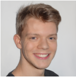
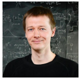
Probabilistic ODE Solutions in Millions of Dimensions



*Nico Krämer** Nathanael Bosch* Jonathan Schmidt* Philipp Hennig

* Equal contribution

June 27, 2022

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

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Chair for the Methods of Machine Learning

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Simulation of ordinary differential equations

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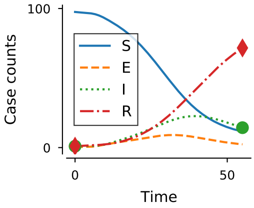

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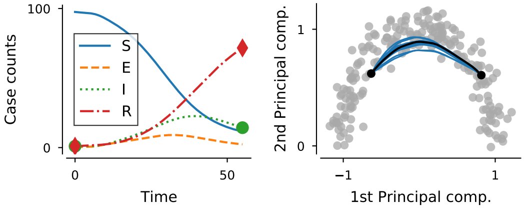

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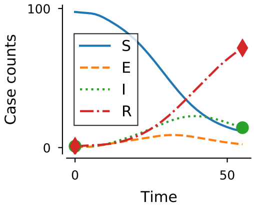
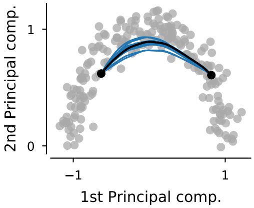
- Many dimensions, $d \gg 100$. Think: $d \sim 10^6$





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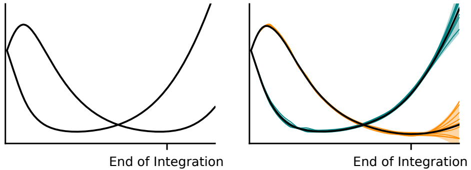

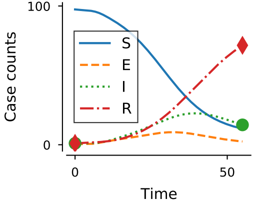

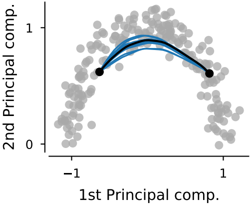

- Any approximation of the posterior distribution

$$p\left(y \;\middle|\; \{\dot{y}(t_n) = f(y(t_n), t_n)\}_{n=0}^{N}, y(t_0) = y_0\right)$$

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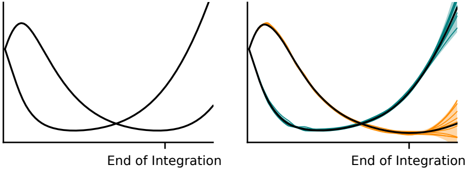

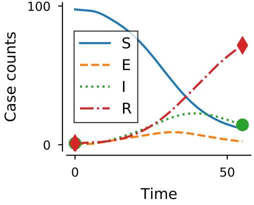

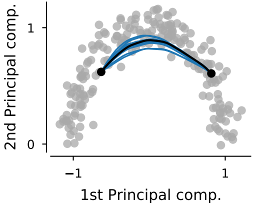

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- *Complexity $O(d^3)$ per step ϟϟ*

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Theorem (Kronecker; simplified)

If the νth-order prior has Kronecker structure, a single step with the prob. ODE solver costs $O(\nu^3 + d\nu^2)$ in float-ops and $O(d\nu + d^2 + \nu^2)$ in memory.

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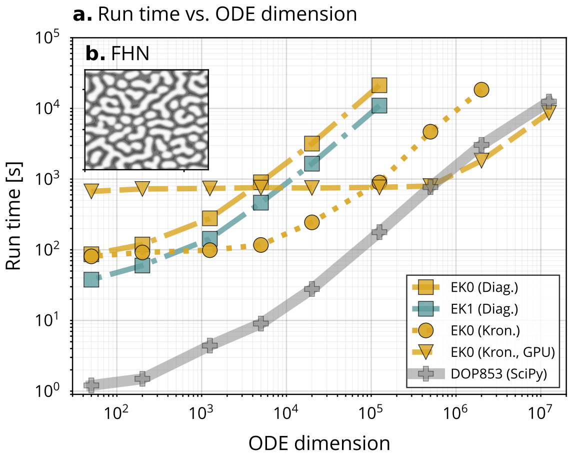

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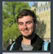
Nathanael Bosch (**`@NathanaelBosch`**)


Jonathan Schmidt


Philipp Hennig (**`@PhilippHennig5`**)

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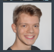
Jonathan Schmidt


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Software: `https://github.com/pnkraemer/tornadox`

```
pip install tornadox
```

Paper: `https://arxiv.org/abs/2110.11812`

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Experiment code