Deep Symbolic Regression for Recurrent Sequences

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LIP6

Meta

Setting

Given the sequence [1,2,3,5,8,13], what is the next term ?

- Numeric answer : 21
- Symbolic answer : $u_n = u_{n-1} + u_{n-2}$

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Hardly studied in the machine learning community, because **symbolic regression is tricky!**

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Typical approach: genetic programming (very slow)

Our approach: seq2seq Transformer (treat math as a language)

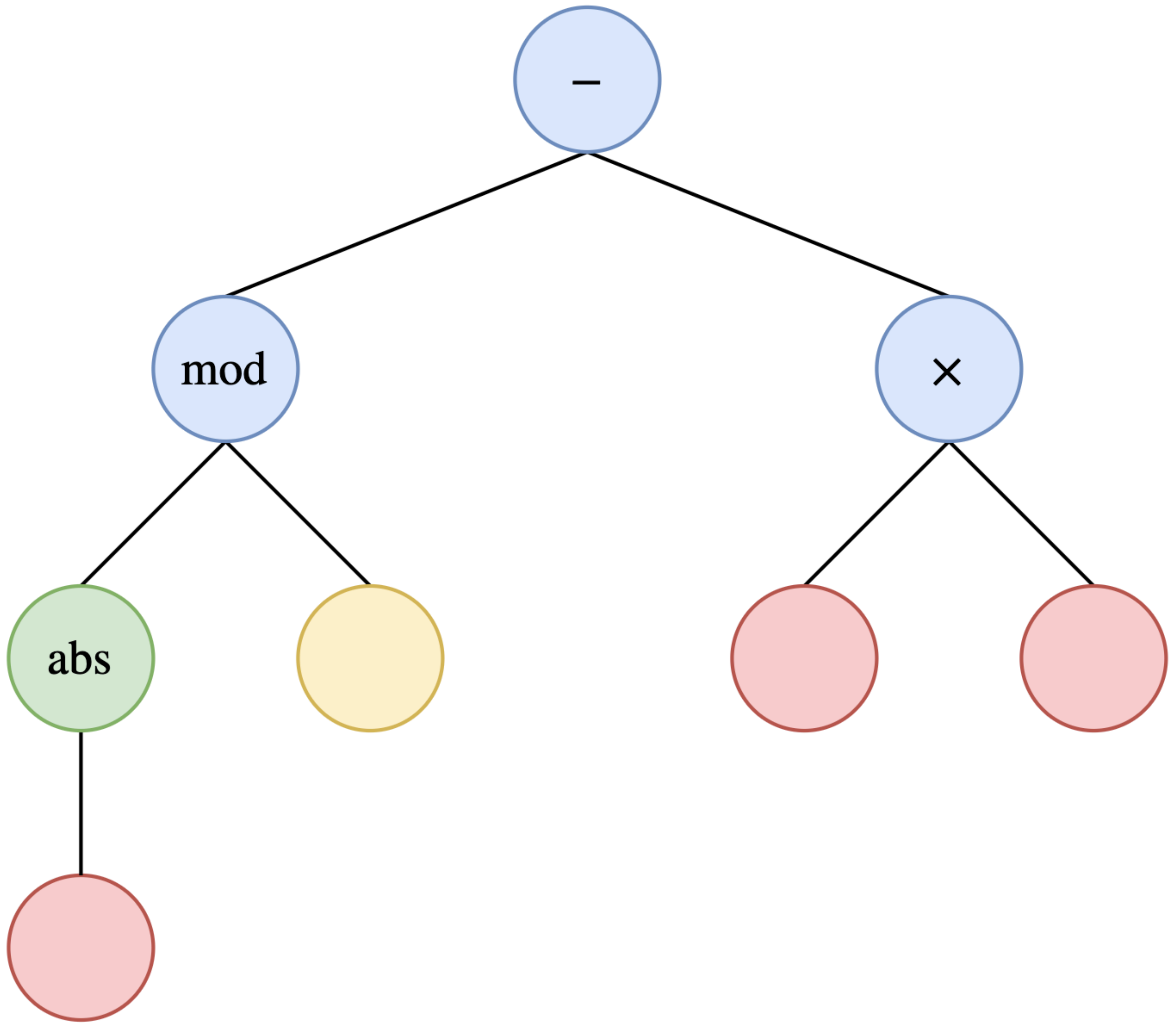
[Valipour et al. 2021]

