Reward-Free RL is No Harder Than Reward-Aware RL in Linear Markov Decision Processes

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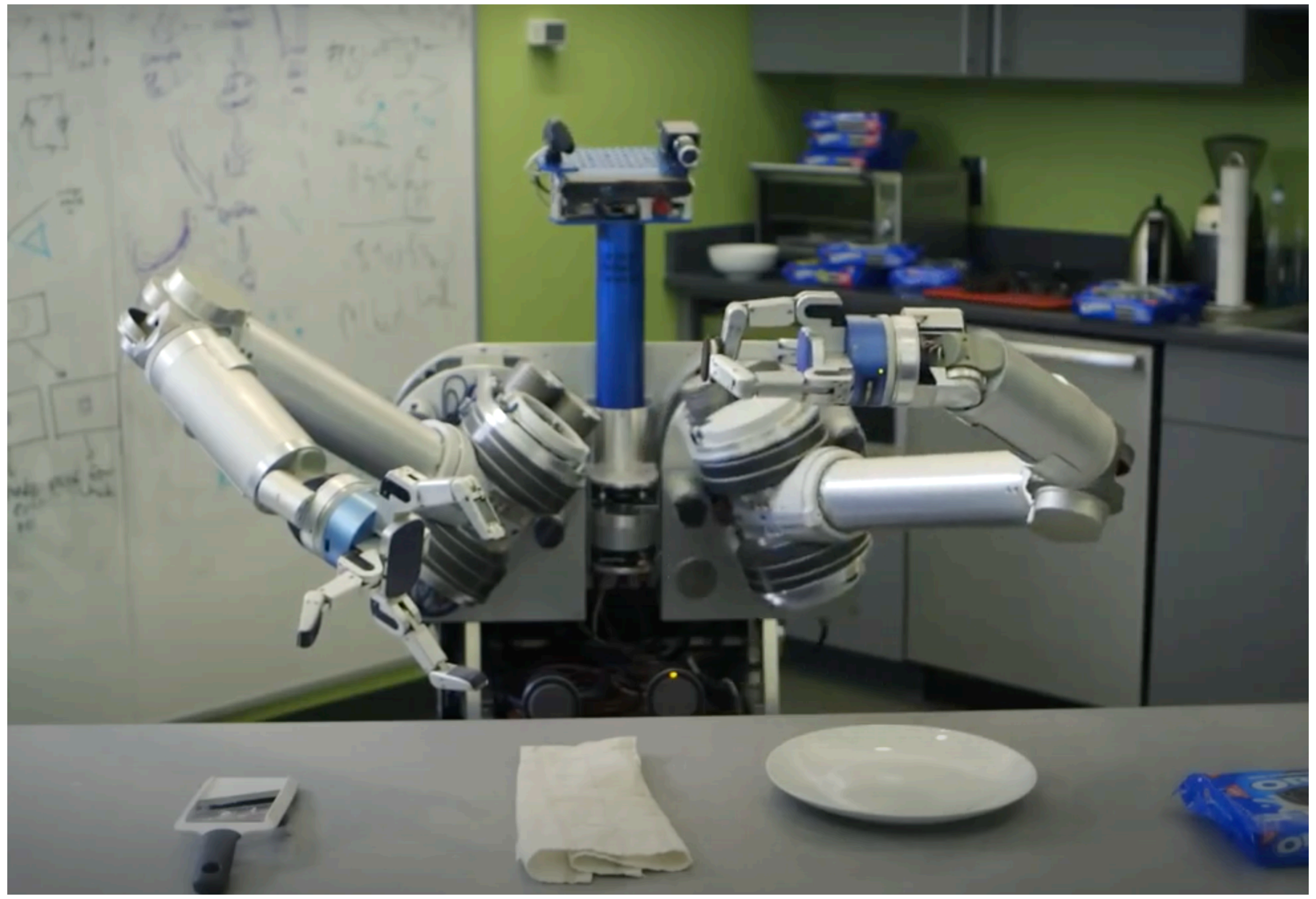

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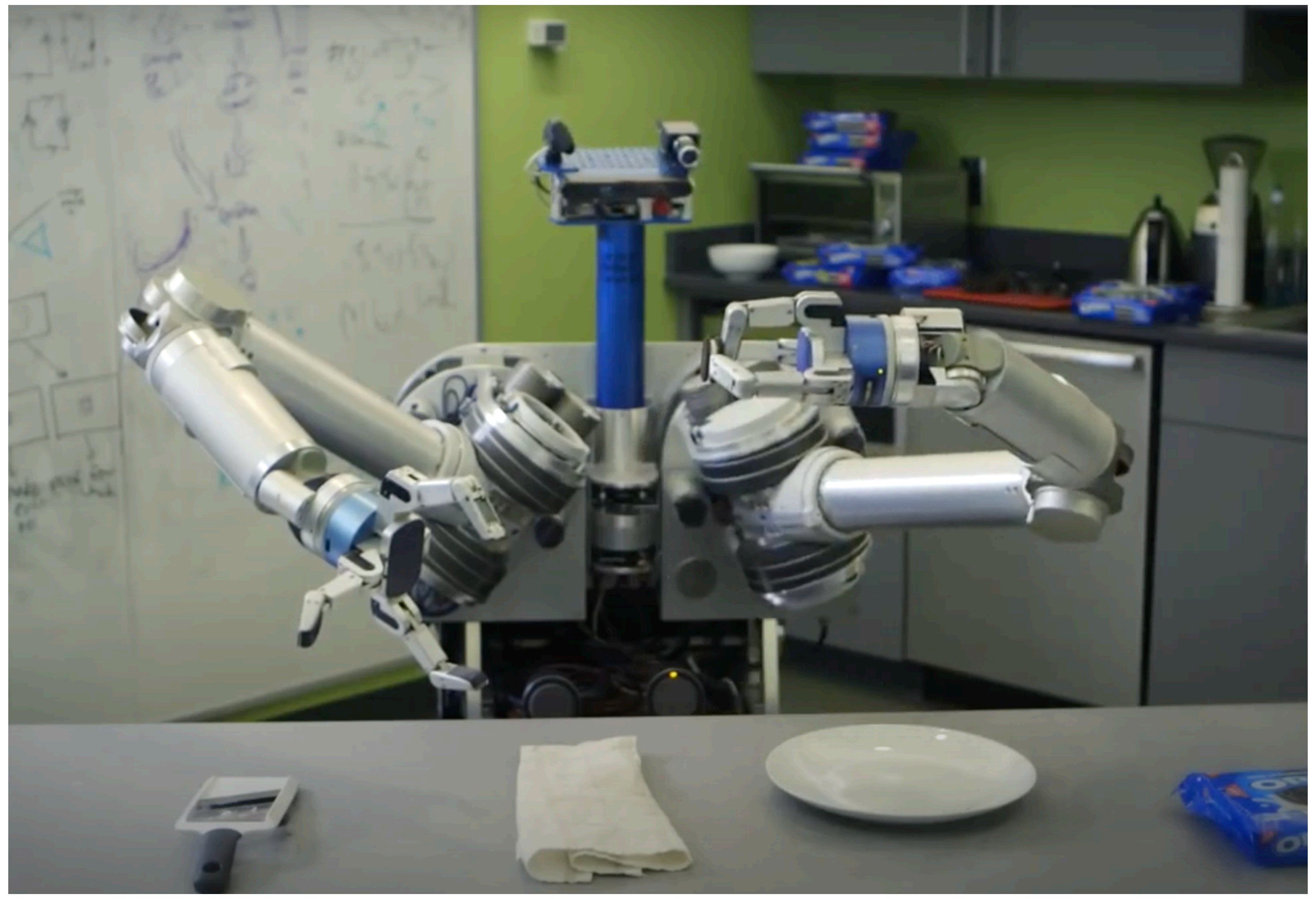
