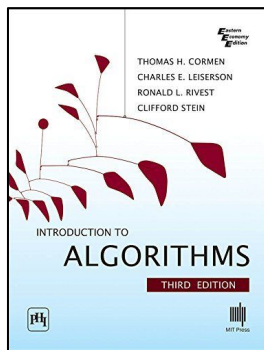# Faster Fundamental Graph Algorithms via Learned Predictions
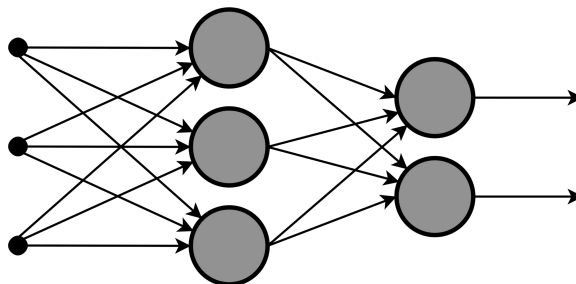
Justin Chen, Sandeep Silwal, Ali Vakilian, Fred Zhang

Learning-Augmented Algorithms

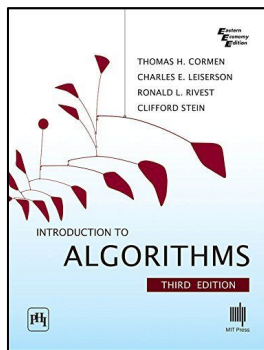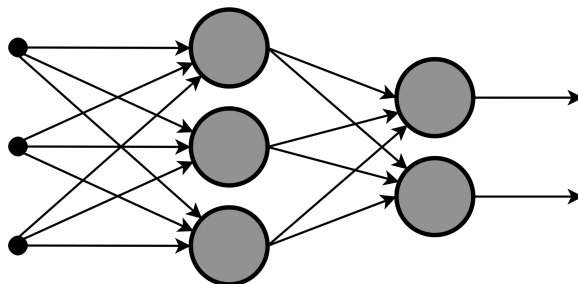New view of algorithm design: rigorous algorithmic analysis with ML help

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Learning-Augmented Algorithms

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Most results have focused on online, streaming, or sampling settings where predictions yield knowledge about the unseen part of the input

Faster Matchings via Learned Duals (Dinitz, Im, Lavastida, Moseley, Vassilvitskii '21)

- ○ Learning-augmented framework applied to runtime analysis for batch graph problems

- ○ Minimum cost matching via the Hungarian algorithm

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○ Used learned predictions of the duals of the matching problem to "warm-start" the Hungarian algorithm

○ Theoretically analyze the complexity of learning a predicted dual, rounding it to feasibility, and the resulting runtime parameterized by the prediction error

Our Results

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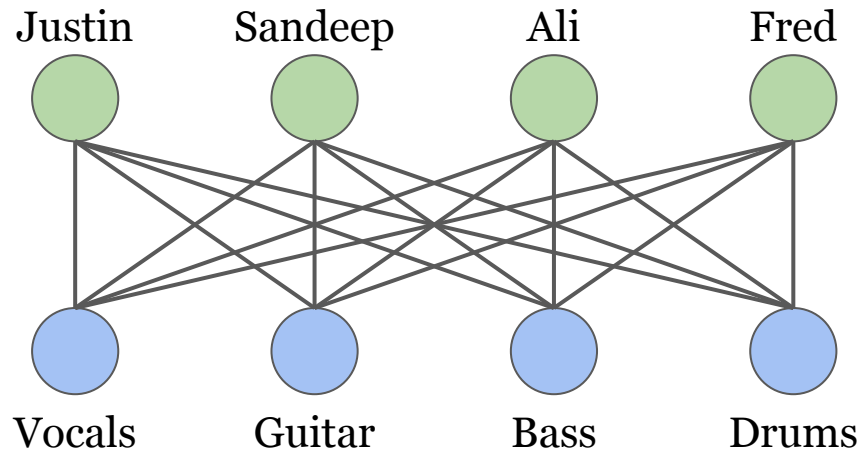
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○ Bipartite graph G = (V, E) with non-negative integral edge-costs

○ Find a perfect matching that minimizes the summed cost
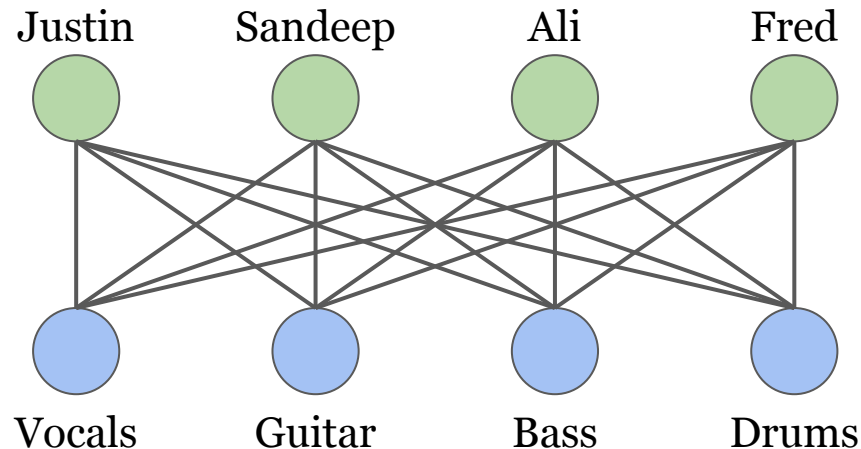
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- ○ Hungarian algorithm (primal-dual): O(mn) runtime

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- Minor change to the algorithm combined with improved analysis

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Check out our paper and poster!