

Sublinear-Time Clustering Oracle for Signed Graphs

Stefan Neumann (KTH) and Pan Peng (USTC)

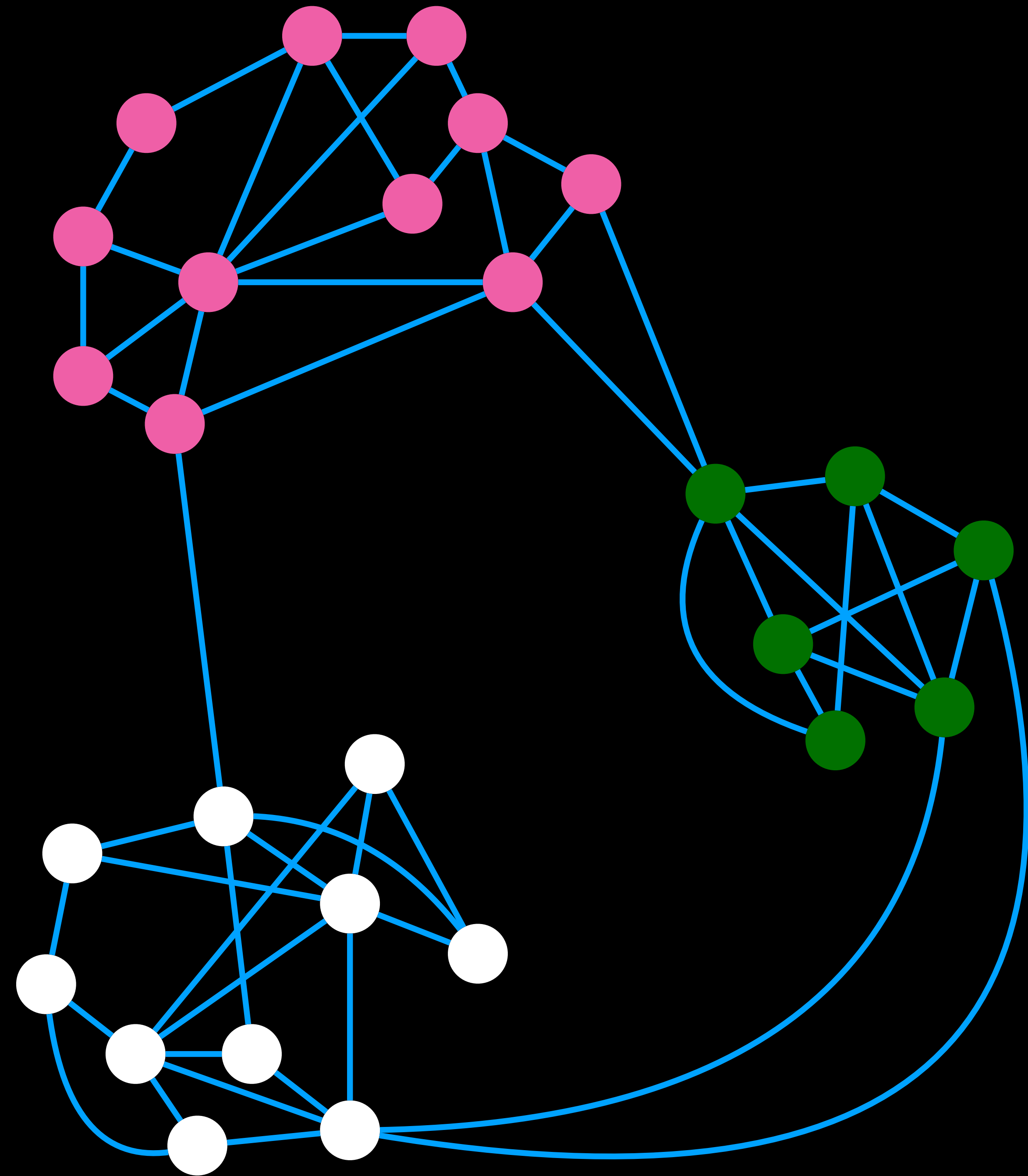
@StefanResearch

ICML'22



