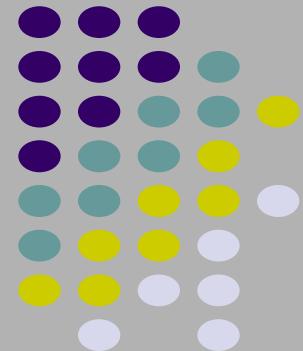
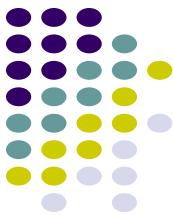


# Detached Error Feedback for Distributed SGD with Random Sparsification

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# Background

## Communication compression for distributed sgd

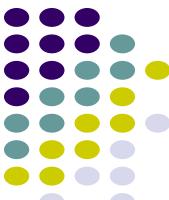
- Computation overheads
  - Top- $\kappa$  :  $O(\kappa \log_2 d)$ , inefficient on GPUs.
  - Random sparsification (RBGS):  $O(1)$ ; ring-allreduce
  - large compression error, inferior performance.

## Error Feedback

- Add current compression error to the next iteration
  - Assumptions 3.1 & 3.2 to bound the compression error

**Assumption 3.1.** (Bounded Variance)  $\forall \theta \in \mathbb{R}^d$ , the variance of the stochastic gradient satisfies  $\mathbb{E}_{\xi \in \mathcal{S}} \|\nabla f(\theta; \xi) - \nabla F_{\mathcal{S}}(\theta)\|_2^2 \leq \sigma^2$ .

**Assumption 3.2.** (Bounded Second Moment)  $\forall \theta \in \mathbb{R}^d$ , the second moment of the full gradient satisfies  $\|\nabla F_{\mathcal{S}}(\theta)\|_2^2 \leq M^2$ .



# DEF

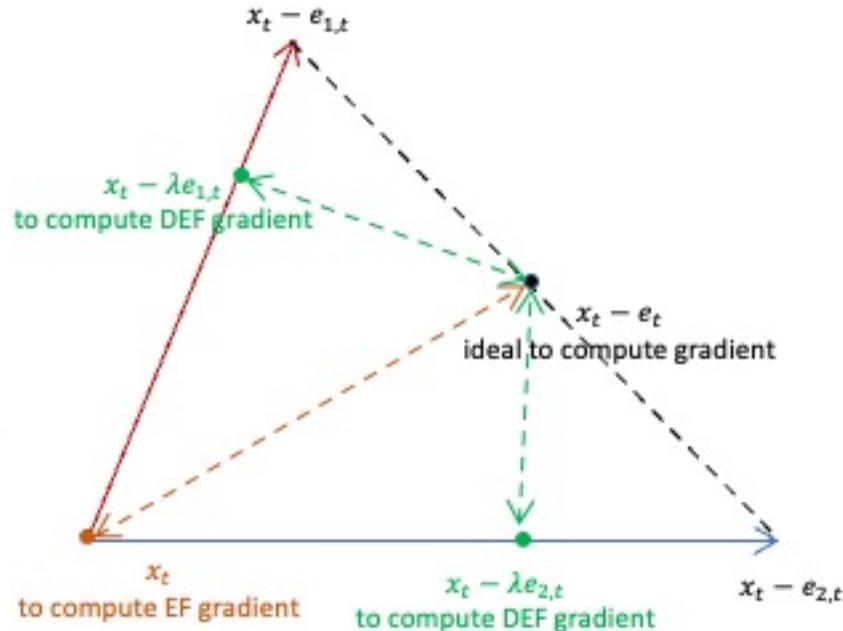
## Detached Error Feedback

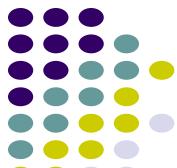
- Trade-off when second-moment cannot be bounded

$$(\alpha\sigma)^2 + ((1-\alpha)M)^2 \stackrel{\alpha = \frac{M^2}{\sigma^2 + M^2}}{\geq} \frac{\sigma^2 M^2}{\sigma^2 + M^2} \stackrel{M \gg \sigma}{=} \sigma^2$$

