Cooperative Online Learning in Stochastic and Adversarial MDPs

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- A fundamental paradigm for sequential decision making.
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Cooperation in RL

- Multiple agent that learn the same environment share information in order to improve performance.
- Applications: communication networks, traffic routing, robotics, etc.

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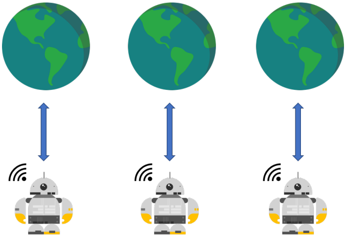
Fresh vs Non-fresh randomness

Fresh randomness

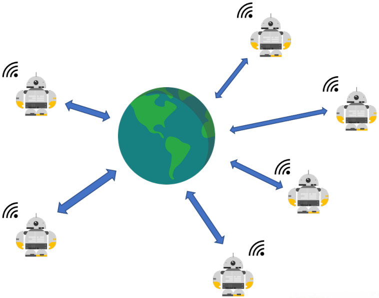
- Duplicates of the same environment - cost and transition to next state is freshly randomized.



E.g., Atari games.

Non-fresh randomness

- The same environment - agents that take the same action in the same state observe the same cost and next state.



Central questions

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\Is there a di erent limit for fresh and non-fresh randomness?"

Related work

- Optimal regret in single-agent stochastic and adversarial MDPs. [Zimin and Neu, 2013, Rosenberg and Mansour, 2019, Jin et al., 2020]

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$\sqrt{H^3SAK}$ (unknown transition bandit feedback)

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- In multi-agent adversarial MAB, one can achieve regret that scales as [Cesa-Bianchi et al., 2019],

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- Cooperation in RL was considered only in the stochastic and fresh randomness case by Lidard et al. [2021],

$$\sqrt{H^4SAK/m}.$$

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Our contribution

- We are the first to study non-fresh randomness, and to face new challenges in this model.
- First to consider adversarial cost in cooperative learning in MDPs.
- Thoroughly analyze all relevant settings, and prove nearly-matching regret lower and upper bounds.

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  - Each agent sample an action $a_h \sim \pi^k(\cdot \mid s_h)$.
  - Agent suffers cost $c_h(s_h, a_h)$ and transition to a new state $s_{h+1} \sim p_h(\cdot \mid s_h, a_h)$, where p is unknown.

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Regret

The performance is measured by the *regret* - the difference between the total agent's cost and the cost of the best policy in hindsight.

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- Compute an optimistic estimate of Q and act greedily with respect to it.
- One can show that the regret scales as the sum of confidence radius on the agent's trajectory.
- With non-fresh randomness we get m times more samples and the confidence radius shrinks faster. With that we can show optimal regret for each agent:

$$R_K \;.\; \sqrt{\frac{H^3SAK}{m}}:$$

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Much more challenging setting:

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Much more challenging setting:

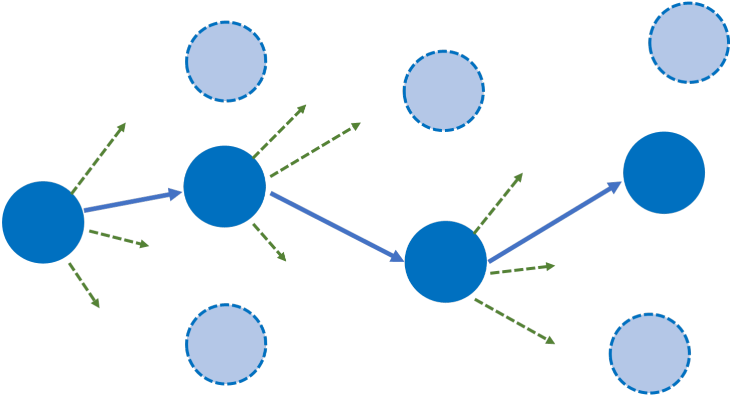
- If agents play a deterministic policy (e.g., optimistic algorithm), then they all follow the same trajectory. Hence, we don't have additional feedback.
- Optimism alone is no longer a good approach.
- In fact, we show a lower bound of $\sqrt{H^2SK}$ regardless on the number of agents!