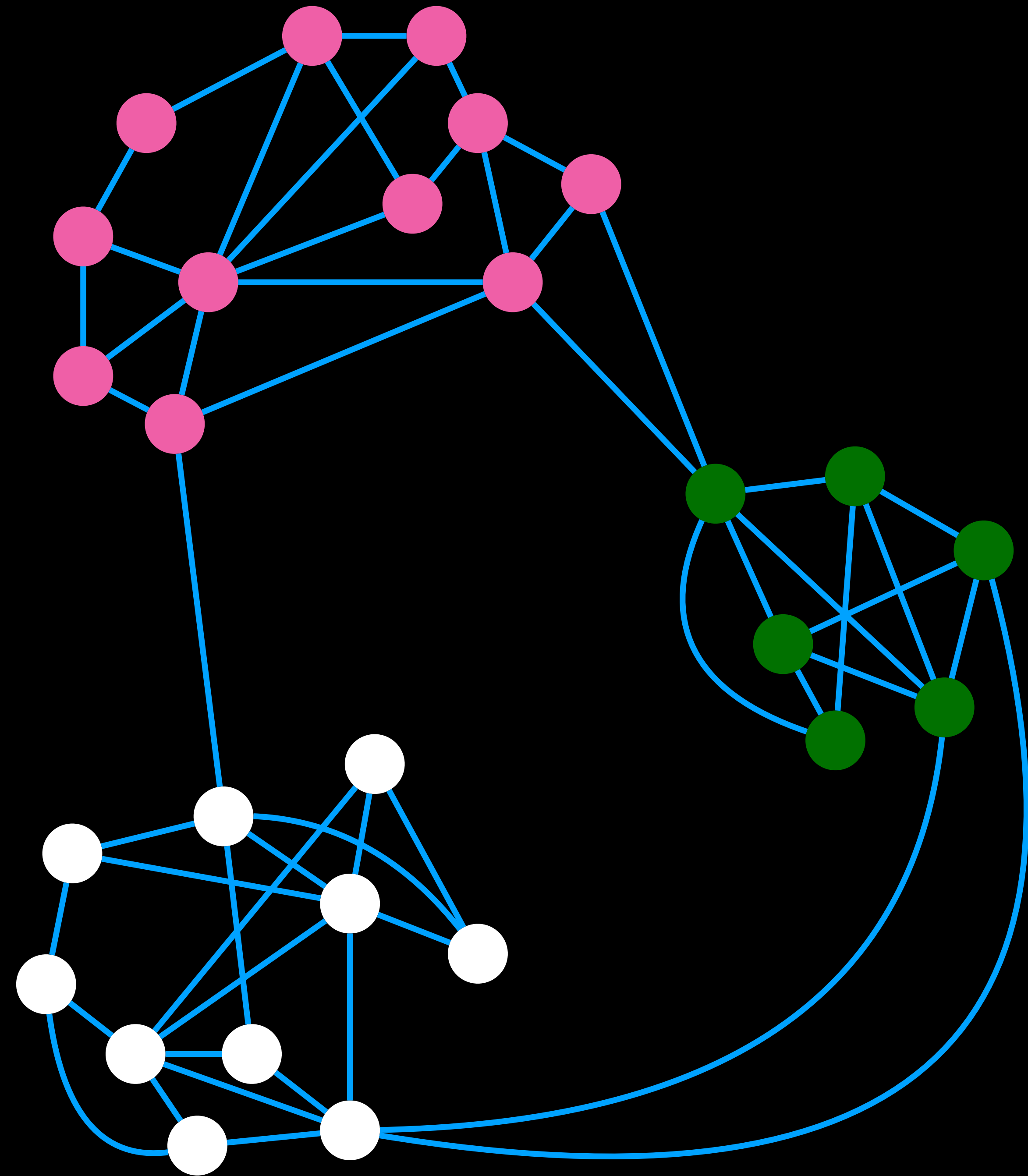
Graph Clustering

- Ubiquitous task in machine learning and data science



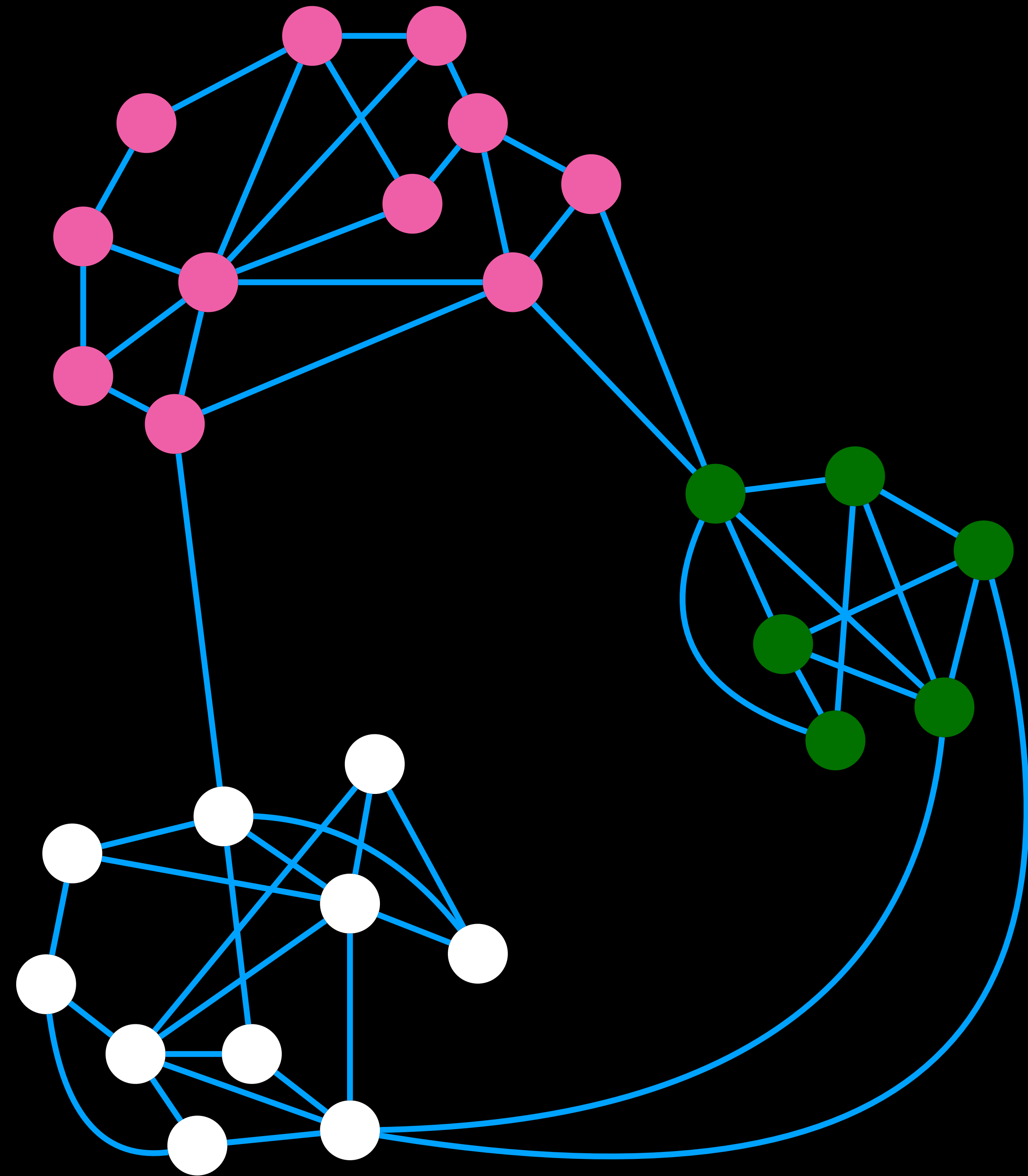