[Biggio et al., 2021]

Generating examples

[Lample & Charton, 2019]

1. Sample operators and build a **tree**



Generating examples

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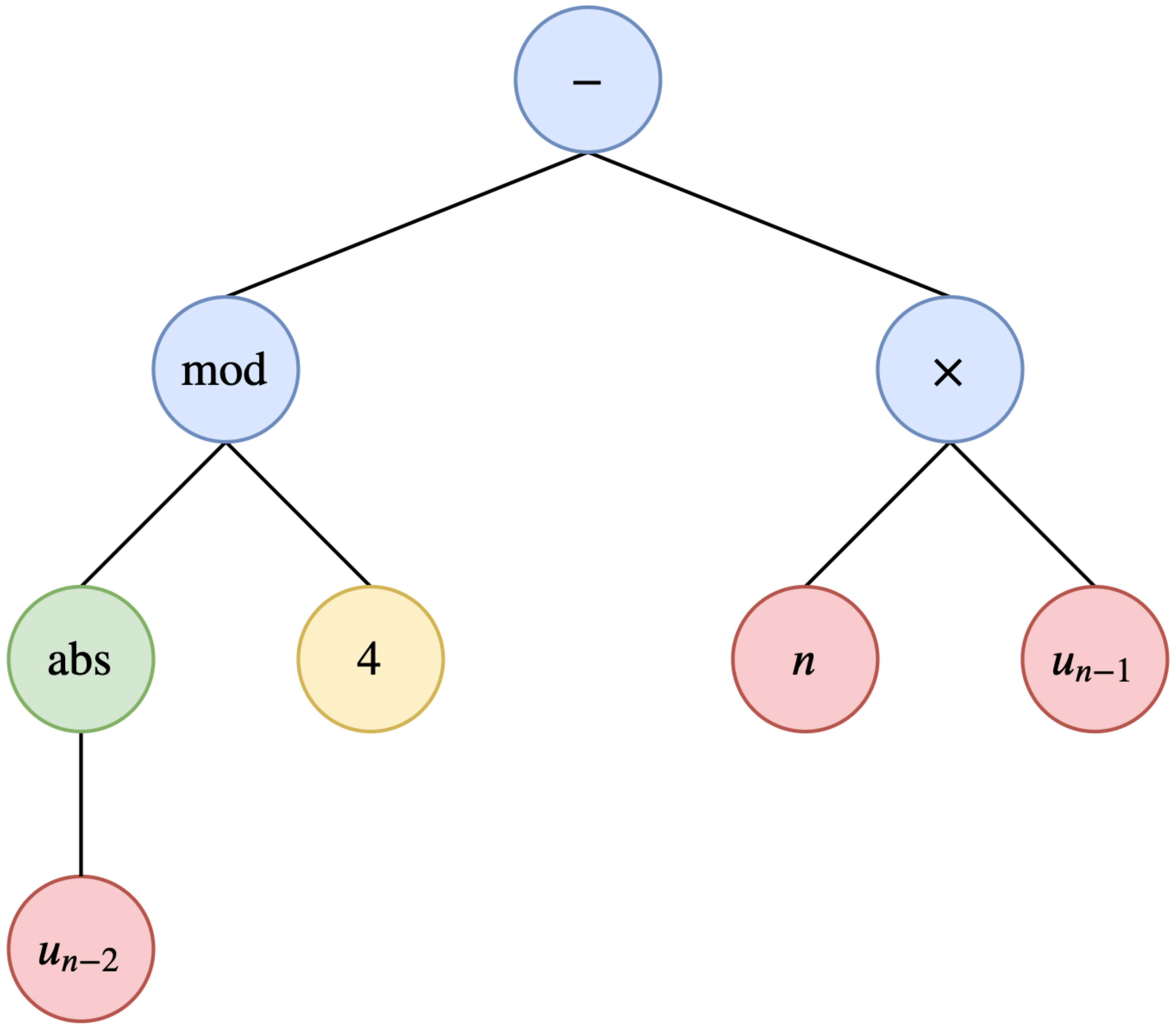
1. Sample operators and build a **tree**