Intuitively, we would expect reward-free RL to be harder than reward-aware RL



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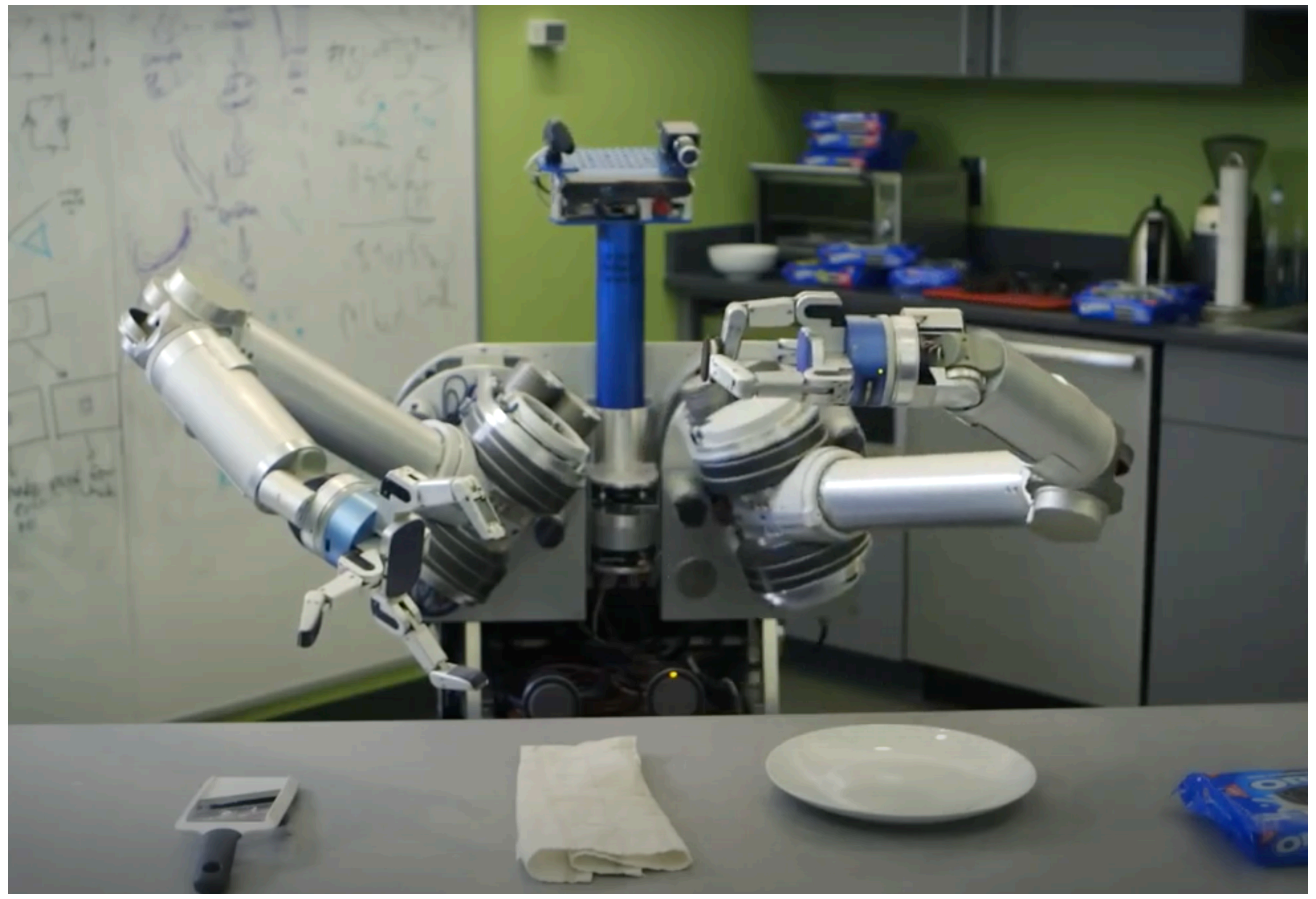

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