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- Ubiquitous task in machine learning and data science
- *Application:*
Find communities in (social) networks



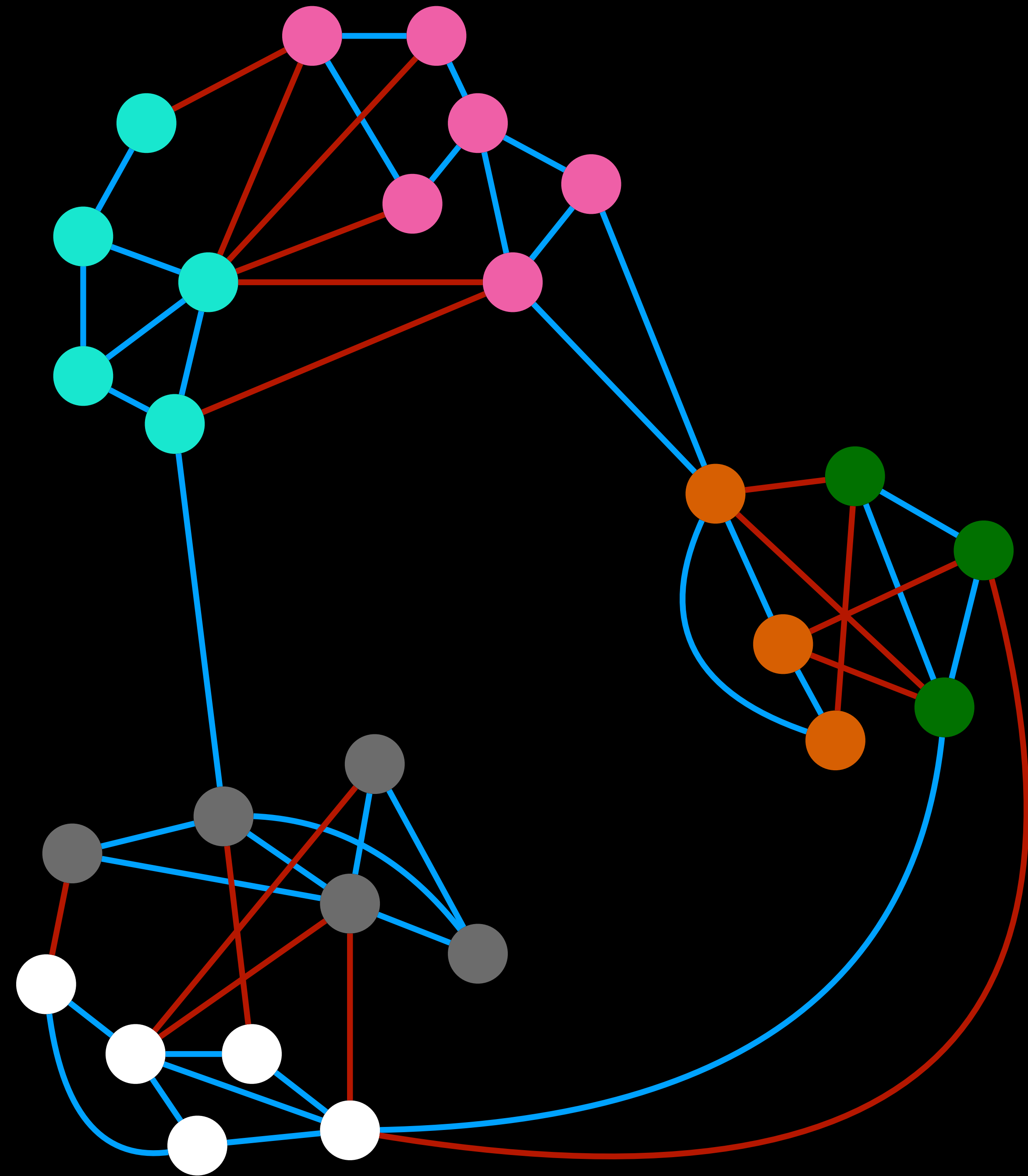
Graph Clustering

- Ubiquitous task in machine learning and data science
- *Application:* Find communities in (social) networks
