Algorithms for the Communication of Samples

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Channel Coding

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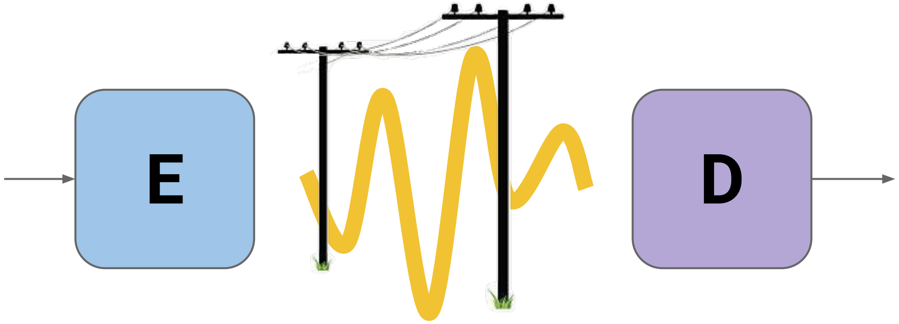

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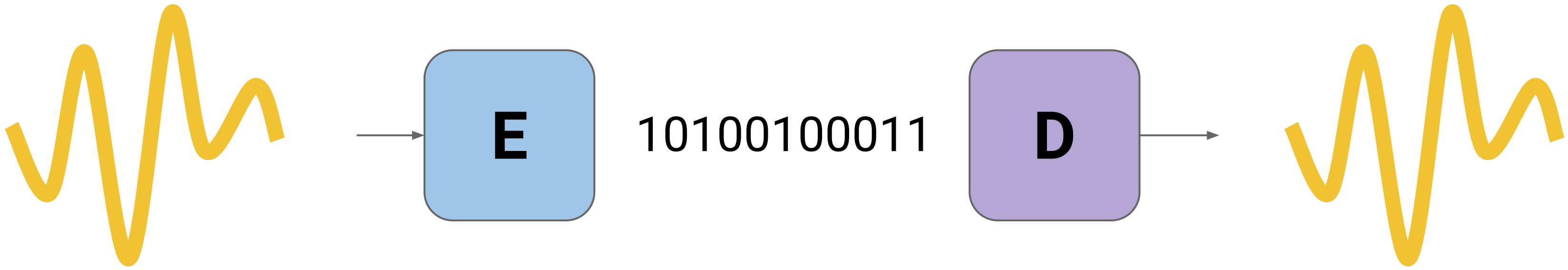
Reverse Channel Coding



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Example: Differential Privacy

Data

Noise

$$\mathbf{Z} = \mathbf{x} + \mathbf{U}$$

Message

Reverse Channel Coding

$$\mathbf{Z} \sim q_{\mathbf{x}}$$

Reverse Channel Coding

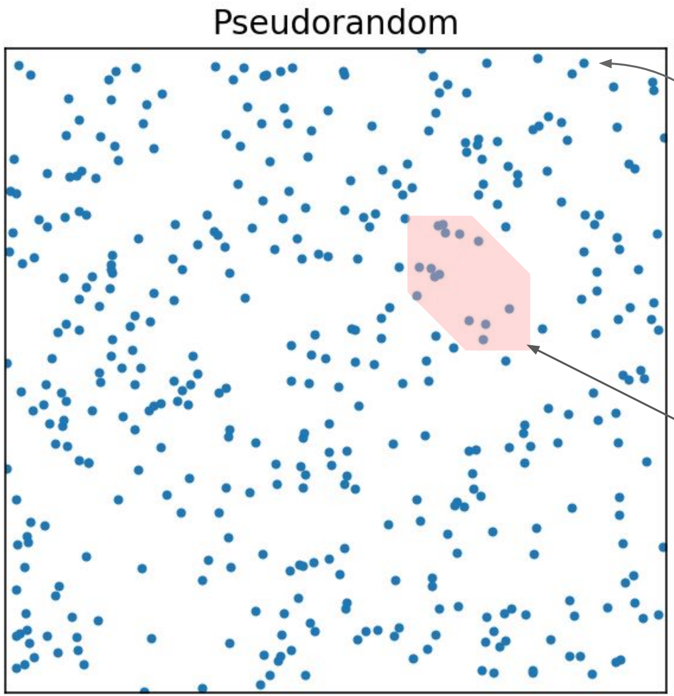
$$I[\mathbf{X}, \mathbf{Z}] = h[\mathbf{Z}] - h[\mathbf{Z} \mid \mathbf{X}] = \mathbb{E}[D_{\mathrm{KL}}[q_{\mathbf{X}} \parallel p]]$$

