Exploiting Independent Instruments: Identification and Distribution Generalization

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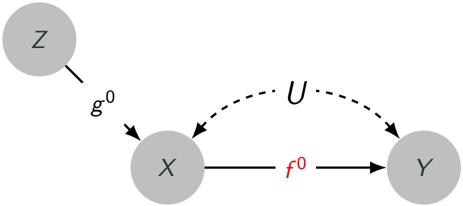
Instrumental Variable (IV) Setting

We consider the following structural causal model M^0

$$
\begin{aligned}
Z &:= \epsilon_Z \\
U &:= \epsilon_U \\
X &:= g^0(Z, U, \epsilon_X) \\
Y &:= f^0(X) + h^0(U, \epsilon_Y)
\end{aligned}
$$



where $Z \in \mathbb{R}^r$ are instruments, $U \in \mathbb{R}^q$ are unobserved variables, $X \in \mathbb{R}^d$ are predictors, $Y \in \mathbb{R}$ is a response, and $(\epsilon_Z, \epsilon_U, \epsilon_X, \epsilon_Y)$ are jointly independent noise variables. The causal function f^0 satisfies independence restriction $Y - f^0(X) \perp\!\!\!\perp Z$.

Identification of f^0: Moment restriction vs Independence restriction

E.g., consider a linear causal function $f^0(x) = x^\top \theta^0$ for some $\theta^0 \in \mathbb{R}^d$.

Classical IV approach

Identification of f^0 is based on the (conditional) moment restriction:

$$\mathbb{E}[Y - X^\top \theta \mid Z] = 0. \tag{1}$$

f^0 is not identifiable when $\mathbb{E}[X \mid Z] = 0$.

Independence-based IV

Identification of f^0 is based on the independence restriction:

$$Y - X^\top \theta \perp\!\!\!\perp Z. \tag{2}$$

We can identify f^0 even if $\mathbb{E}[X \mid Z] = 0$.

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The independence restriction (2) yields

(i) Strictly stronger identifiability results.

(ii) (in some settings) More efficient estimators (e.g., under weak instruments).

Independence-based IV with HSIC

Given $(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z})$, our method aims to find a function $\hat{f}$ that minimizes the dependency between the residuals $\boldsymbol{Y} - \hat{f}(\boldsymbol{X})$ and the instruments $\boldsymbol{Z}$.

We propose the HSIC-X ('X' for 'exogenous') estimator:

$$\hat{f} := \arg\min_{f \in \mathcal{F}} \widehat{\mathrm{HSIC}}(\boldsymbol{Y} - f(\boldsymbol{X}), \boldsymbol{Z}), \tag{3}$$

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Two heuristics to alleviate the non-convexity issue:

(i) Initialize the parameters in the first trial at the OLS/2SLS solutions.

(ii) Restarting heuristic: Test for the independence restriction at the solution. If the test is rejected, randomly re-initialize the parameters and restart the optimization.

Under-identified IV and Distribution Generalization

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Theorem [Generalization to interventions on Z]

Let $\ell : \mathbb{R} \to \mathbb{R}$ be a convex loss function and $\mathcal{I}$ be a set of interventions on Z. If the interventions $\mathcal{I}$ is 'strong enough', then

$$\inf_{f \in \mathcal{F}_{\text{inv}}} \mathbb{E}_{M^0}\big[\ell(Y - f(X))\big] = \inf_{f \in \mathcal{F}} \sup_{i \in \mathcal{I}} \mathbb{E}_{M^0(i)}\big[\ell(Y - f(X))\big], \tag{4}$$

where $\mathcal{F}_{\text{inv}} := \{f_\diamond \in \mathcal{F} \mid Z \perp\!\!\!\perp Y - f_\diamond(X) \text{ under } \mathbb{P}_{M^0}\}$ is the space of invariant functions.

Under-identified IV and Distribution Generalization

Motivated by (4), we propose the HSIC-X-pen ('pen' for 'penalization') estimator:

$$\hat{f}^{\lambda} = \arg\min_{f \in \mathcal{F}} \widehat{\text{HSIC}}(\boldsymbol{Y} - f(\boldsymbol{X}), \boldsymbol{Z})) + \lambda \sum_{i=1}^{n} \ell(Y_i - f(X_i)), \quad (5)$$

where the tuning parameter $\lambda \in [0, \infty)$ is selected as the largest possible value for which an HSIC-based independence test between the residuals and the instruments is not rejected.

Contributions

Three contributions:

(i) We discuss the use of the independence restriction $Y - f(X) \perp\!\!\!\perp Z$ in IV estimation and its implication on the identifiability of f^0.

(ii) We propose HSIC-X, a gradient-based learning method that exploits the independence restriction to estimate f^0 and prove its consistency.

(iii) We propose to use the independence restriction for distribution generalization and prove theoretical guarantees.

Have some questions? See you all at the poster session: Tue 19 Jul 6:30 p.m.