- Typically studied for *unsigned* graphs



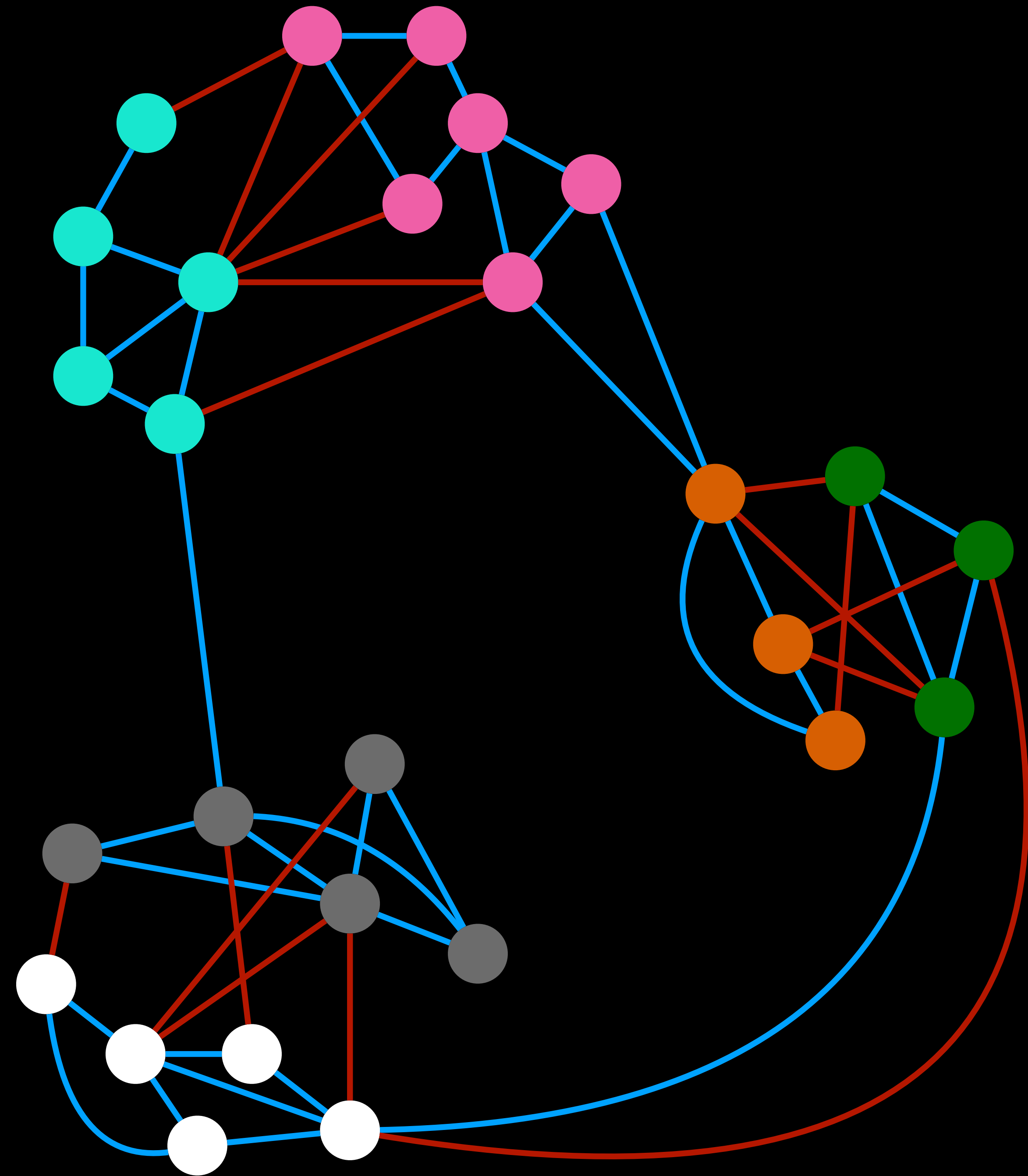
Signed Graphs

- Social networks can be seen as *signed* networks



