Streaming Algorithms for Support-Aware Histograms

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Histograms in Data Streams

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- Succinctly capture locality in the distribution and are easily interpretable
- Let H(k) be the set of all k-piece histograms over [n]
- **Goal:** Using small space, find a $f \in H(k')$ s.t.

$$\mathrm{err}(f, P) < \min_{f^* \in H(k)} \mathrm{err}(f^*, P) + \varepsilon$$

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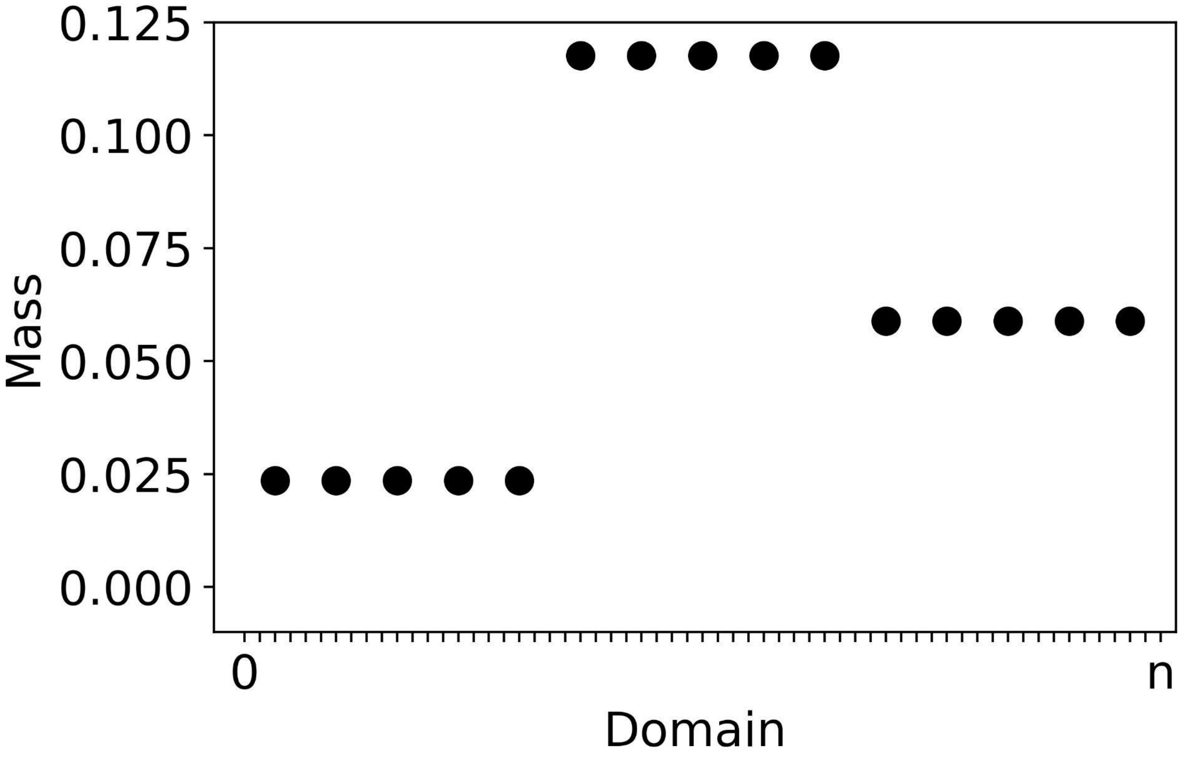
    - “Support-aware” error is a natural definition that captures simple structure in sparse data

A simple example (3-piece histogram)



