







# Fast and Reliable Evaluation of Adversarial Robustness with Minimum-Margin Attack

Ruize Gao<sup>1</sup> Jiongxiao Wang<sup>1</sup> Kaiwen Zhou<sup>1</sup> Feng Liu<sup>2</sup> Binghui Xie<sup>1</sup> Gang Niu<sup>3</sup> Bo Han<sup>4</sup> James Cheng<sup>1</sup>

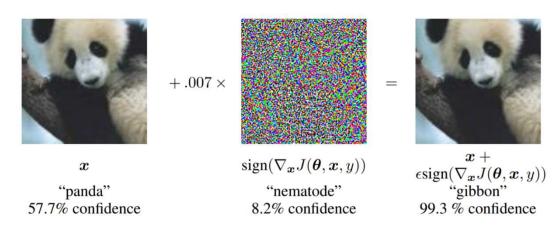
<sup>1</sup>Department of Computer Science and Engineering, The Chinese University of Hong Kong, HKSAR, China <sup>2</sup>School of Mathematics and Statistics, The University of Melbourne, Melbourne, Australia

<sup>3</sup>RIKEN Center for Advanced Intelligence Project (AIP), Tokyo, Japan

<sup>4</sup>Department of Computer Science, Hong Kong Baptist University, HKSAR, China



### The Deep Neural Networks are Vulnerable to Adversarial Examples



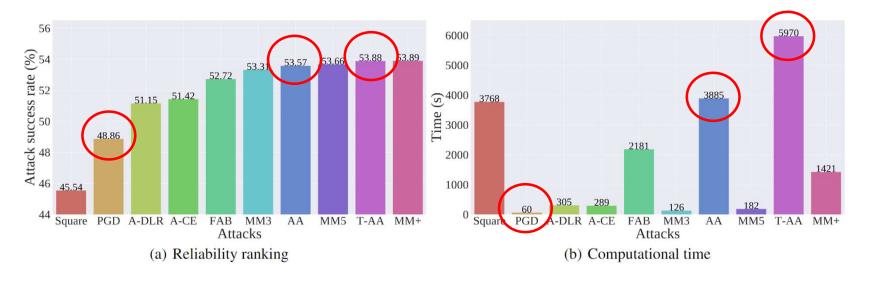
From: Explaining and Harnessing Adversarial Examples. In ICLR, 2015.

It's necessary to find a reliable way to evaluate adversarial robustness of a DNN.

Adversarial Attack: The Dilemma between Reliability and Computational Efficiency.

Benchmark 1: Projected Gradient Descent Attack (PGD), high computational efficiency but low reliability

Benchmark 2: The attack ensemble AutoAttack, high reliability but low computational efficiency



Note: MM3, MM5 and MM+ are different versions of our provided MM attack.

The necessary and sufficient condition to the complete robustness of the classifier.

**Condition 1.** Given a natural example x with its true label y, the K-class classifier f satisfies

$$\forall x' \in \mathcal{B}_{\epsilon}[x], z_y(x') - \max_{i \neq y} z_i(x') \ge 0,$$

where 
$$\mathcal{B}_{\epsilon}[x] = \{x' \mid d_{\infty}(x, x') \leq \epsilon\}; \ z_{y}(x') = f(x')_{y}; \ z_{i}(x') = f(x')_{i}.$$

According the **condition 1**, we define the most adversarial example.

**Definition 1** (The most adversarial example). Given a natural example x with its true label y, the most adversarial example  $x^*$  within  $\mathcal{B}_{\epsilon}[x]$  is defined as:

$$\forall x' \in \mathcal{B}_{\epsilon}[x], x^* = \underset{x'}{\operatorname{arg\,max}} -(z_y(x') - \underset{i \neq y}{\operatorname{max}} z_i(x')),$$

where  $\mathcal{B}_{\epsilon}[x] = \{x' \mid d_{\infty}(x, x') \leq \epsilon\}$  is the closed ball of radius  $\epsilon > 0$  centered at x;  $z_y(x') = f(x')_y$ ;  $z_i(x') = f(x')_i$ .

### Using margin to identify the "most adversarial example"

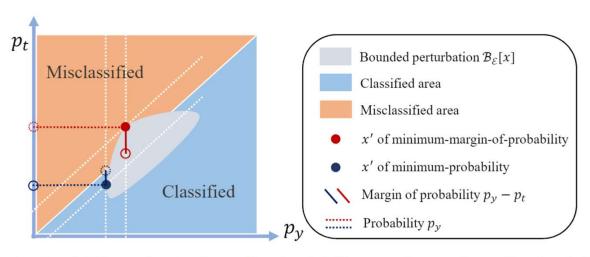


Figure 2. Minimum margin of probability. p denotes the predicted probability,  $p_y$  and  $p_t$  are the predicted probability on the true label y and a targeted false label t. The gray shape is the image of the adversarial variants x' within the bounded perturbation ball  $\mathcal{B}_{\epsilon}[x]$  under the mapping of the network onto  $(p_y, p_t)$ ; the orange area  $(p_t > p_y)$  indicates the region where the adversarial variants are misclassified, or to say a successful attack, while the blue area  $(p_t < p_y)$  indicates the region where the adversarial variants do not attack successfully.