2. Fill in the **leaves**

$$u_n = \text{abs}(u_{n-2}) \bmod 4 - n u_{n-1}$$



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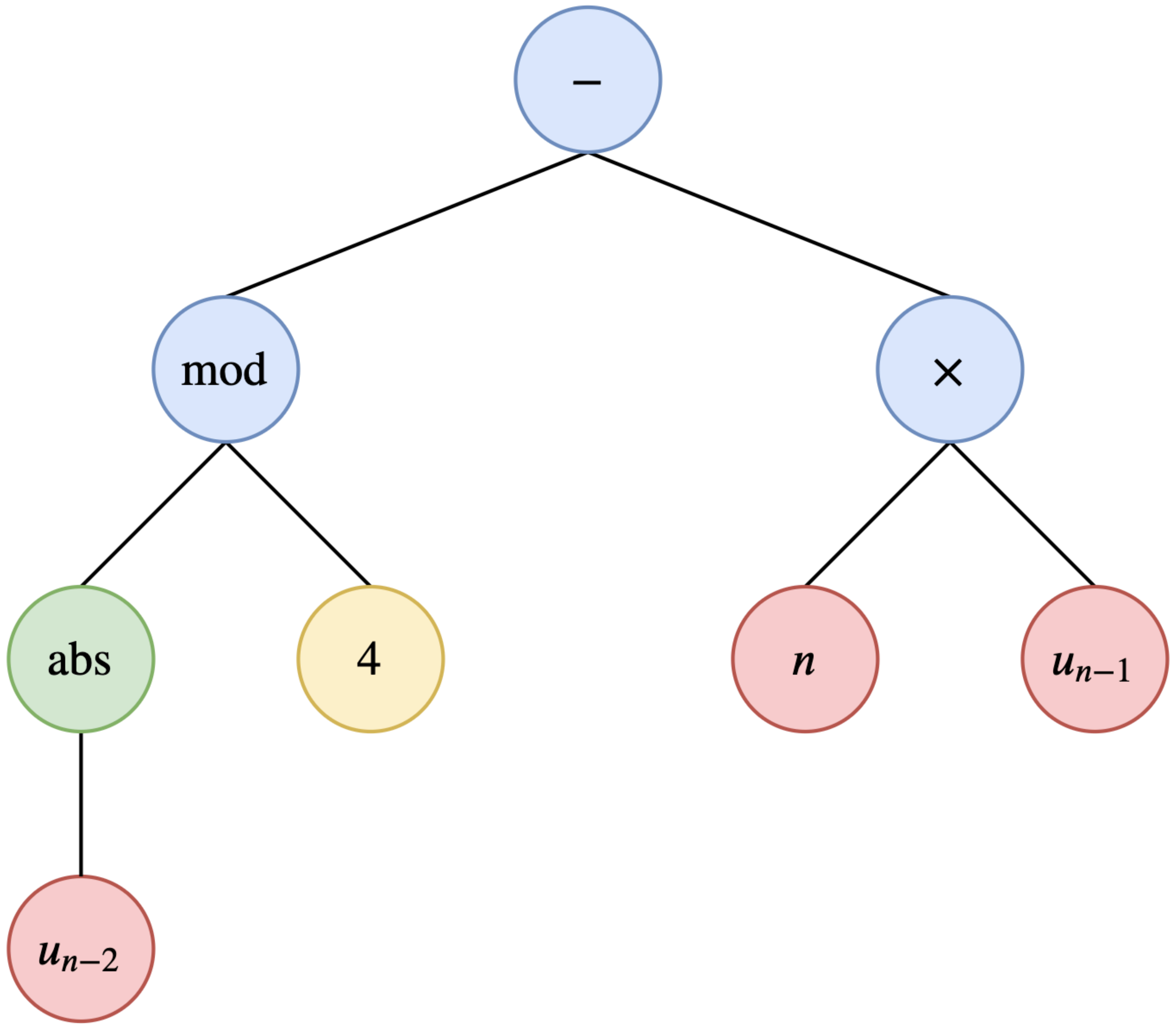

[1, -3, 2, ...