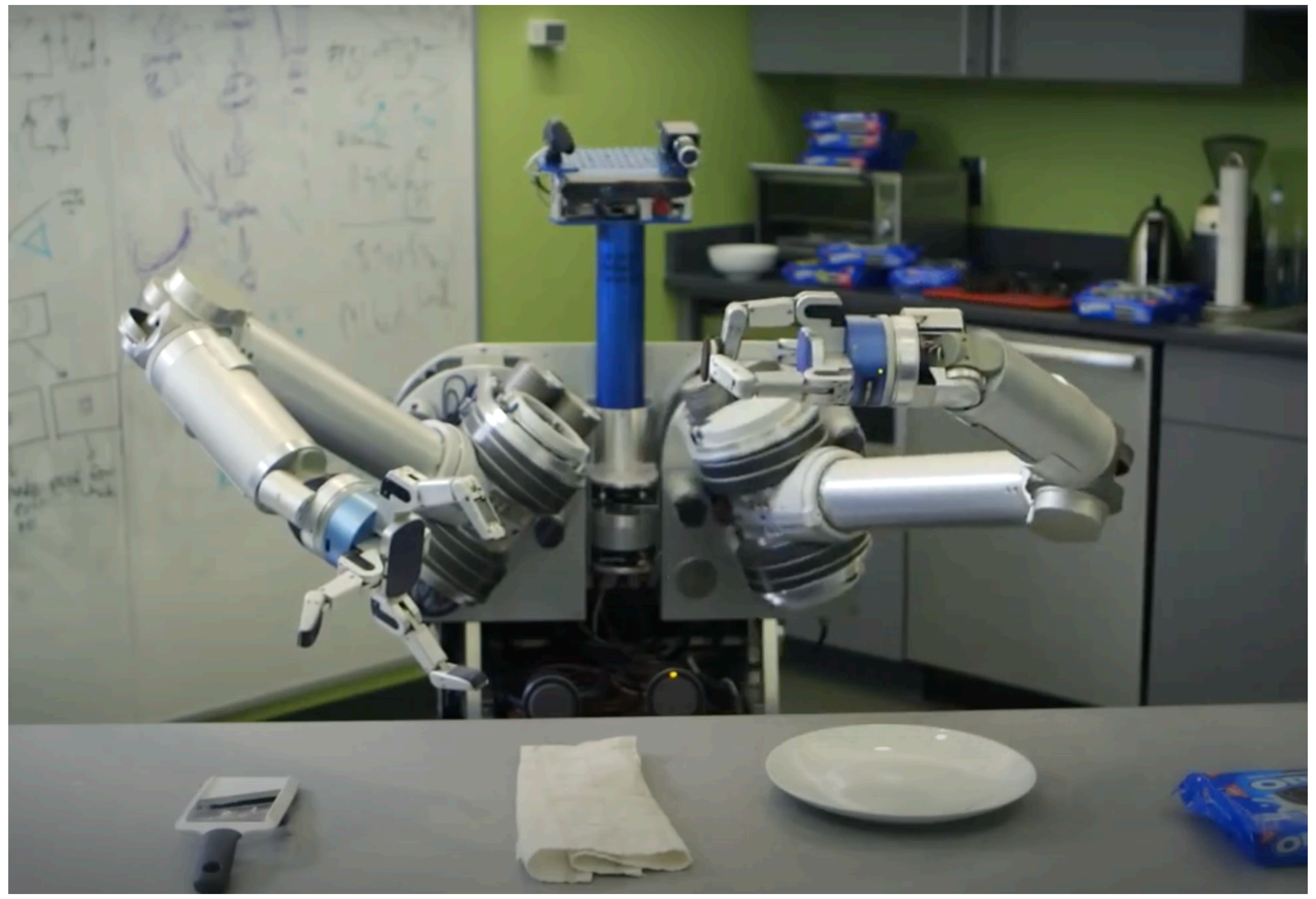
Optimal rate for **reward-aware** RL = $\Theta\left(\frac{SA}{\epsilon^2}\right)$



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In **tabular** MDPs:

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