Simultaneous Graph Signal Clustering and Graph Learning

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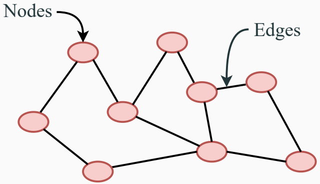
Graph Learning

- Graphs are used to study relational data.



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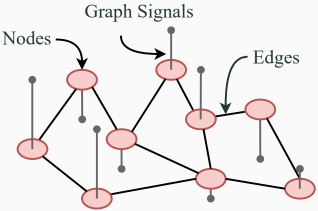

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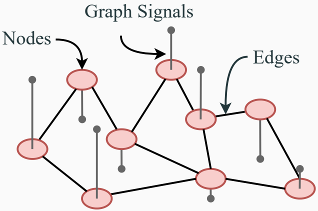
- Graphs are used to study relational data.
- Graph Signal Processing aims to extend ML/SP concepts to graph signals.
  - Signal recovery, sampling, filtering...



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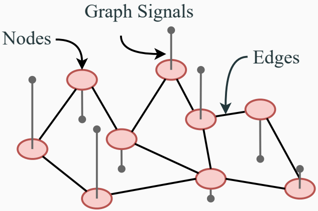

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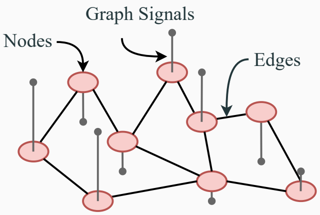

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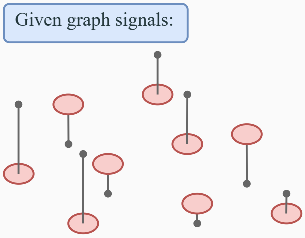

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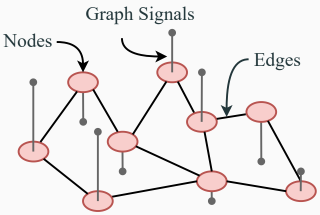

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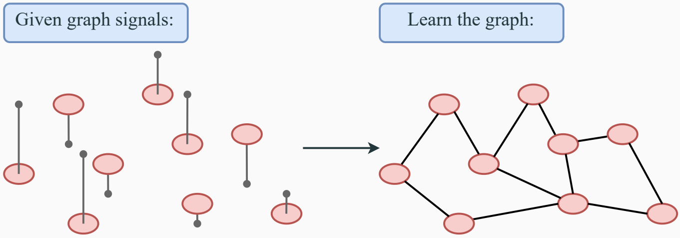

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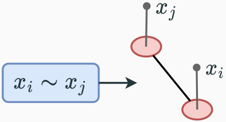

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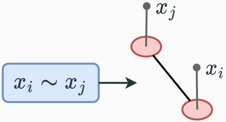
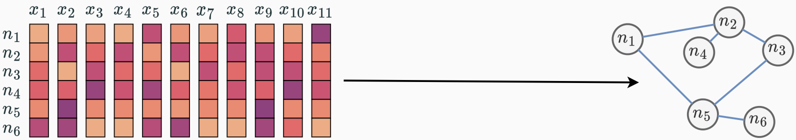


- These works mostly assume that data is homogeneous.



Graph Signals

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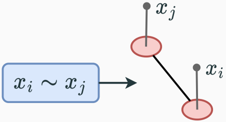
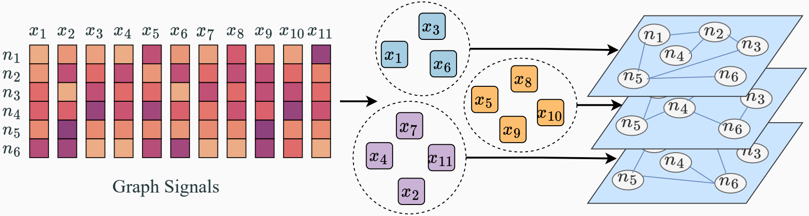


- However, many problems include heterogeneous data.