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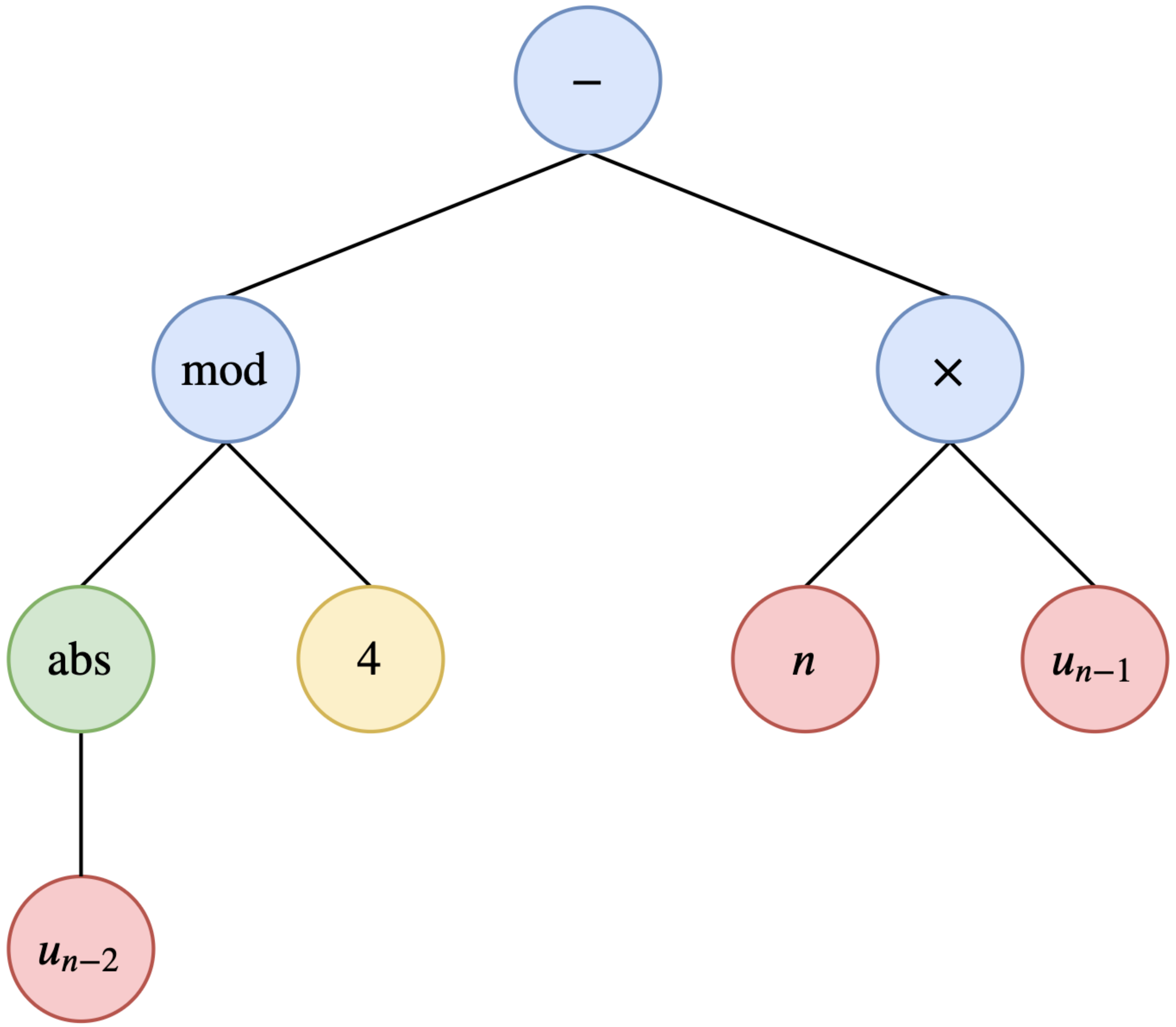

[1, -3, 2, ... -5, 27, 135, ...]