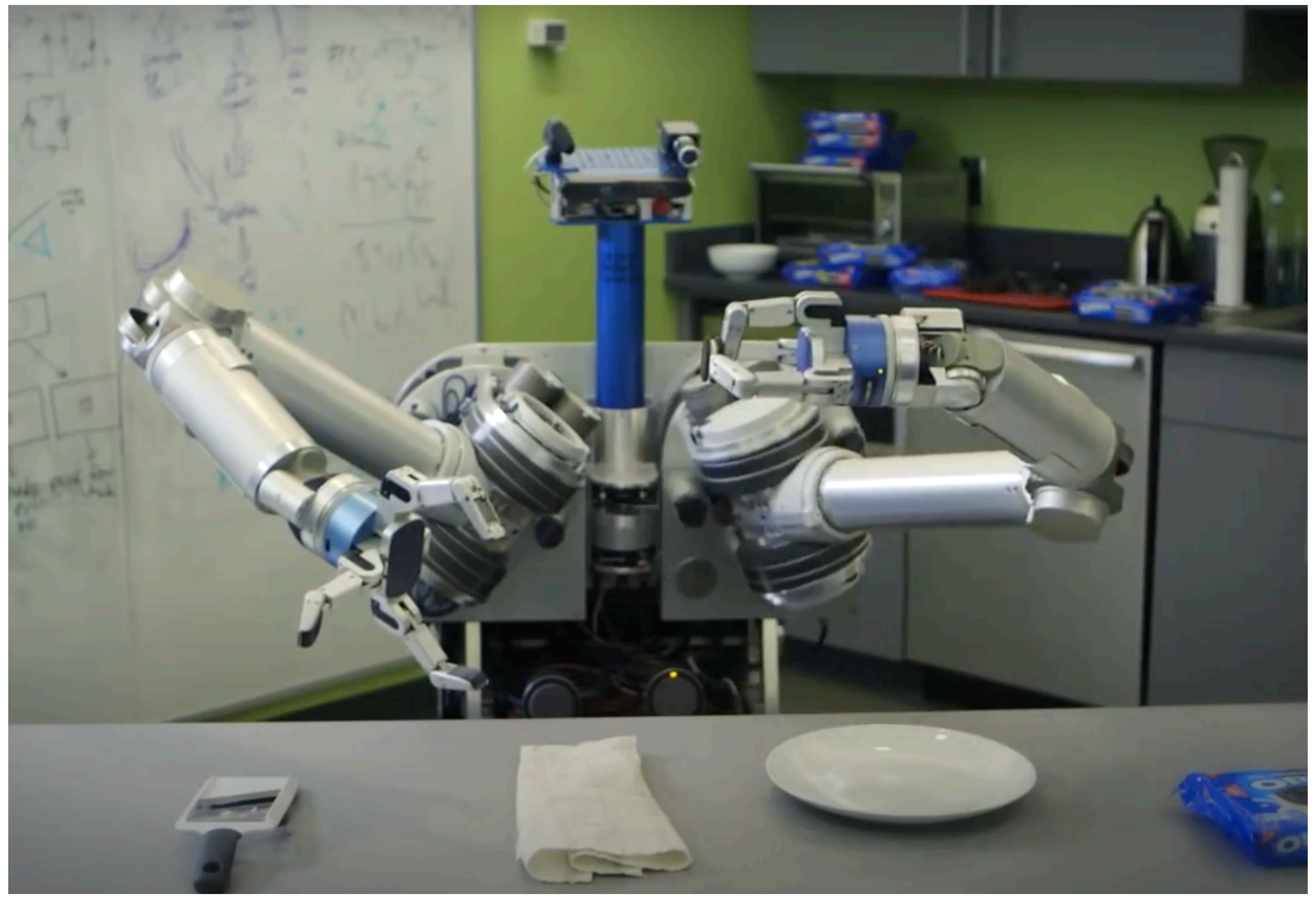
Optimal rate for **reward-free** RL = $\Theta\left(\frac{S^2A}{\epsilon^2}\right)$



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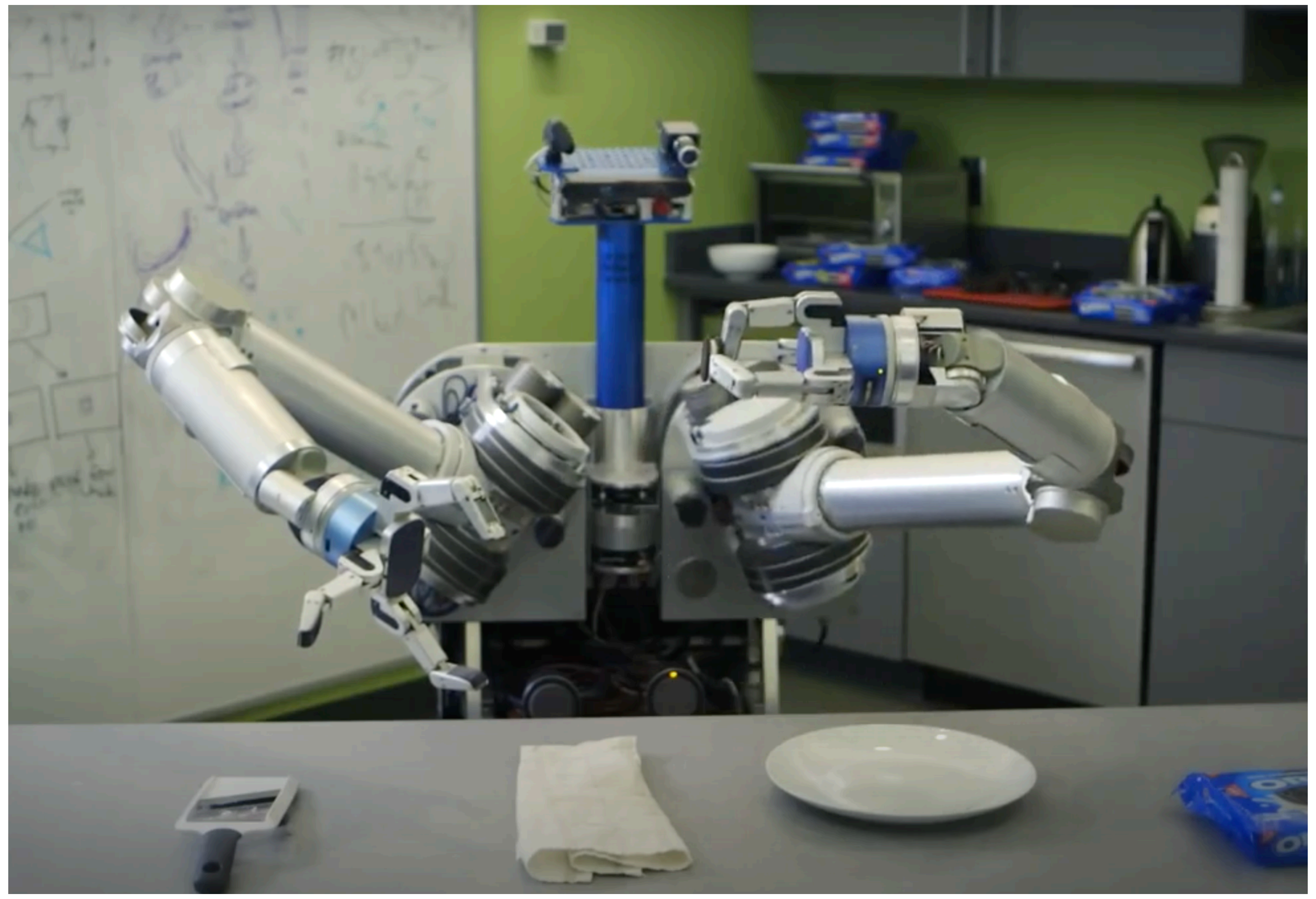
