A Random Matrix Analysis of Data Stream Clustering: Coping With Limited Memory Resources

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Random matrix framework for data stream clustering

- Observed data: $\mathbf{x}_1; \mathbf{x}_2; \dots; \mathbf{x}_t; \dots \in \mathbb{R}^p$ $\quad \mathbf{x}_t \overset{\text{i.i.d.}}{\sim} N(\quad ; \mathbf{I}_p)$

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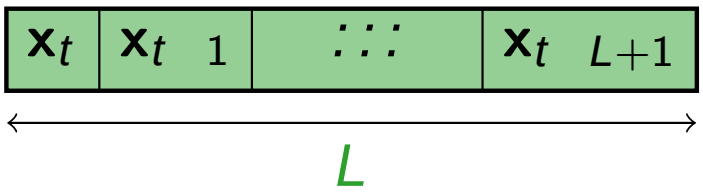
- Memory



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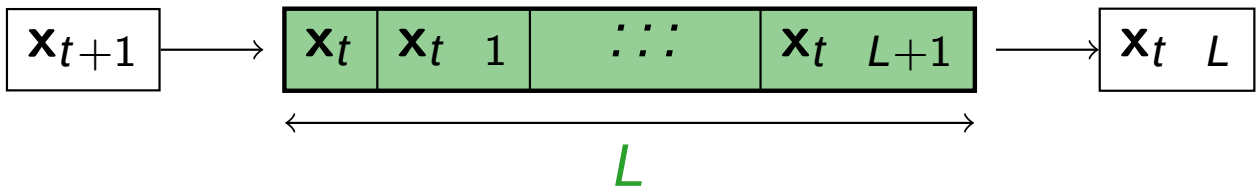

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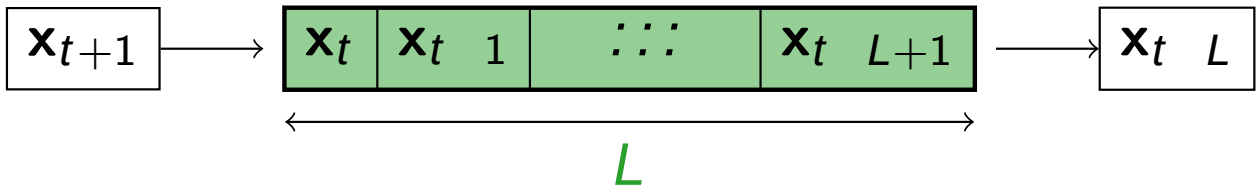

- **Clustering?**

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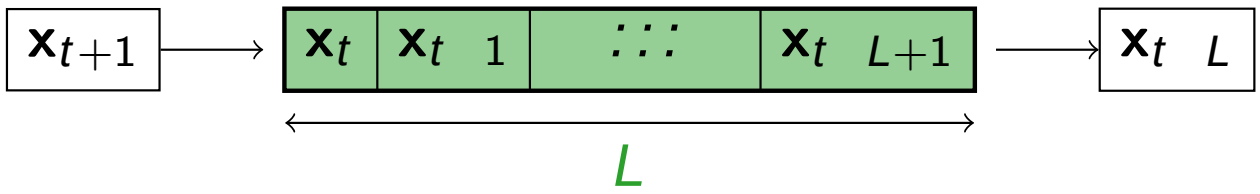

- **Clustering** on the $n > L$ previous points
  $\mathbf{X} = \mathbf{x}_{t\ n+1}\ \mathbf{x}_{t\ n+2}\ \ldots\ \mathbf{x}_{t\ 2} \in \mathbb{R}^{p\ n}$

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- Observed data: $\mathbf{x}_1; \mathbf{x}_2; \ldots; \mathbf{x}_t; \ldots\ 2\ \mathbb{R}^p$ $\quad$ $\mathbf{x}_t \ \overset{\text{i.i.d.}}{}\ N(\quad; \mathbf{I}_p)$
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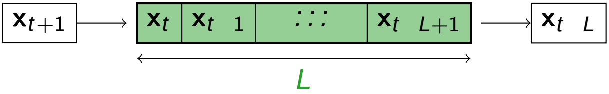

$n; p; L \quad ! \quad +1$

$p{=}n \quad ! \quad c$

$(2L \quad 1){=}n \quad ! \quad "$

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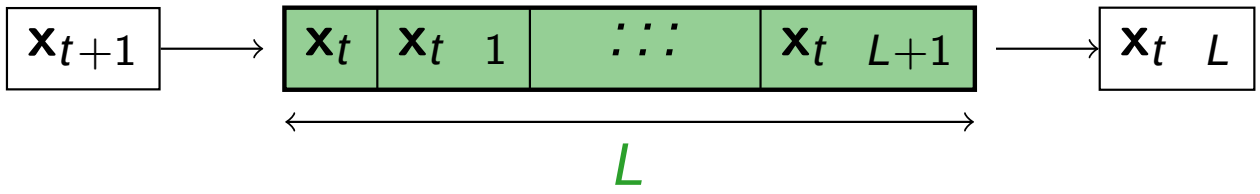