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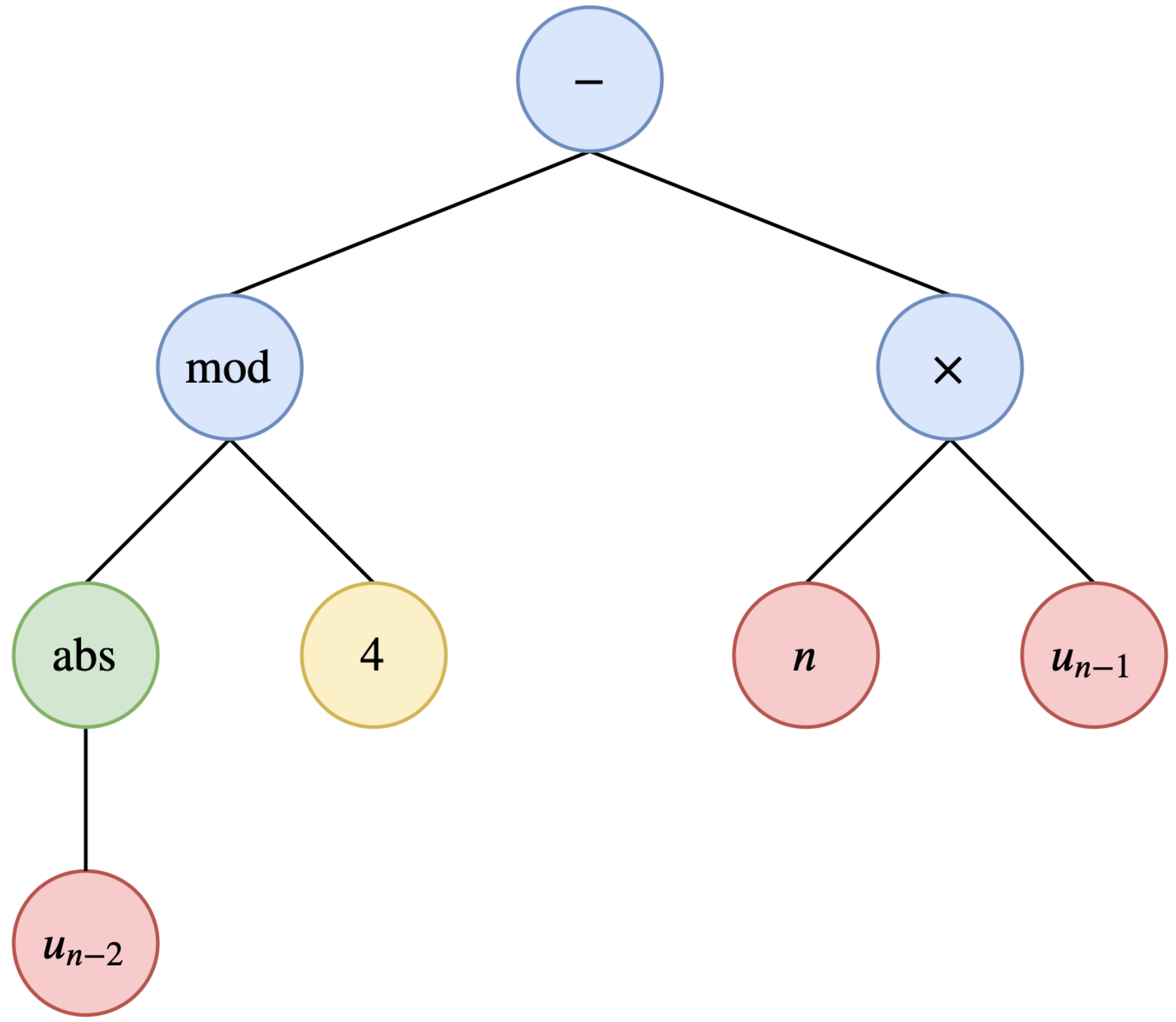

Input [1, -3, 2, … -5, 27, 135, …]

Target [sub, mod, abs, u_{n-2}, 4, mul, n, u_{n-1}]