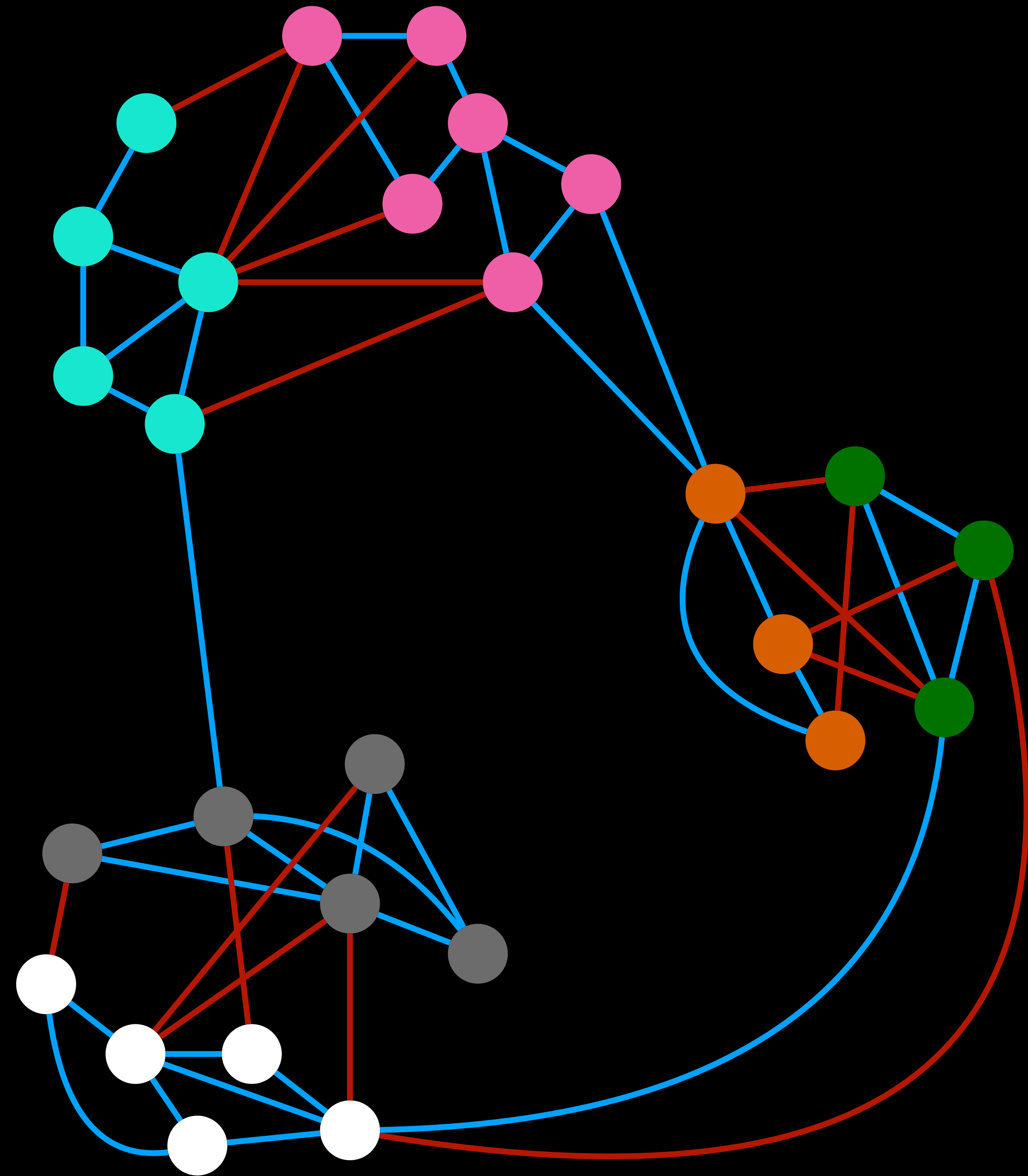
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- Each edge has a sign $+$ or $-$ indicating whether