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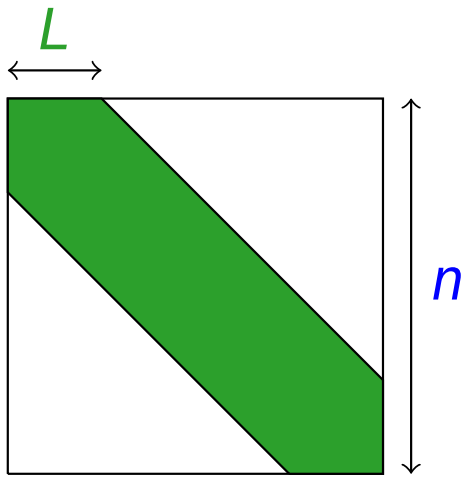
- Kernel matrix



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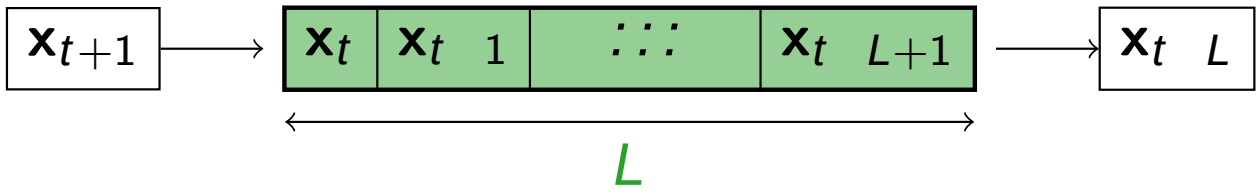

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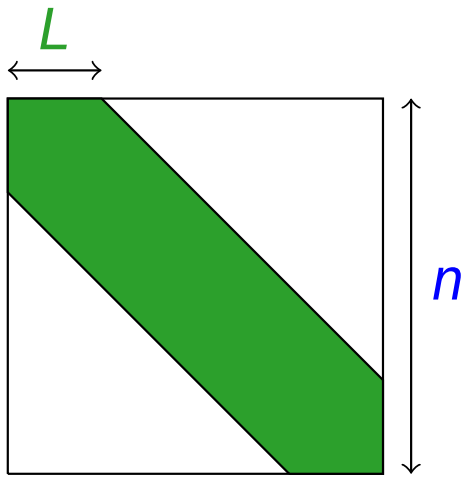

$$\mathbf{K} = \frac{\mathbf{X}^{>}\mathbf{X}}{p}$$

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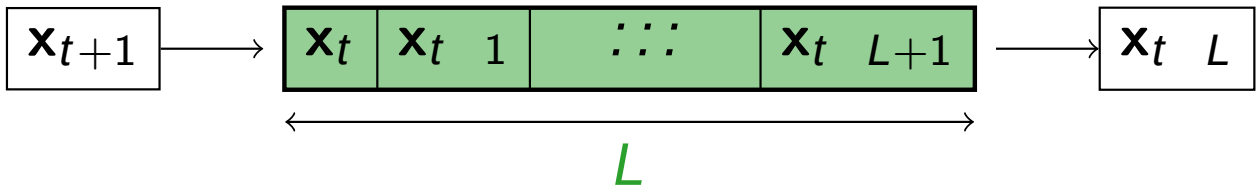

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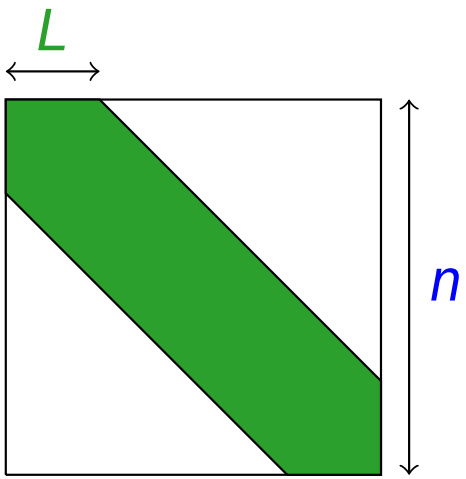

$$\mathbf{K}_L = \frac{\mathbf{X}^\top \mathbf{X}}{p}$$

$$\begin{bmatrix} 2 & 1 & \cdots & 1 & 0 & 3 \\ 6 & \ddots & & & \ddots & 7 \\ 6 & & & & & 7 \\ 6 & & & & & 7 \\ 4 & 1 & & & 1 & 5 \\ & \ddots & & & \ddots & \vdots \\ | & \underbrace{0 \qquad 1 \quad \cdots \quad 1}_{T} & & & & \} \end{bmatrix}$$

Data stream clustering: improving over batch clustering

- Size-L memory *batch clustering* vs. $\mathbf{K}_L = \frac{1}{p}\mathbf{X}^{\top}\mathbf{X}$ T