Generating examples

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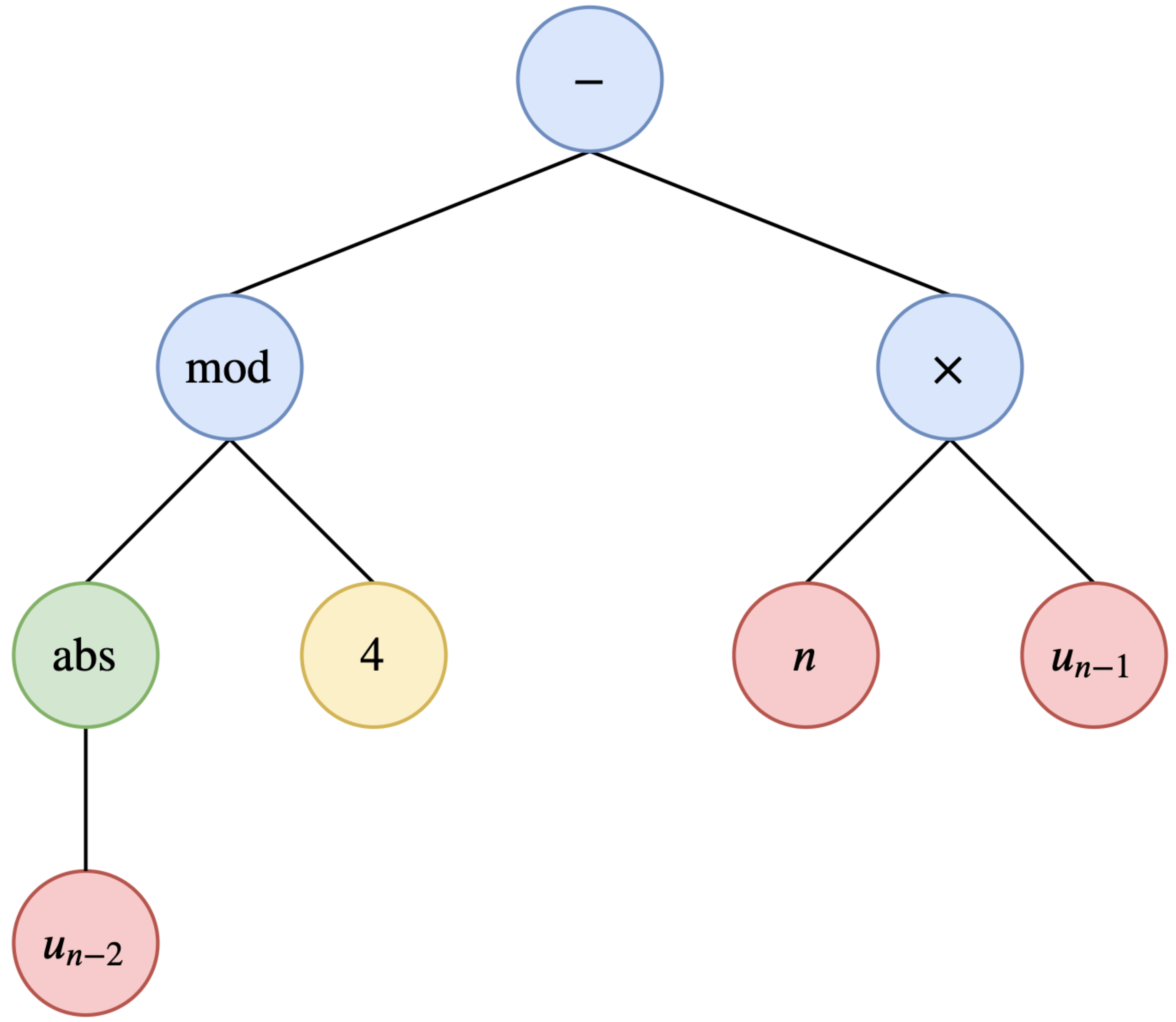
	Integer	Float
Unary	`abs, sqr, sign, step`	`abs, sqr, sqrt, inv, log, exp sin, cos, tan, atan`
Binary	`sum, sub, mul, intdiv, mod`	`sum, sub, mul, div`