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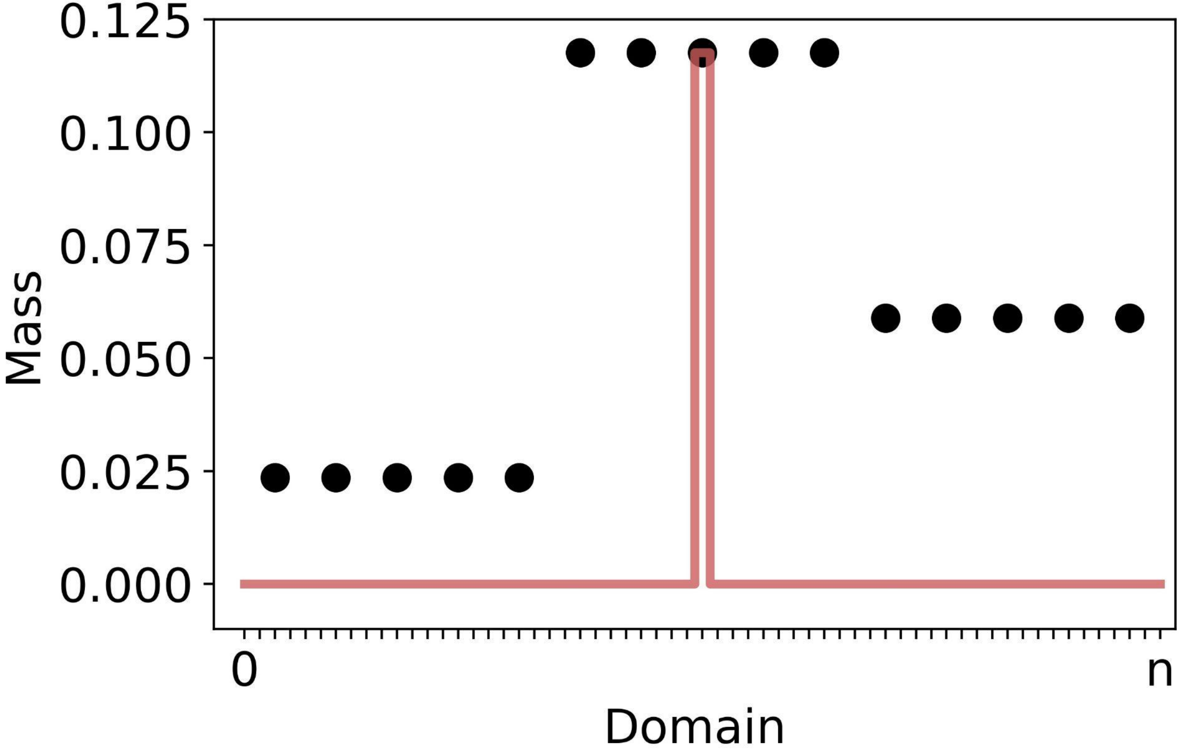

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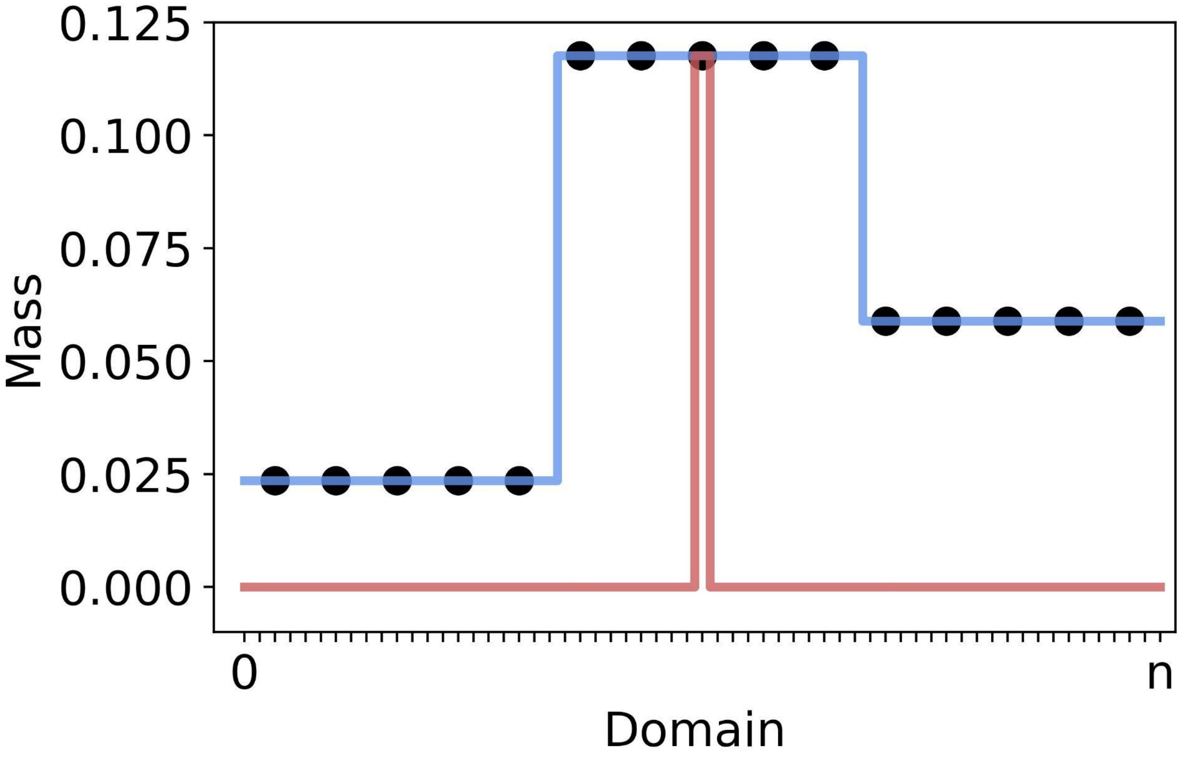

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Check out our paper and poster!