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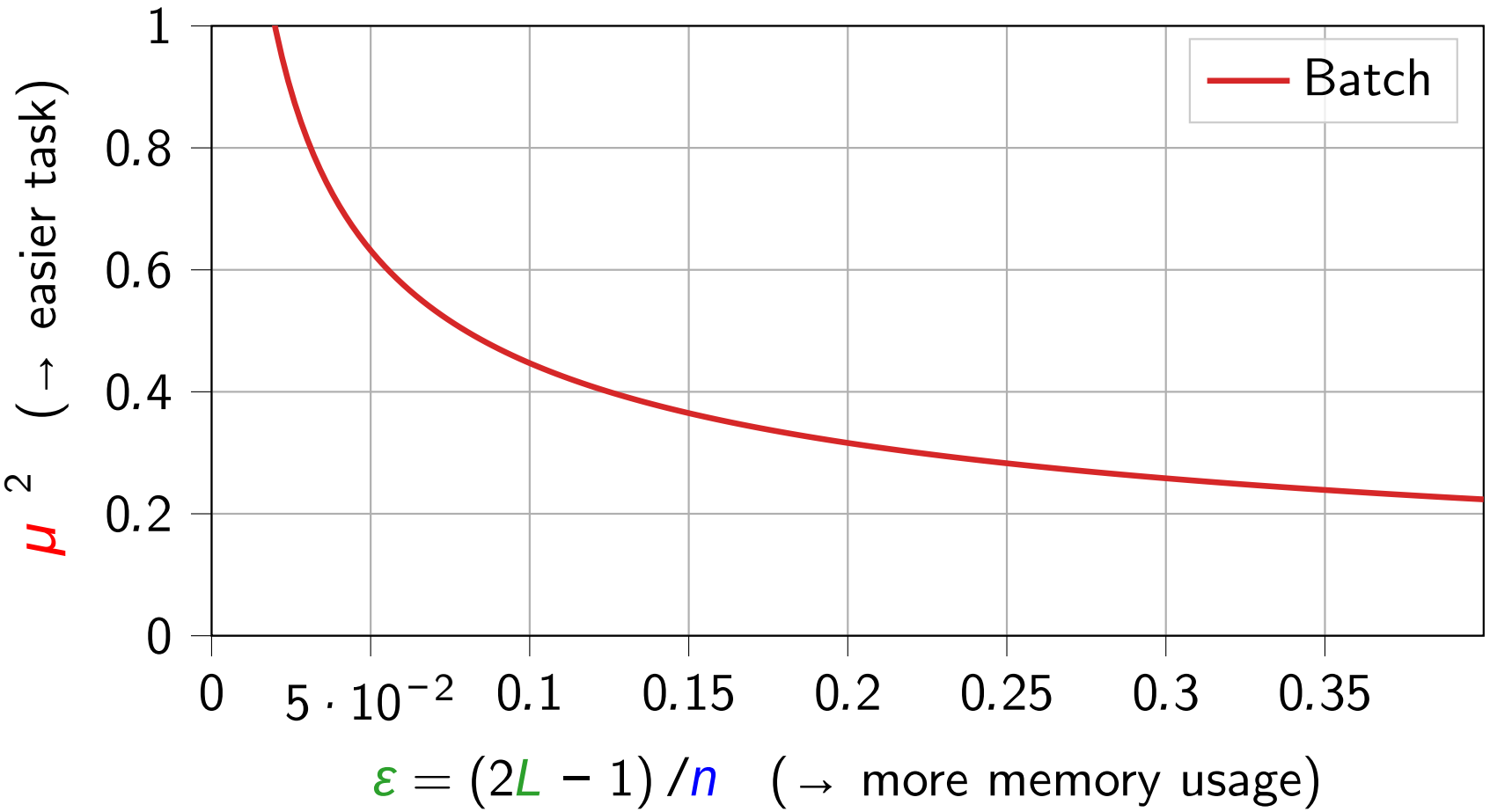
- **Spectral clustering phase transition** (n=$p = 100$ ( ) $c = 0.01$)