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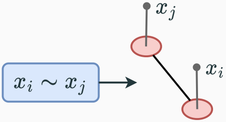

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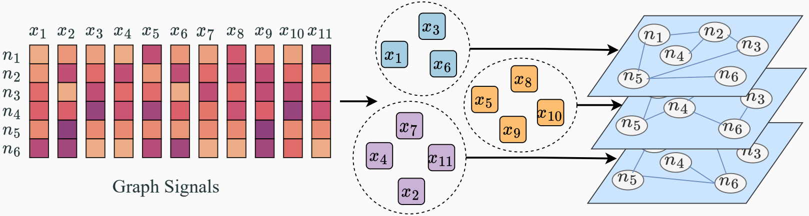
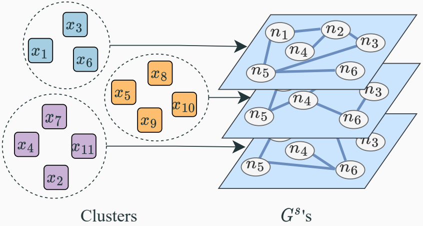


- Applications:
  - scRNAseq: Cluster cells while learning GRNs,
  - Recommendation: Cluster users while learning item graphs.

GRASCale

- Existing works use smoothness of the signals to perform clustering.



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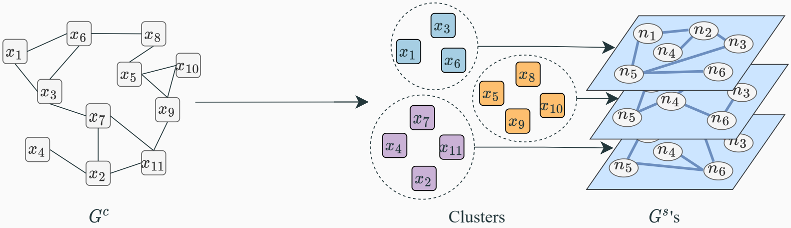

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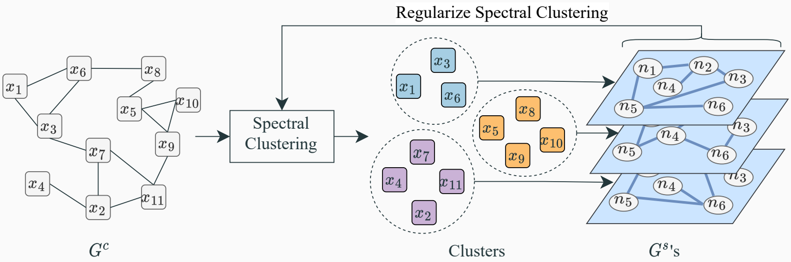

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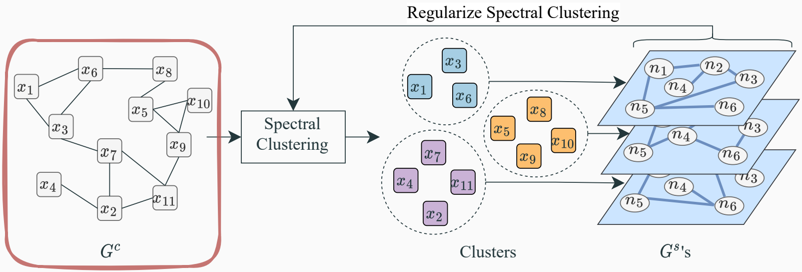

$$\underset{\mathsf{Z}\in\mathbb{R}^{n\times k}}{\text{minimize}} \quad \underbrace{\operatorname{tr}(\mathsf{Z}^\top \mathsf{L}^c \mathsf{Z})}_{\text{Graph Cut}} \qquad (1)$$