$$u_n = \mathrm{abs}(u_{n-2}) \bmod 4 - n u_{n-1}$$



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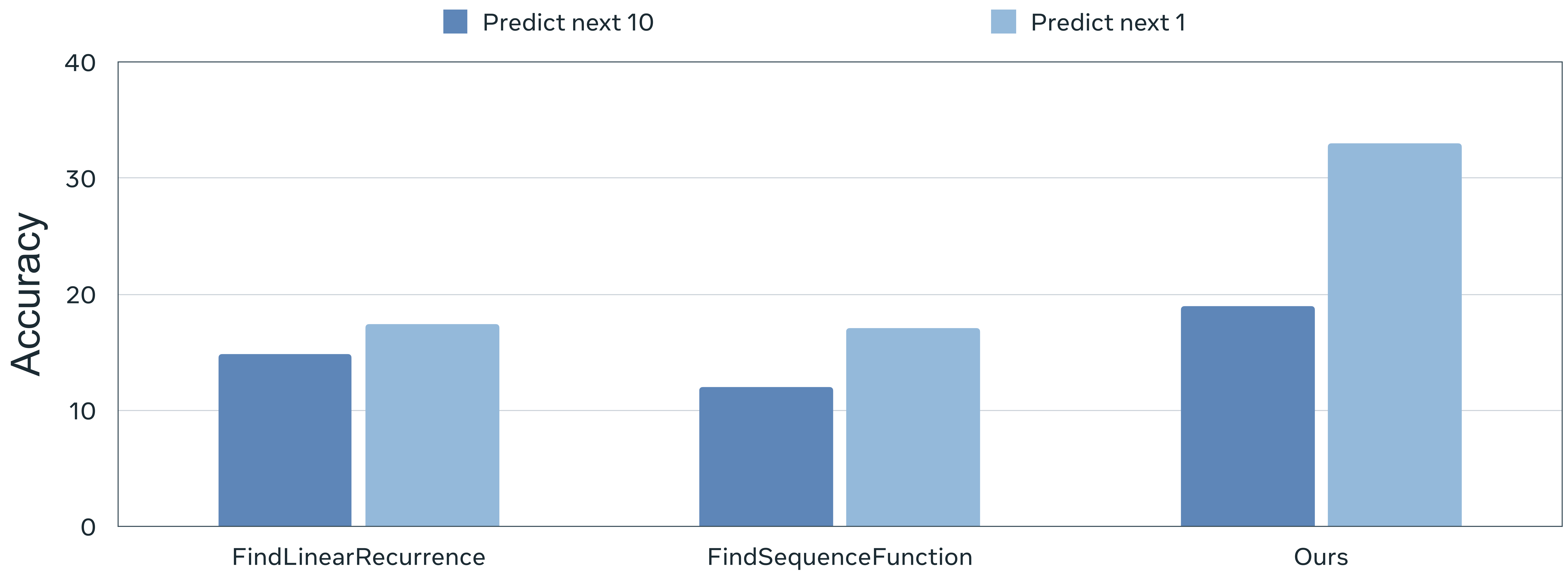
Predicting OEIS sequences



Our models outperform Mathematica both at recurrence prediction and extrapolation !

By-products

Constant	Approximation	Rel. error
0.3333	$(3+\exp(-6))^{-1}$	10^{-5}
0.33333	$1/3$	10^{-5}
3.1415	$2\arctan(\exp(10))$	10^{-7}
3.14159	π	10^{-7}
1.6449	$1/\arctan(\exp(4))$	10^{-7}
1.64493	$\pi^2/6$	10^{-7}
0.123456789	$10/9^2$	10^{-9}
0.987654321	$1-(1/9)^2$	10^{-11}

Approximating constants

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Approximating constants

Expression u_n	Approximation $\hat{u}_n$
$\mathrm{arcsinh}(n)$	$\log(n+\sqrt{n^2+1})$
$\mathrm{arccosh}(n)$	$\log(n+\sqrt{n^2-1})$
$\mathrm{arctanh}(1/n)$	$\frac{1}{2}\log(1+2/n)$
$\mathrm{catalan}(n)$	$u_{n-1}(4-6/n)$
$\mathrm{dawson}(n)$	$\frac{n}{2n^2-u_{n-1}-1}$
$\mathrm{j0}(n)$ (Bessel)	$\frac{\sin(n)+\cos(n)}{\sqrt{\pi n}}$
$\mathrm{i0}(n)$ (mod. Bessel)	$\frac{e^n}{\sqrt{2\pi n}}$

Approximating functions

Thank you !

🕹️ **Interactive demo** https://symbolicregression.metademolab.com

🧑‍💻 **Open-source code** https://github.com/facebookresearch/recur

🧑‍🏫 **Poster session** Hall E #433