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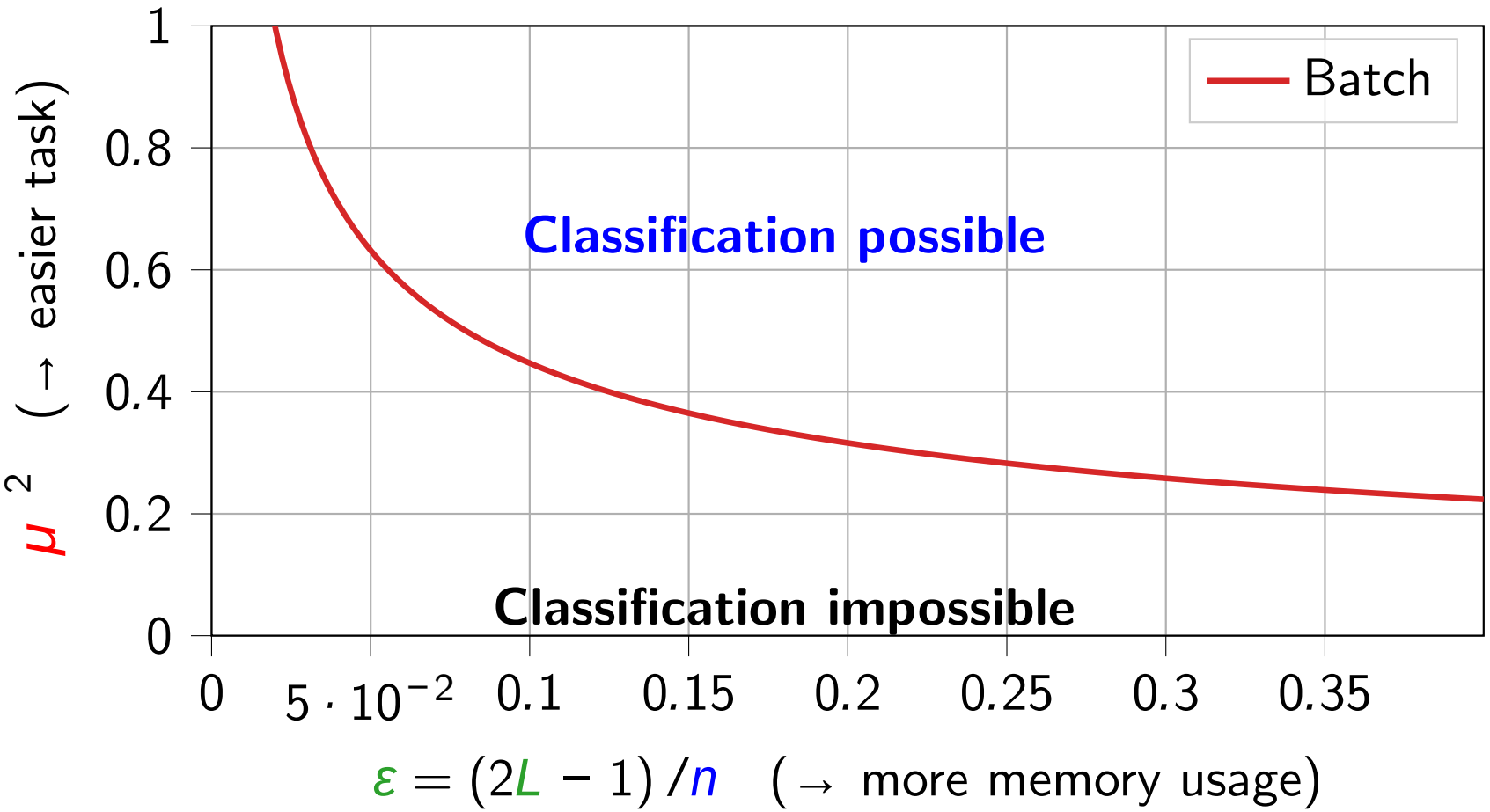

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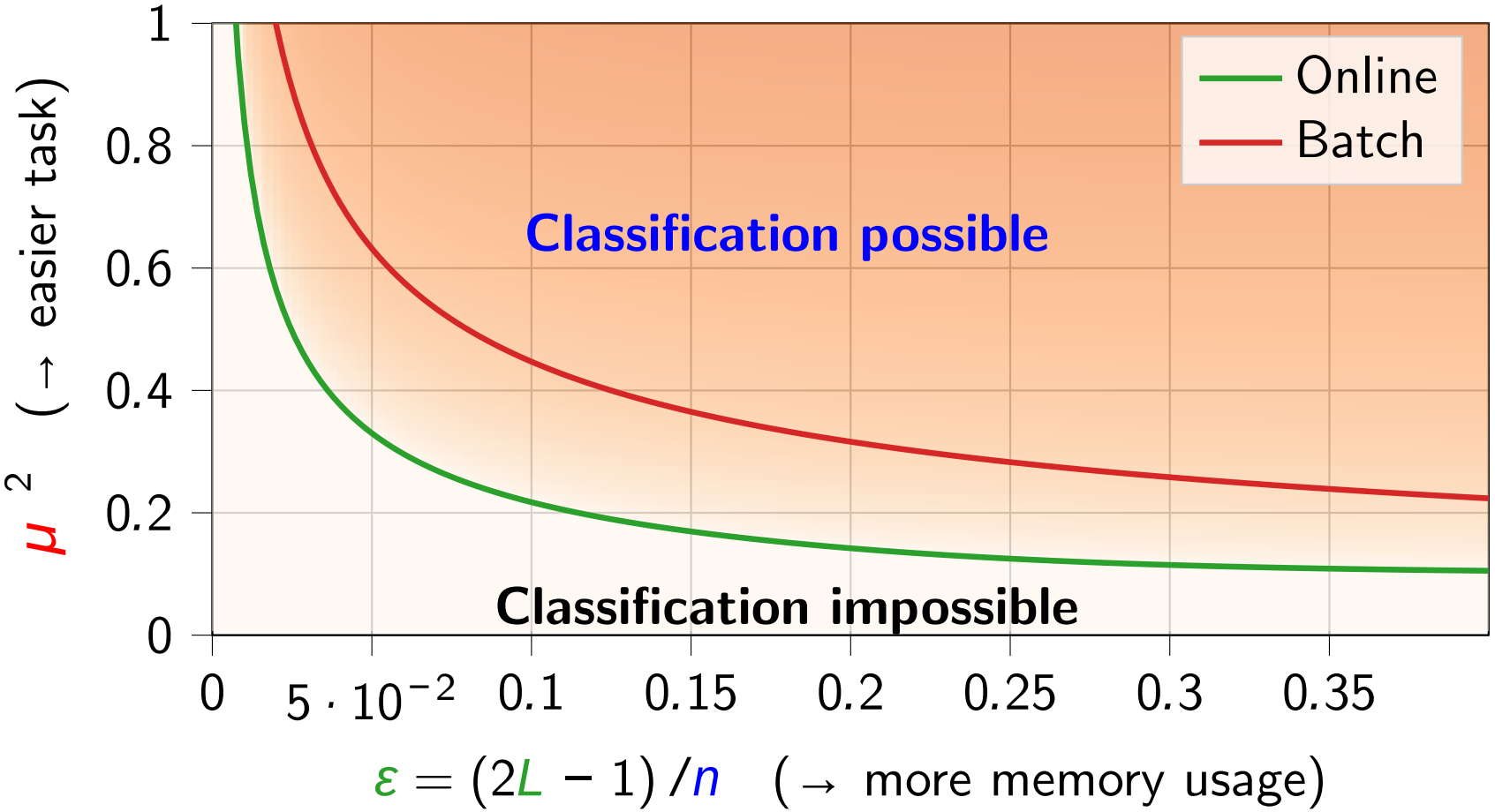

