

FedNew: A Communication-Efficient and Privacy-Preserving Newton-Type Method for Federated Learning

Anis Elgabli, Chaouki Ben Issaid, Amrit Bedi, Ketan Rajawat, Mehdi Bennis, and Vaneet Aggarwal

The FL Problem :

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) := \frac{1}{n} \sum_{i=1}^n f_i(x) \right\}$$

Algorithms

First order approach

$$x^{k+1} = x^k - \alpha \nabla f(x^k) = x^k - \frac{\alpha}{n} \sum_{i=1}^n \nabla f_i(x^k)$$

+Pos

- Lower computation cost per iteration
- Lower communication cost per iteration

- Neg

- Slow convergence speed (depends on the condition number)

Second order approach: Standard Newton Method

$$x^{k+1} = x^k - \left(\sum_{i=1}^n \nabla^2 f_i(x^k) \right)^{-1} \sum_{i=1}^n \nabla f_i(x^k)$$

+Pos

- Fast convergence
- Local quadratic convergence rate (independent on the condition number)

-Neg

- Higher computation cost per iteration
- Higher communication cost per iteration

Newton direction is the solution to:

$$z(x^k) = \arg \min_{y \in \mathbb{R}^d} \frac{1}{2} y^T \nabla^2 f(x^k) y - y^T \nabla f(x^k).$$

 **Our Approach :**

$$\begin{aligned} \min_{y_i, y} \quad & \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{2} y_i^T (\nabla^2 f_i(x^k) + \alpha I) y_i - y_i^T \nabla f_i(x^k) \right) \\ \text{s.t.} \quad & y_i = y, \quad \forall i \in [n], \end{aligned}$$

- One-pass ADMM to approximate the newton direction followed by inexact Newton update.
- Only one (model sized) vector is transmitted at each iteration (Low comm cost).
- Privacy is preserved. No explicit transmission of the Hessian/gradient.

1. At each client i:

$$y_i^k = (H_i^k + \alpha I + \rho I)^{-1} (g_i^k - \lambda_i^{k-1} + \rho y^{k-1})$$