(Bennett & Shor, 2002)

Source Coding ($q_{\mathbf{X}} = \delta_{\mathbf{X}}$)

$$I[\mathbf{X}, \mathbf{X}] = H[\mathbf{X}] - \cancel{H[\mathbf{X} \mid \mathbf{X}]} = \mathbb{E}[D_{\mathrm{KL}}[\delta_{\mathbf{X}} \parallel p]]$$

Reverse Channel Coding



Approach:

1) Generate $\mathbf{Z}_i \sim p$

Target distribution $q_{\mathbf{X}}$

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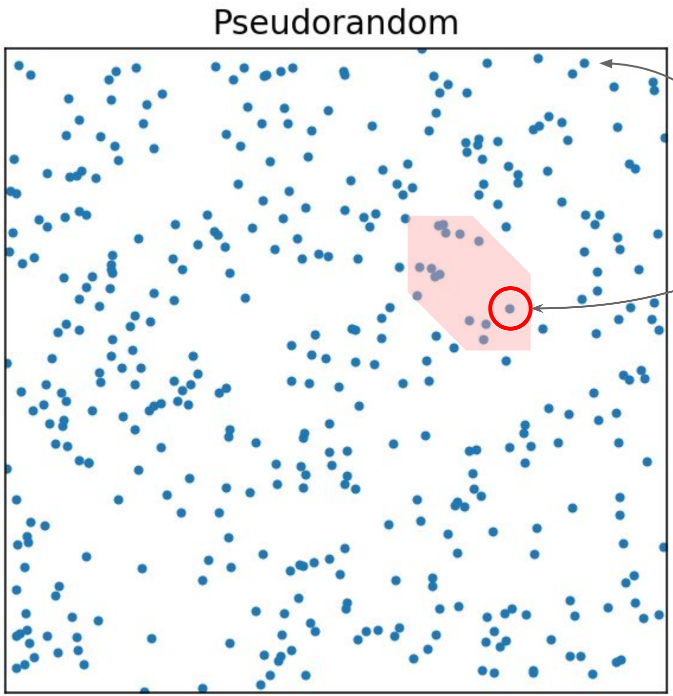
Reverse Channel Coding



Approach:

1) Generate $\mathbf{Z}_i \sim p$

2) Pick K such that $\mathbf{Z}_K \sim q_{\mathbf{x}}$

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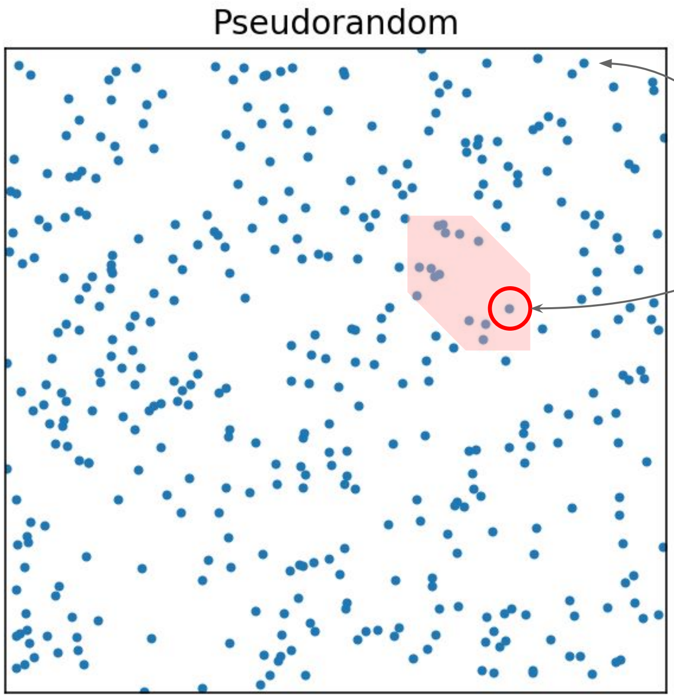
Reverse Channel Coding



Approach:

1) Generate $\mathbf{Z}_i \sim p$
2) Pick K such that $\mathbf{Z}_K \sim q_{\mathbf{x}}$
3) Encode K