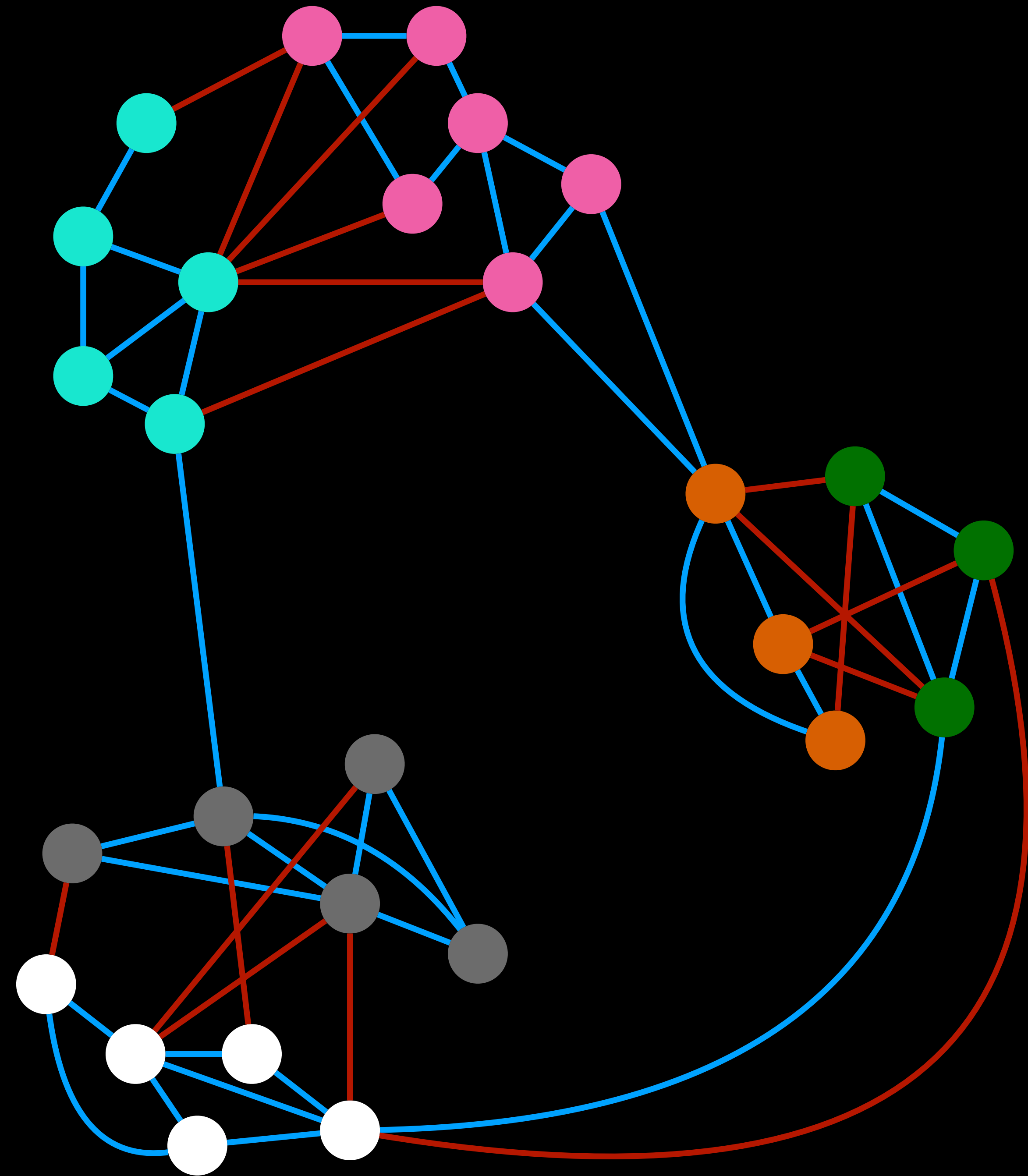
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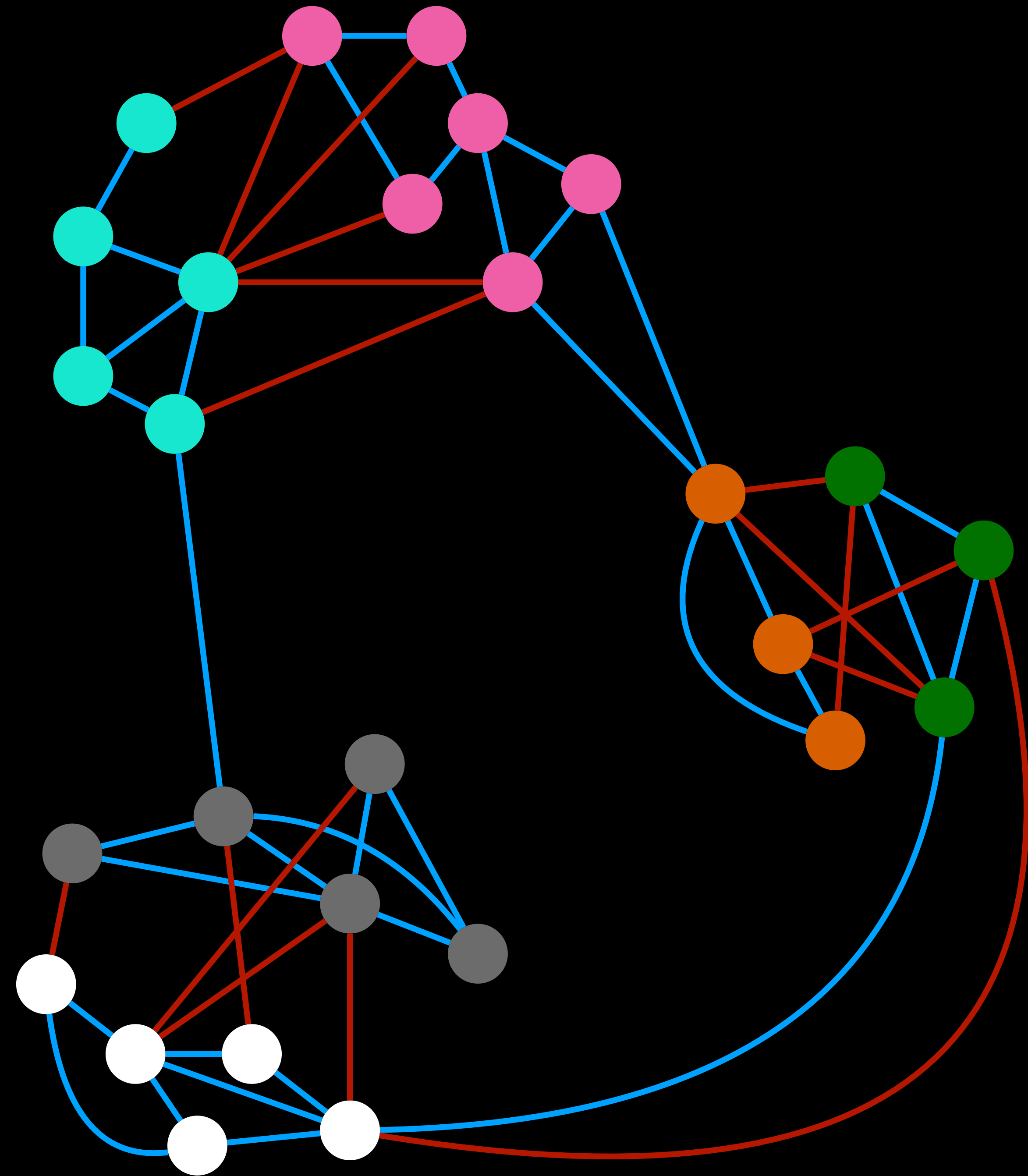
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