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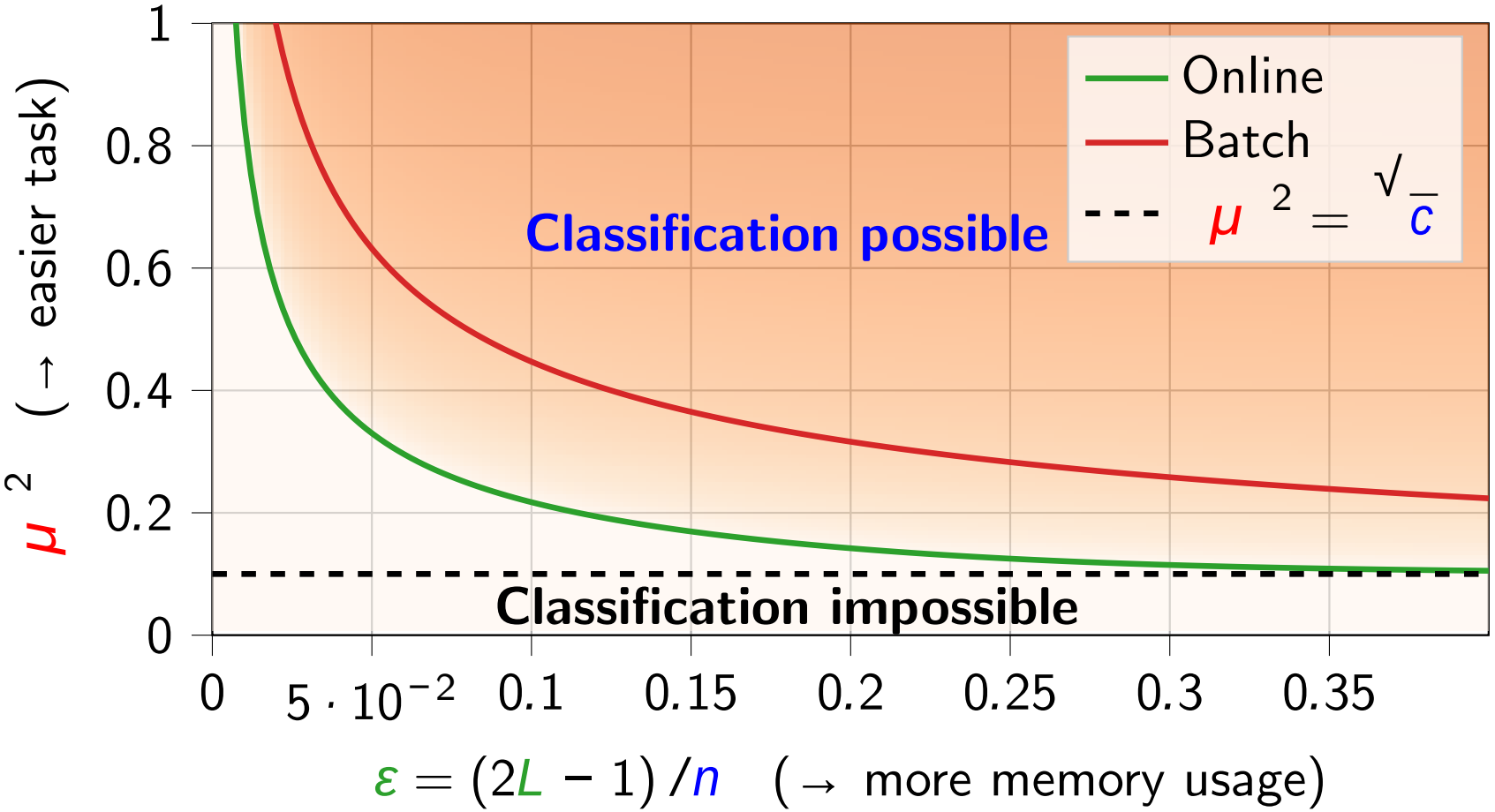

From theory to practice: online kernel spectral clustering

- **BigGAN images** (VGG features, $p = 4\,096$) $T = 20\,000$

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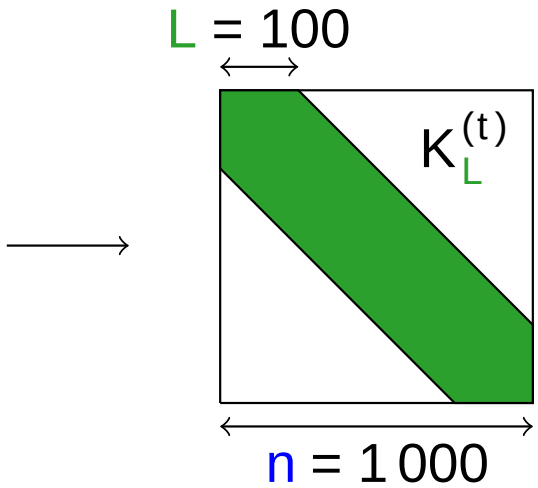