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Minimal Random Coding (MRC)

$$\mathbf{Z}_1, \ldots, \mathbf{Z}_N \sim p$$

$$\pi_{\mathbf{x}}(k) \propto \frac{q_{\mathbf{x}}(\mathbf{Z}_k)}{p(\mathbf{Z}_k)}$$

$$K \sim \pi_{\mathbf{x}}$$

(Havasi et al., 2019)

Ordered Random Coding (ORC)

$$K_{\mathrm{MRC}} = \mathrm{argmax}_k \ \ln q_{\mathbf{x}}(\mathbf{Z}_k) - \ln p(\mathbf{Z}_k) + G_k$$

$$G_n \sim \mathrm{Gumbel}(0, 1)$$

Ordered Random Coding (ORC)

$$K_{\text{ORC}} = \text{argmax}_k \; \ln q_{\mathbf{x}}(\mathbf{Z}_k) - \ln p(\mathbf{Z}_k) + \tilde{G}_k$$

$$\tilde{G}_1 \geq \tilde{G}_2 \geq \cdots \geq \tilde{G}_N$$

Ordered Random Coding (ORC)

$$\mathbf{z}_{K_{\mathrm{ORC}}} \sim \mathbf{z}_{K_{\mathrm{MRC}}}$$

$$H[K_{\mathrm{ORC}}] \leq H[K_{\mathrm{MRC}}] = \log N$$

$$H[K_{\mathrm{ORC}}] \leq I[\mathbf{X}, \mathbf{Z}] + \log(I[\mathbf{X}, \mathbf{Z}] + 1) + 4$$

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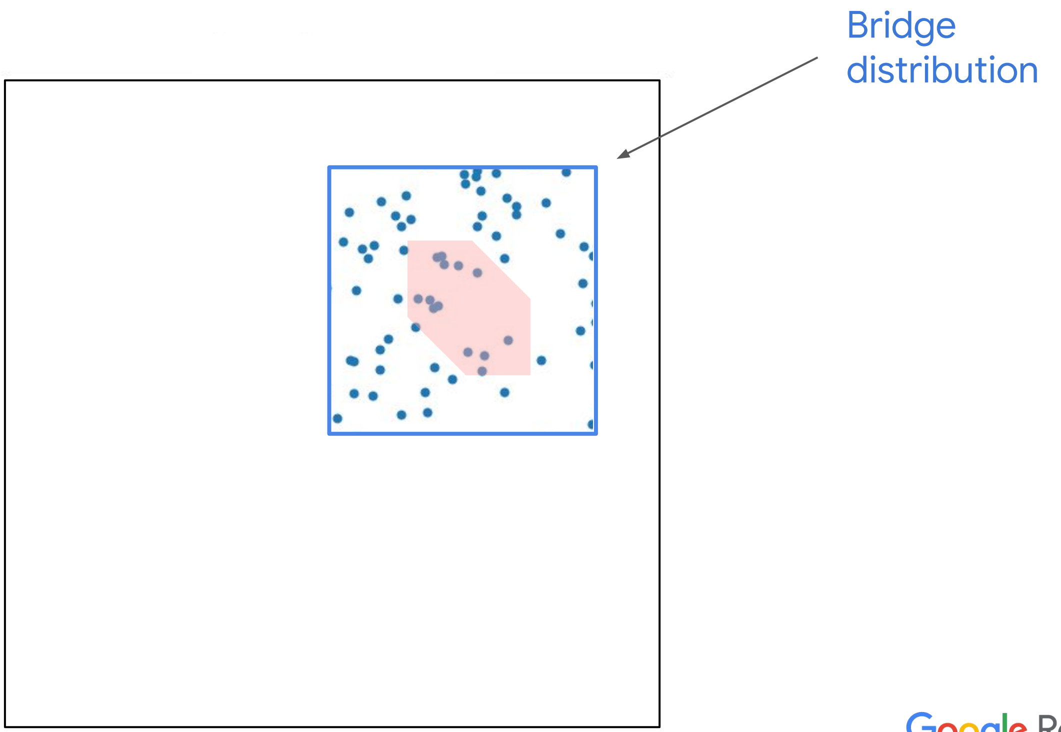
Hybrid coding