$$\text{s. t.} \quad \mathsf{Z} \in \mathbb{D}$$

- Z: Relaxation of 0-1 clustering indicator matrix,
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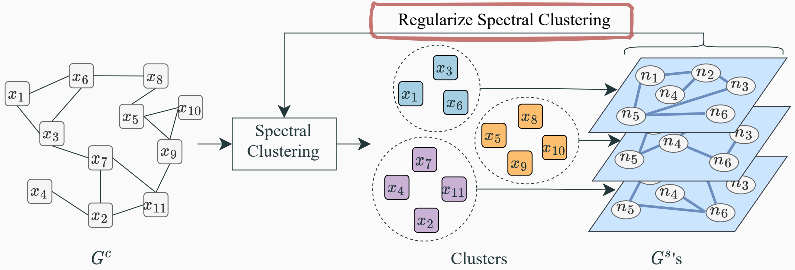

$$\begin{aligned} \underset{\mathbf{Z}\in\mathbb{R}^{n\times k}}{\text{minimize}} \quad & \underbrace{\mathrm{tr}(\mathbf{Z}^\top \mathbf{L}^c \mathbf{Z})}_{\text{Graph Cut}} + \alpha_1 \sum_{s=1}^{k} \underbrace{\mathrm{tr}(\mathrm{dg}(\mathbf{Z}_{\cdot s})\mathbf{X}^\top \mathbf{L}^s \mathbf{X})}_{\text{Smoothness}} \\ \text{s. t.} \quad & \mathbf{Z} \in \mathbb{D} \end{aligned} \tag{1}$$

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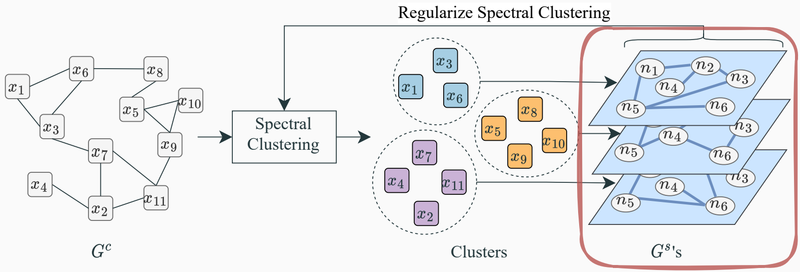

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$$\text{s. t.} \quad \mathbf{Z} \in \mathbb{D},\; \mathbf{L}^s \in \mathbb{L},\; \mathrm{tr}(\mathbf{L}^s) = 2n \;\forall s \in \{1, \ldots, k\}$$

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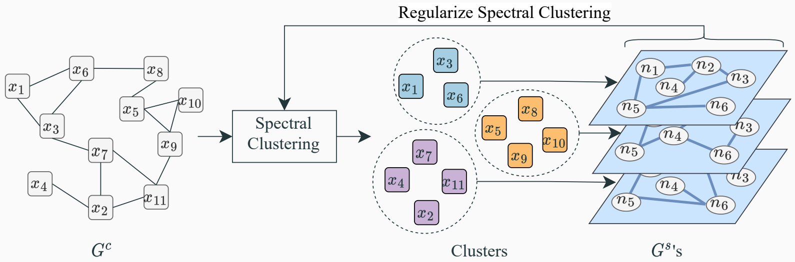

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- Solve with Block Coordinate Descent with prox-linear updates.

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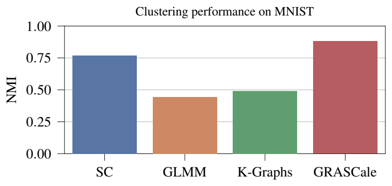

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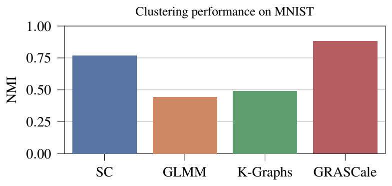

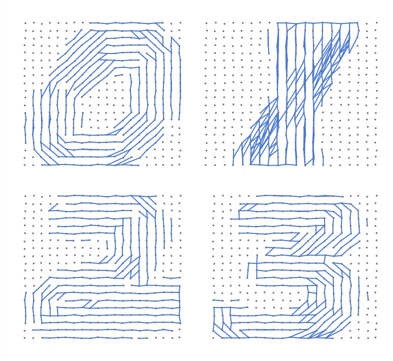

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  - Smoothness of the signals with respect to graphs associated with clusters.