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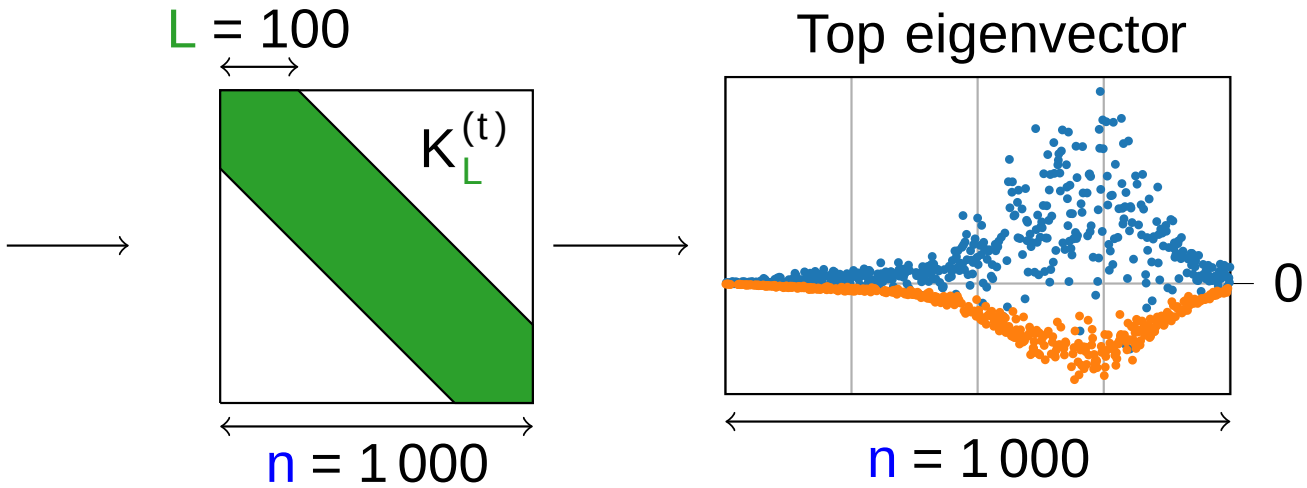

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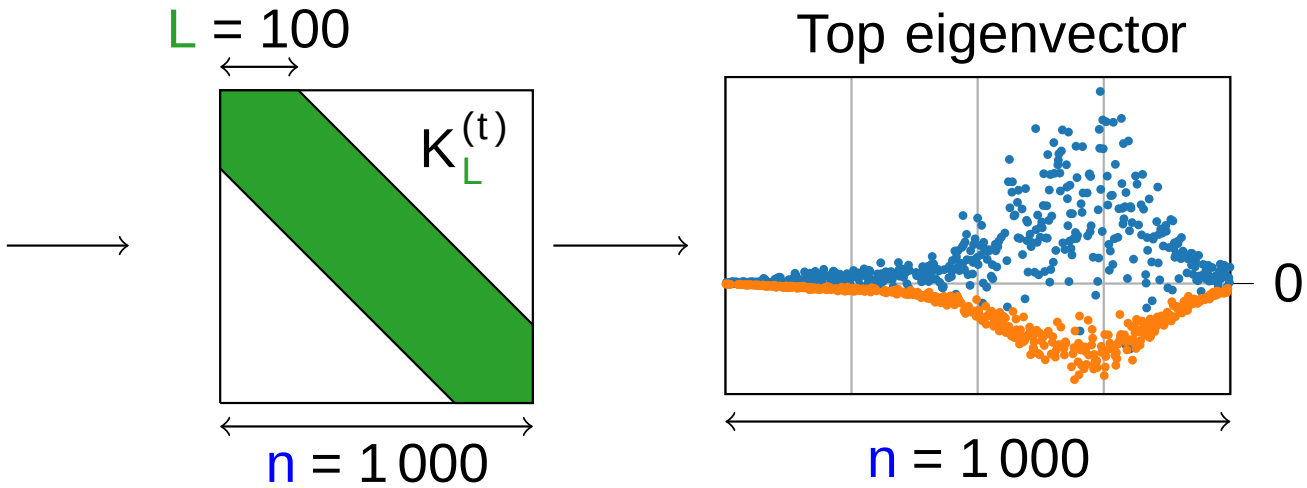