3. The dual variables' update

$$\lambda_i^k = \lambda_i^{k-1} + \rho (y_i^k - y^k).$$

2. At the PS:

$$y^k = \frac{1}{n} \sum_{i=1}^n (y_i^k + \lambda_i^{k-1} / \rho)$$

4. Newton update

$$x^{k+1} = x^k - y^k,$$

Simulation Results

Fig.1

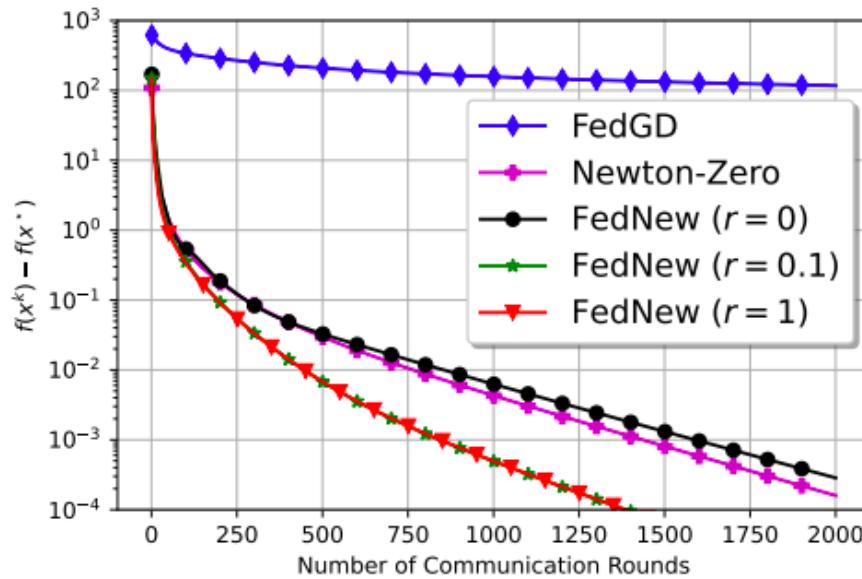
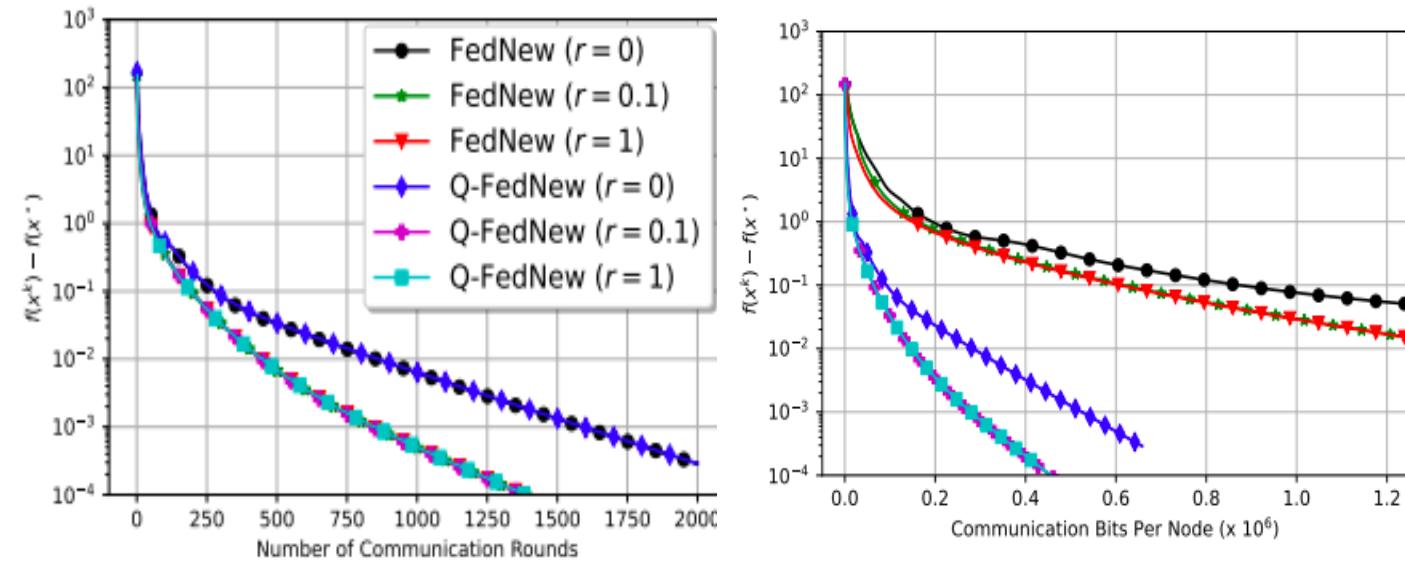


Fig.2



Dataset: a1a

Key messages from Fig 1:

- FedNew when using Zeroth hessian at each iteration (no hessian update to avoid matrix inversion, $r=0$) is as fast as Newton zero with better privacy guarantees.
- FedNew when updating the hessian at each iteration ($r=1$) converges faster at higher computation cost.
- FedNew with hessian update at every 10-th iterations is as fast as FedNew ($r=1$) with 10 times less computation cost.

Key message from Fig. 2

- Quantization leads to significant reduction in terms of the number of transmitted bits while achieving the same convergence speed of FedNew.

Conclusion and Future work

- We proposed a novel communication-efficient, and privacy-preserving federated learning framework based on Newton and ADMM methods.
- The proposed approach (FedNew) ensures privacy by hiding the gradient and the Hessian information.
- FedNew achieves the same communication-efficiency of first-order methods **per iteration** while enjoying **faster convergence** and **preserving privacy**.
- FedNew is proved to follow the inexact Newton direction asymptotically.
- A further reduction in communication overhead is achieved by utilizing stochastic quantization.
- Numerical results show the superiority of FedNew compared to existing methods in terms of communication costs while ensuring privacy.
- **Future works**
 - Convergence rate analysis of FedNew and Q-FedNew.
 - Extension of the current framework to fully decentralized topology.

Questions

- For any questions, please email me at: **anis.elgabli@oulu.fi**