Non-fresh - Stochastic MDP

Algorithm (COOP-ULCAE):

- Maintain upper and lower confidence bounds on Q .
- Eliminate arms a such that $\underline{Q}_h^k(s;a) > \overline{Q}_h^k(s;a')$.
- With probability 1 play the optimistic policy.
- With probability :
  - Sample random h.
  - At time h take a random active action.
  - At the rest of the time play the optimistic policy.

Analysis



- On the optimistic policy path we obtain m times more feedback.
- Hence, the regret in these rounds is at most $\sqrt{\frac{SAK}{m}}$.

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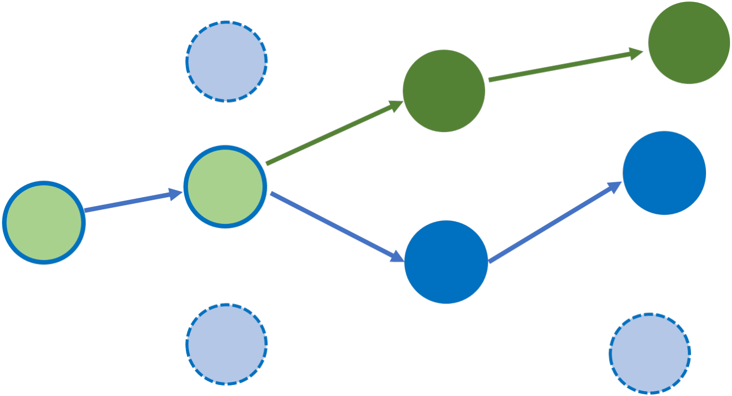

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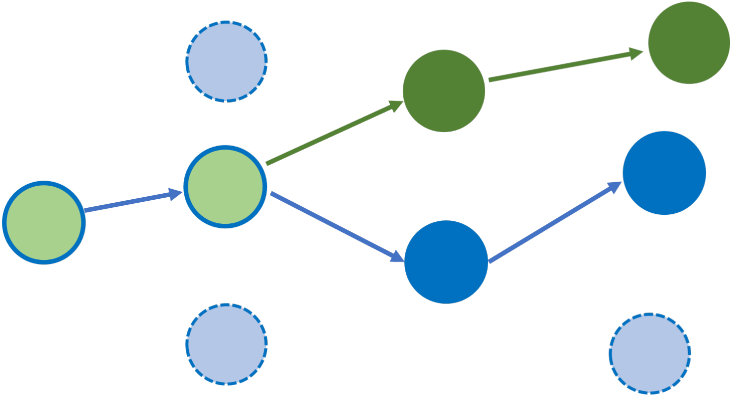

- We take an active action with the exploration policy. Using that, we can show that the regret is similar to a single-agent regret.
- Each agent explores only $O(\sqrt{K})$ episodes.
- Hence, the total regret from these rounds is $\sqrt{SAK}$

Non-fresh - Stochastic MDP

Setting properly allows us to prove the following regret bound.

Theorem

Under non-fresh randomness and stochastic costs COOP-ULCAE guarantees individual regret of,

$$R_K \lesssim \sqrt{H^5SK} + \sqrt{\frac{H^7SAK}{\sqrt{m}}}.$$

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- More challenging analysis under non-fresh randomness.

Summary of our results

Setting	Regret	Lower Bound
Fresh, stochastic, unknown p	$\sqrt{\frac{H^3SAK}{m}}$	$\sqrt{\frac{H^3SAK}{m}}$
Fresh, adversarial, known p	$\sqrt{H^2K} + \sqrt{\frac{H^2SAK}{m}}$	$\sqrt{H^2K} + \sqrt{\frac{H^2SAK}{m}}$
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