In real-world settings, state spaces are often large or infinite, and we must turn to **function approximation**



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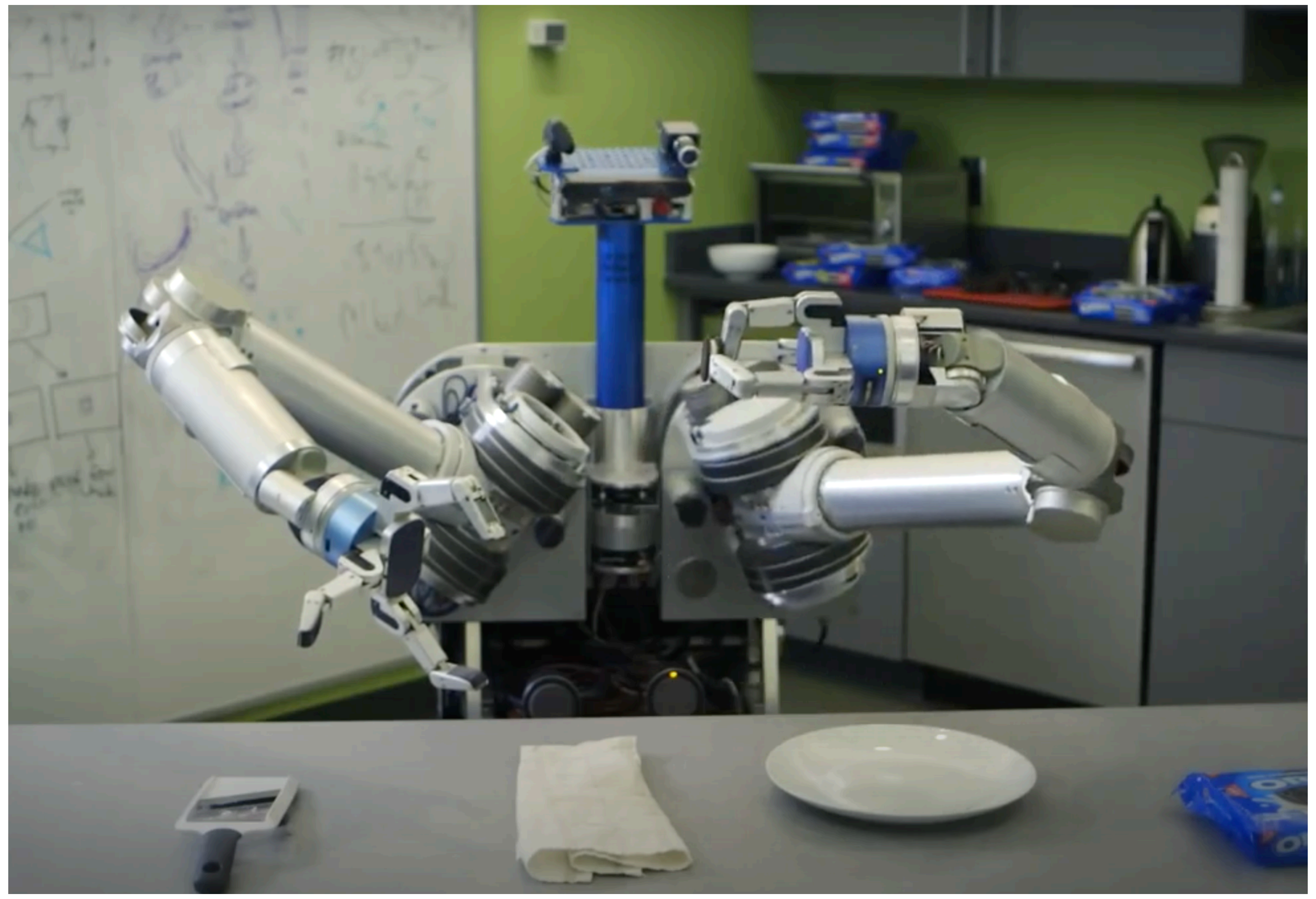

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