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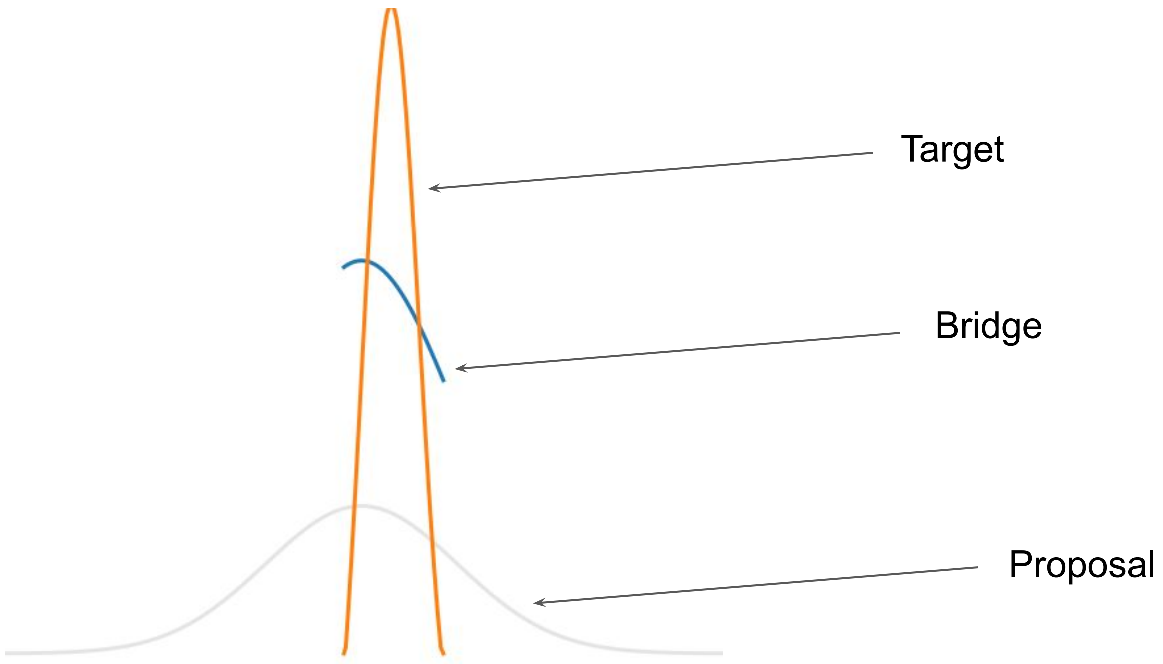
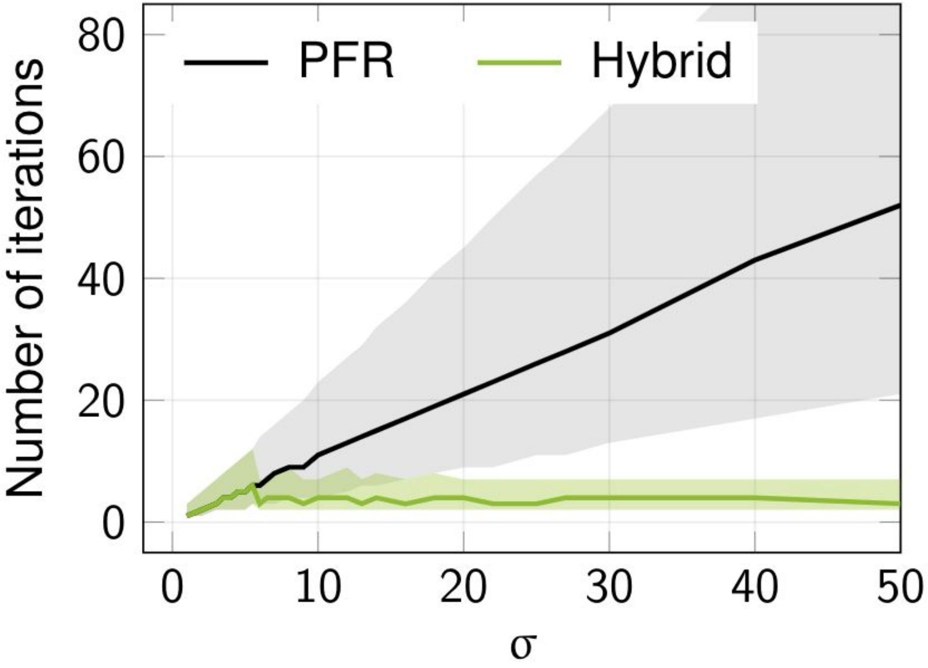
Hybrid coding



Hybrid coding



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Thank You

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