- Compute gradient at a different point

$$\frac{1}{K} \sum_{k=1}^K \|x_t - \lambda e_{k,t} - y_t\|_2^2 = \frac{1}{K} \sum_{k=1}^K \|e_t - \lambda e_{k,t}\|_2^2.$$





# Theoretical Results

## Trade-off for better convergence bounds without bounding second moment

- compression error  $O\left(\frac{2\sigma^2 M^2}{\sigma^2 + M^2}\right)$

## Trade-off for better excess risk

**Theorem 4.6.** (Excess Risk Error of DEF(-A), Appendix B)  
 Let Assumptions 3.1, 3.2, 3.3, 4.1 and 4.2 hold. Suppose  $\eta = \frac{c}{t+1}$ , where  $c > 0$  is some constant.

(1) The generalization error of DEF

$$\epsilon_{gen} = \mathcal{O}(T^{(1-\frac{K}{N})Lc}/((1-\frac{K}{N})Lc+1)). \quad (22)$$

(2) Suppose  $\eta \leq \frac{1}{4L}$ . The optimization error of DEF

$$\epsilon_{opt} = \tilde{\mathcal{O}}(T^{-\frac{\mu c}{2}} + T^{-1}). \quad (23)$$

(3) For RBGS, the generalization error of DEF-A

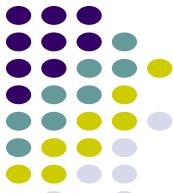
$$\epsilon_{gen} = \mathcal{O}(T^{(1-\frac{K}{N})\delta^{\frac{1}{2}}Lc}/((1-\frac{K}{N})\delta^{\frac{1}{2}}Lc+1)). \quad (24)$$

(4) Suppose  $\eta \leq \frac{1}{8L}$ . The optimization error of DEF-A

$$\epsilon_{opt} = \tilde{\mathcal{O}}(T^{-\frac{\mu\delta c}{2}} + T^{-1} + (1/\sqrt{1-\delta} - 1)^{-2}). \quad (25)$$

## Extends to iterate averaging (IA)

- IA is a special case of DEF-A with compressor  $C(\Delta) = \delta\Delta$
- Excess risk error analysis for IA
  - Trade-off for better excess risk of IA than SGD



# Experiments

Table 2. The ImageNet test accuracy (%) comparison under various compression ratio settings with ResNet-50.

Ratio	SGD	EF	SAEF	PSync	DEF	DEF-A
1	76.04	—	—	—	—	—
16	—	75.29 (↓ 0.75)	75.83 (↓ 0.21)	75.63 (↓ 0.41)	<u>75.98</u> (↓ 0.06)	<b>76.10</b> (↑ 0.06)
64	—	73.05 (↓ 2.99)	74.65 (↓ 1.39)	74.84 (↓ 1.20)	<u>76.16</u> (↑ 0.12)	<b>76.37</b> (↑ 0.33)
128	—	63.80 (↓ 12.2)	74.26 (↓ 1.78)	74.12 (↓ 1.92)	<b>76.17</b> (↑ 0.13)	<u>76.14</u> (↑ 0.10)
256	—	diverge	73.83 (↓ 2.21)	73.02 (↓ 3.02)	<u>75.71</u> (↓ 0.33)	<b>76.00</b> (↓ 0.04)
512	—	diverge	73.00 (↓ 3.04)	72.60 (↓ 3.44)	<u>75.52</u> (↓ 0.52)	<b>75.77</b> (↓ 0.27)
1024	—	diverge	71.89 (↓ 4.15)	71.82 (↓ 4.22)	<b>75.64</b> (↓ 0.40)	<u>75.57</u> (↓ 0.47)



## Take-aways

A new communication-efficient distributed training method DEF without bounding second moment.

First excessive risk analysis for communication-efficient distributed training.

Analysis can be extended to iterate averaging.

Empirical test accuracy improvement.

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