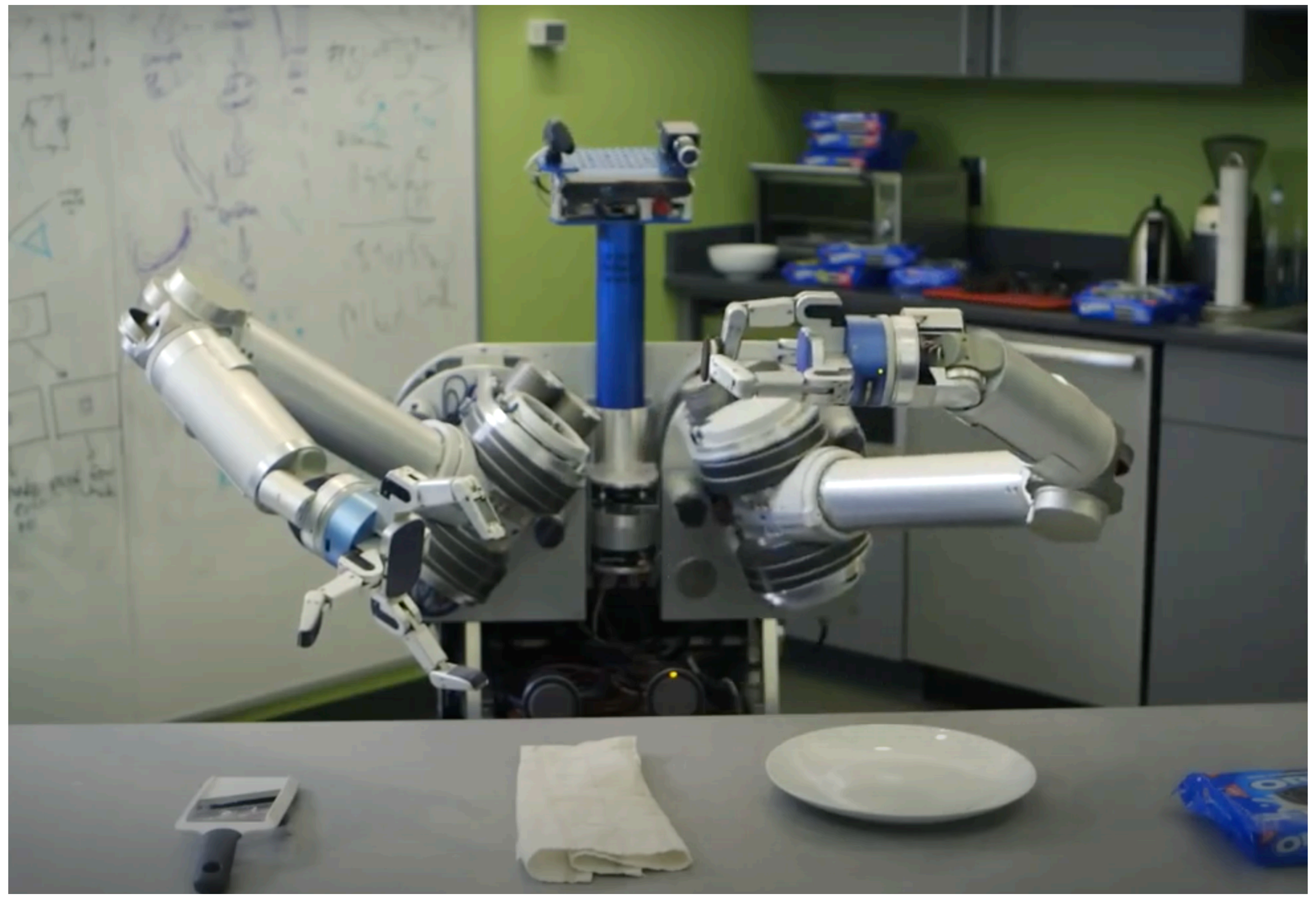
We consider **linear MDPs** (Jin et al., 2020), parameterized by d-dimensional feature vectors ϕ:

$$P_h(s' | s, a) = \langle \phi(s, a), \mu_h(s') \rangle, \quad r_h(s, a) = \langle \phi(s, a), \theta_h \rangle$$



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In the setting of Linear MDPs, we develop a computationally efficient **reward-free** algorithm with complexity of:

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Our results are the first **dimension-optimal, computationally efficient** bounds for linear MDPs

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Given data from exploring, construct an "optimistic" policy using a least-squares value-iteration procedure

Thanks!