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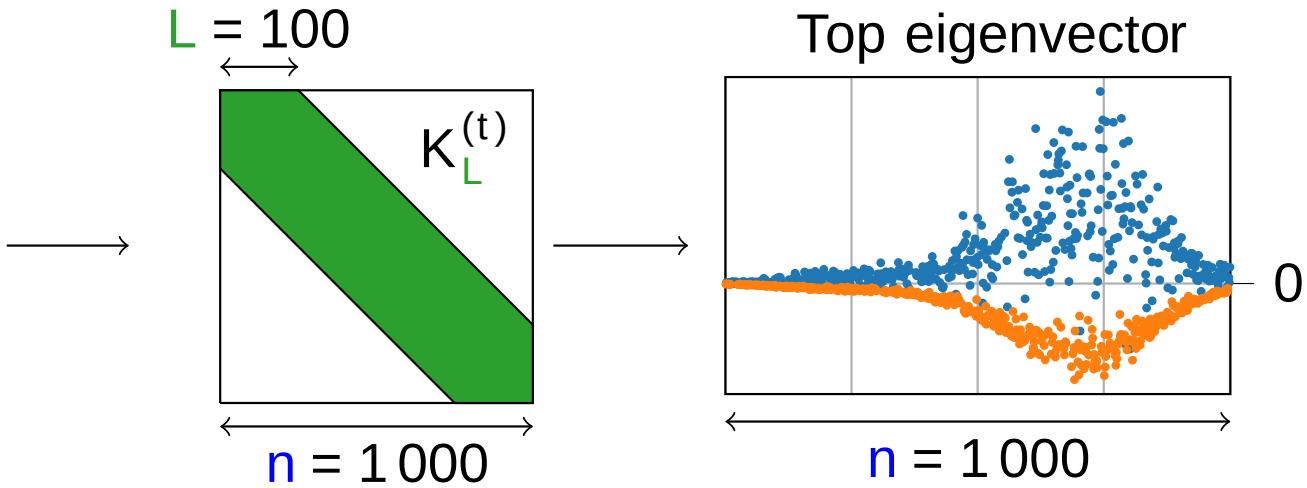

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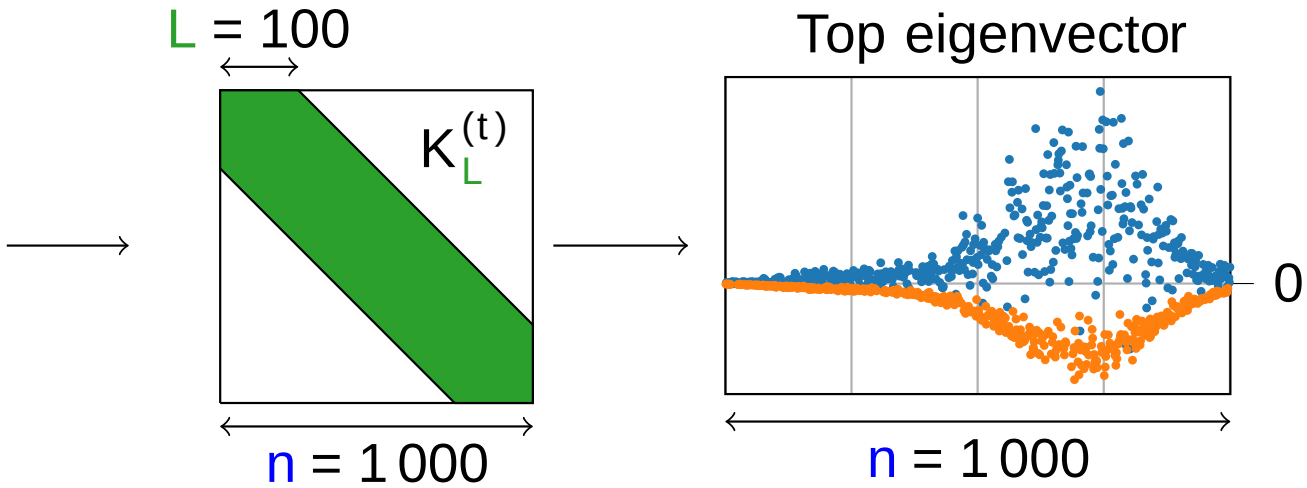

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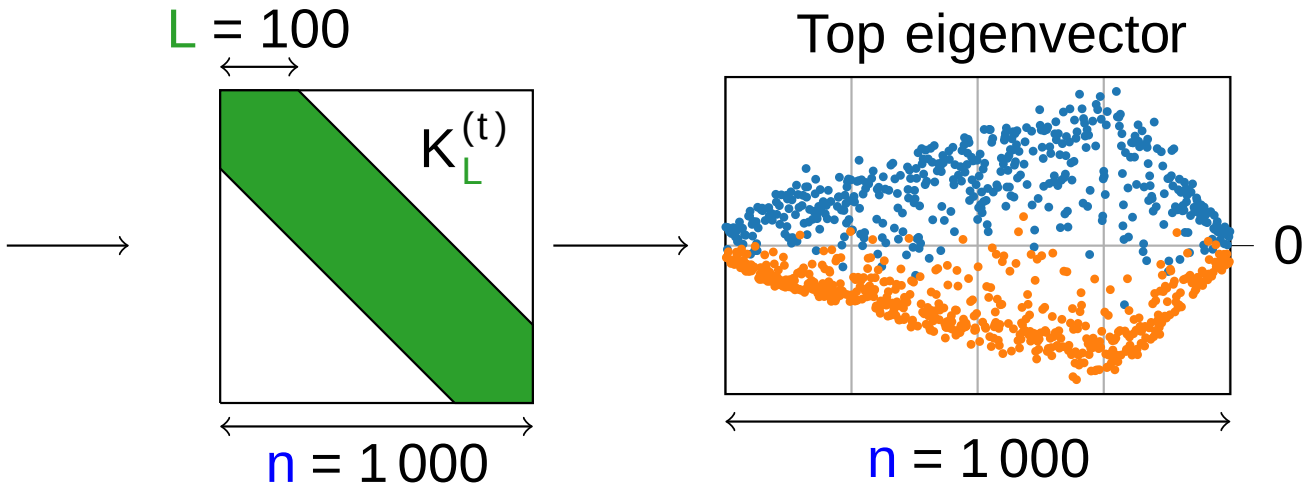
Fashion-MNIST images (raw, $p = 784$) $T = 14\,000$