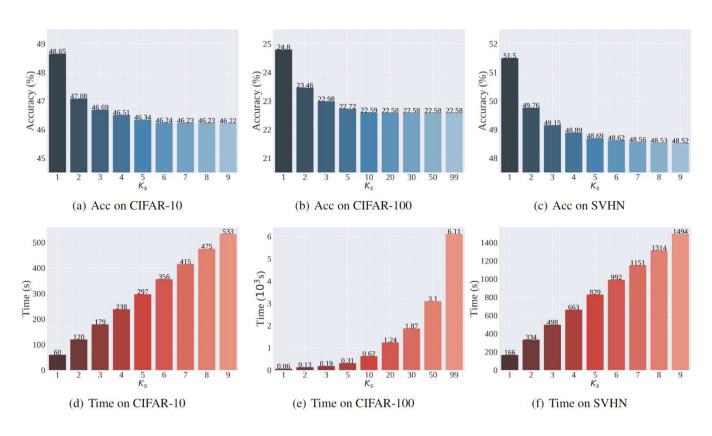
### Sequential TArget Ranking Selection (STATS)

#### 1)Pre-selecting-Targets Strategy:

Selecting partial targets achieves comparable performance.

#### 2) Ranking-Sequential-Attack Strategy:

Consider the false target with the highest predicted probability first; if the attack succeeds, then terminate attacks on other targets; otherwise, continue considering the false target with the second highest predicted probability.



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With the mentioned strategies, we summarize our scheme of MM attack.

Condition 2 and Condition 3 follow the setting of the adaptive step size selection in [1]:

Condition 2. 
$$\sum_{i=w_{j-1}}^{w_j-1} 1_{f(x'_{i+1}) > f(x'_i)} < \beta \cdot (w_j - w_{j-1}).$$

**Condition 3.**  $\alpha^{w_{j-1}} \equiv \alpha^{w_j}$  and  $f_{max}^{w_{j-1}} \equiv f_{max}^{w_j}$ .

#### Reference:

[1]:Croce, F. and Hein, M. Reliable evaluation of adversarial robustness with an ensemble of diverse parameter-free attacks. In ICML, 2020.

#### Algorithm 1 MM Attack

1: **Input:** natural data x, true label y, set of false labels C, model f, loss function  $\ell_{MM}$ , maximum number of PGD steps N, perturbation bound  $\epsilon$ , initial step size  $\alpha$ , the number of classes K, targets selection number  $K_s$ , checkpoints set W;

```
2: Output: adversarial data x';
 3: while K_s > 0 do
        x_0' \leftarrow x;
        x'_{max} \leftarrow x;
        f_{max} \leftarrow f(x_0');
 7: c = \arg\max_{i \in C} f(x)_i;
        for k = 0 to N - 1 do
           x'_{k+1} \leftarrow \prod_{\mathcal{B}_{\epsilon}[x]} (x'_k + \alpha sign(\nabla_{x'_k} \ell_{MM}(f(x'_k), y, c));
            if f(x'_{k+1}) > f_{max} then
10:
               x'_{max} \leftarrow x'_{k+1};
11:
              f_{max} \leftarrow f(x'_{k+1});
12:
13:
            if k \in W and (Condition 2 or Condition 3) then
14:
15:
               \alpha \leftarrow \alpha/2;
              x'_{k+1} \leftarrow x'_{max};
16:
17:
            end if
18:
        end for
19: C \leftarrow C \setminus \{c\};
        if \arg \max_{i \in C} f(x')_i \neq y then
            K_s \leftarrow 0;
21:
         end if
        K_s \leftarrow K_s - 1;
24: end while
```

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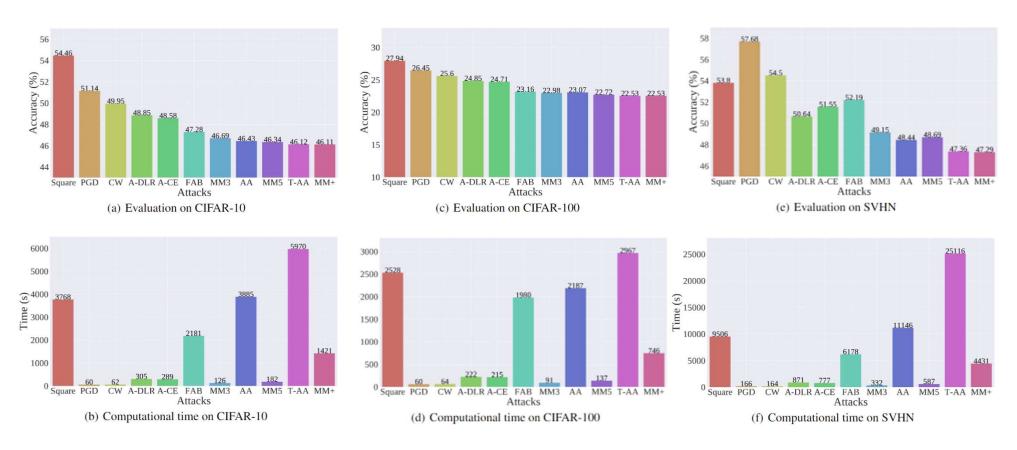
### Baselines:

- PGD: Projected Gradient Descent Attack [1]
- CW: Carlini and Wagner attack [2]
- A-DLR: PGD with adaptive step size and DLR loss [3]
- A-CE: PGD with adaptive step size and CE loss [3]
- FAB: A component of the AutoAttack ensemble [3]
- Square: A component of the AutoAttack ensemble [3]
- AA: AutoAttack with untargeted version [3]
- T-AA: AutoAttack with targeted version [3]

#### Reference:

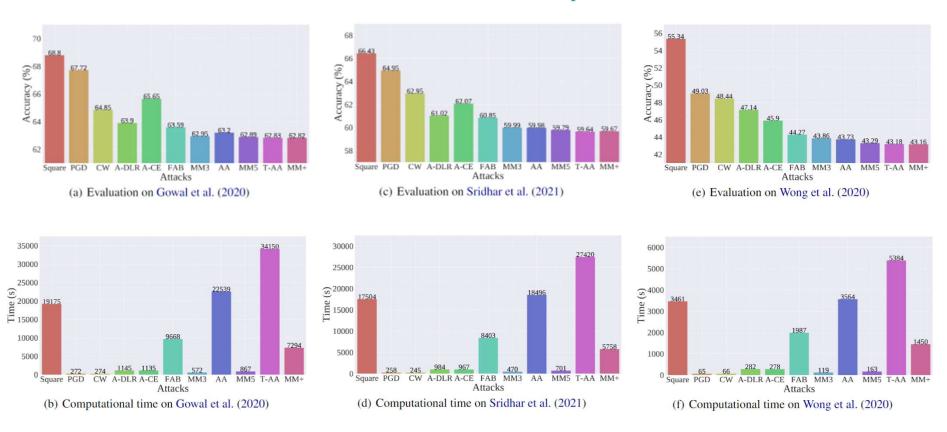
- [1]:Madry, A., Makelov, A., Schmidt, L., Tsipras, D., and Vladu, A. Towards deep learning models resistant toadversarial attacks. In ICLR, 2018.
- [2]:Carlini, N. and Wagner, D. Towards evaluating the robustness of neural networks. In CVPR, 2017.
- [3]:Croce, F. and Hein, M. Reliable evaluation of adversarial robustness with an ensemble of diverse parameter-free attacks. In ICML, 2020.

#### Main results on different datasets.



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### Main results on different well-trained models provided in RobustBench.



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# **Experiments**

### Adversarial Training with MM Attack.

#### Main results on CIFAR-10

Methods	PGD	Diff.	CW	Diff.	MM3-F10	Diff.	MM3-F20	Diff.	MM3	Diff.
PGD (Test)	51.14	-4.10	51.47	-3.77	54.96	-0.28	55.24	0.00	55.04	-0.20
CW (Test)	49.95	-1.89	53.26	0.00	51.18	-2.08	51.16	-2.10	51.84	-1.42
A-CE (Test)	48.58	-3.92	48.16	-4.34	51.55	-0.95	52.50	0.00	52.22	-0.28
A-DLR (Test)	48.85	-1.44	52.76	0.00	49.78	-2.98	49.88	-2.88	50.29	-2.47
FAB (Test)	47.28	-1.22	47.13	-1.37	47.83	-0.67	48.28	-0.22	48.50	-0.00
Square (Test)	54.46	-0.66	55.32	0.00	54.80	-0.52	54.83	-0.49	55.12	-0.20
AA (Test)	46.43	-1.85	46.36	-1.92	47.62	-0.66	47.84	-0.44	48.28	-0.00
T-AA (Test)	46.12	-0.97	45.26	-1.83	46.39	-0.70	46.73	-0.36	47.09	-0.00
MM3 (Test)	46.69	-1.17	46.77	-1.09	47.20	-0.66	47.48	-0.38	47.86	-0.00
MM9 (Test)	46.21	-0.95	45.36	-1.80	46.49	-0.67	46.82	-0.34	47.16	-0.00
MM+ (Test)	46.12	-0.90	45.22	-1.80	46.39	-0.63	46.68	-0.34	47.02	-0.00

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## Thank You!