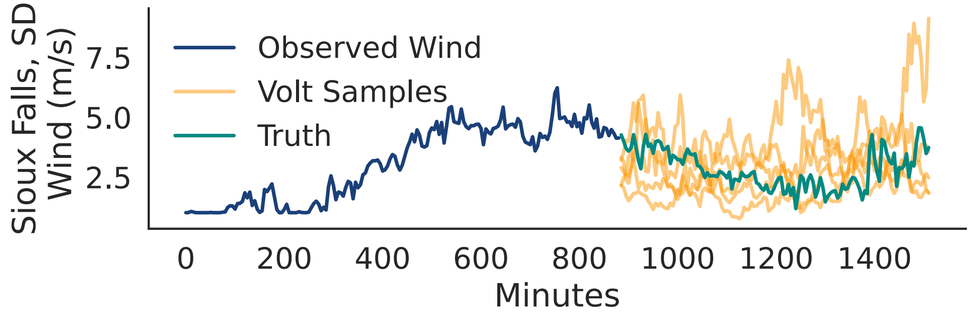


Calibration: Wind Speed

- U.S. Historical Climatological Network windspeed data at 154 spatial locations
- Aggregate hundreds of thousands of forecasts with prediction horizons up to 100 time steps
- Volt and Magpie produce nearly perfectly calibrated distributions
  - Practical use as a stochastic weather generator, or risk forecasting tool

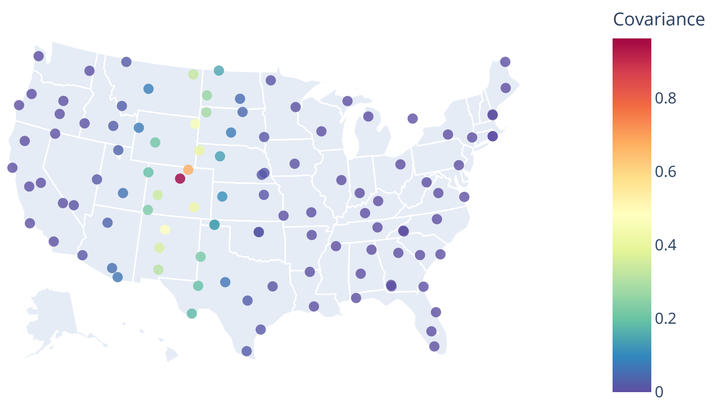


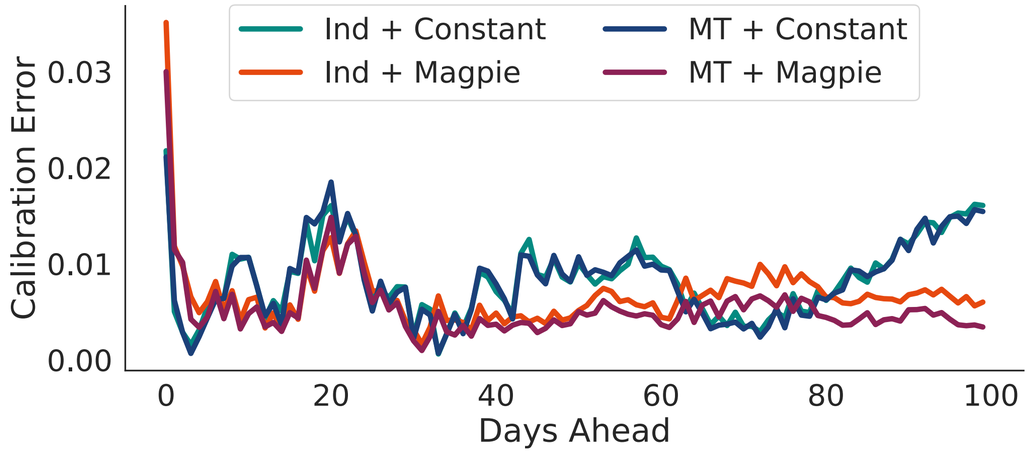




Multitask Volatility GPs

- With Volt we construct a novel multitask model through sharing the latent volatility across tasks
- Assume that the volatility of two processes are drawn from the same latent GP, then jointly forecast the volatility and compute the corresponding kernels
- Gives scientific insight into the relation between processes, and reduces calibration error at long horizons





Thank You

https://github.com/g-benton/volt

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