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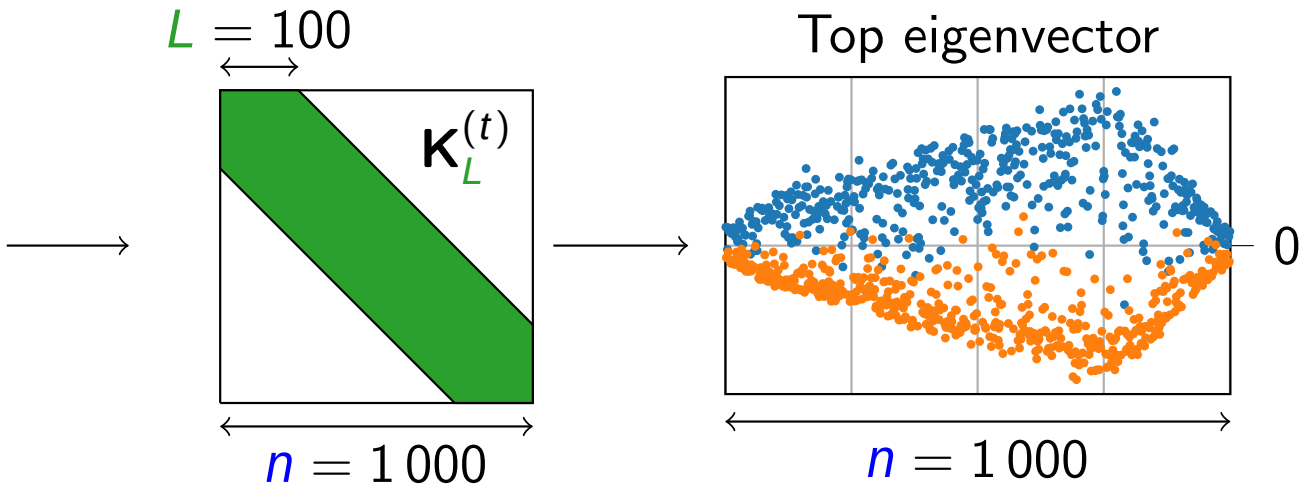

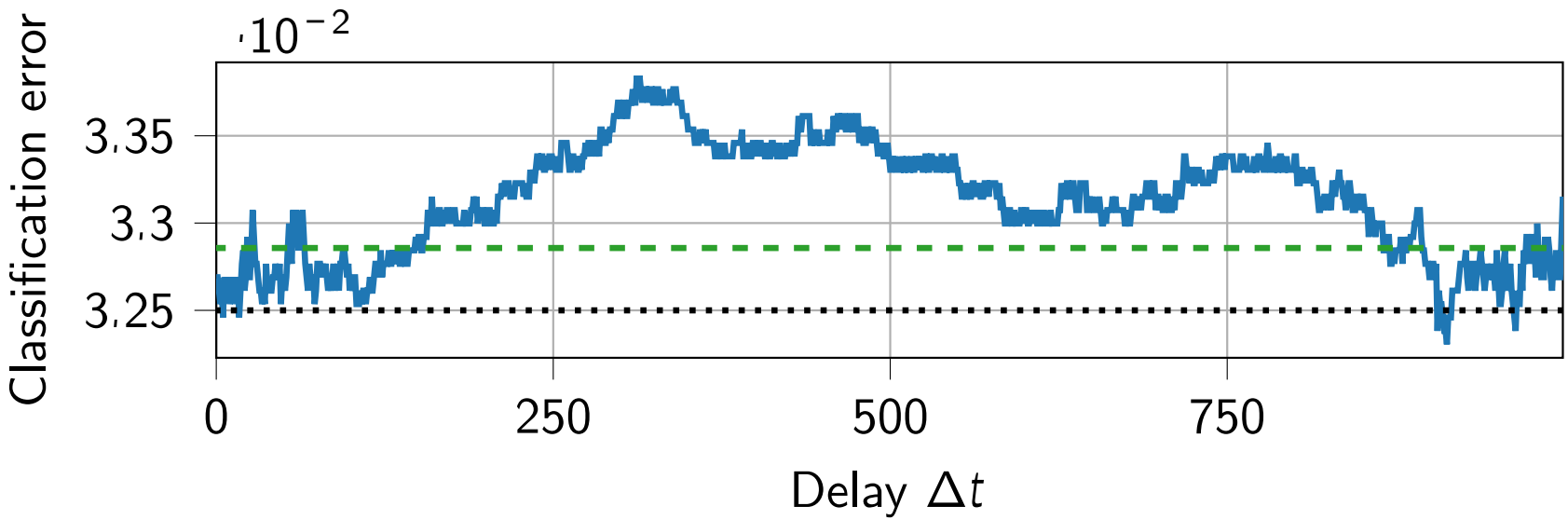

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