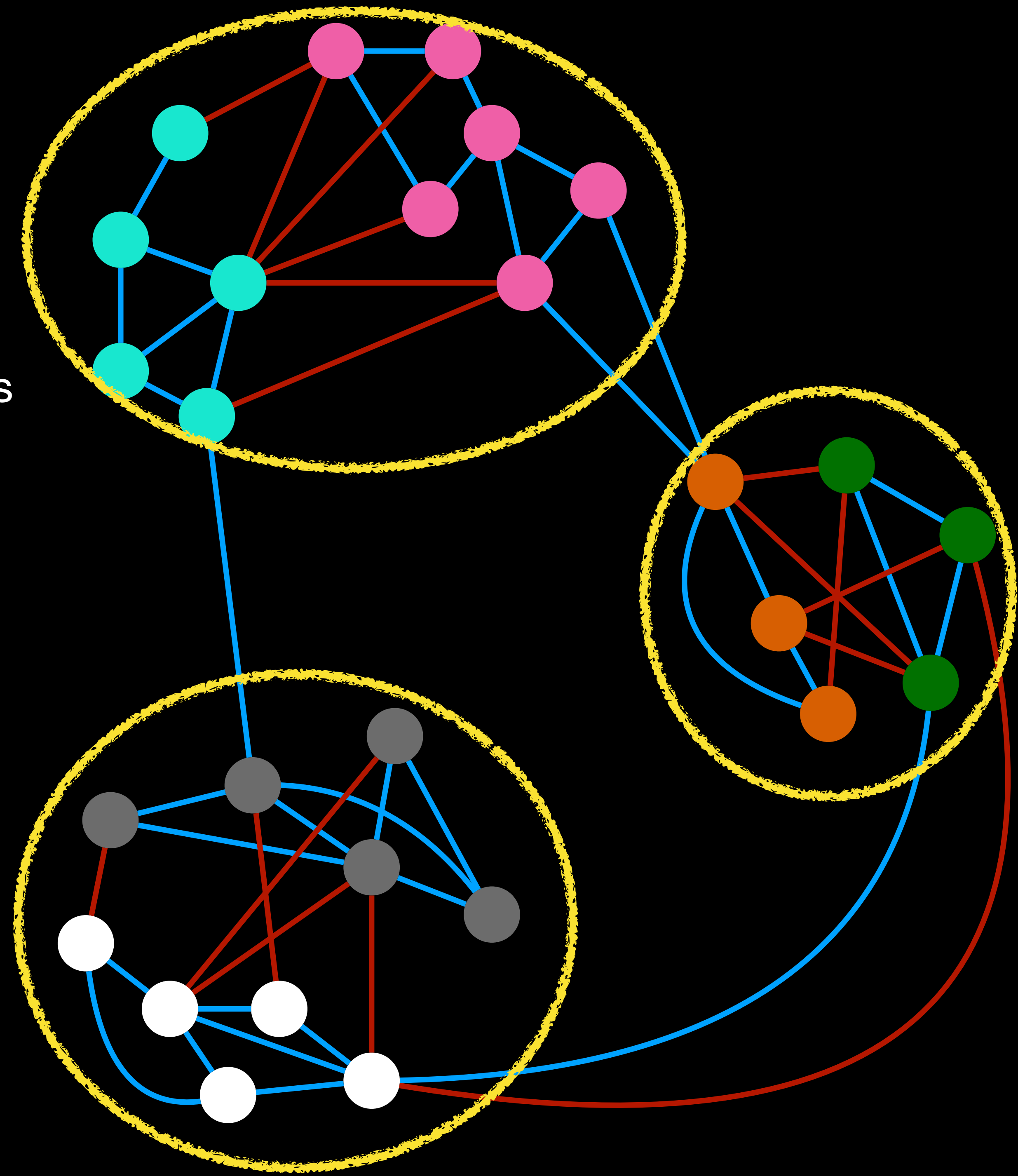
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- Allows to *detect conflicting groups* in social networks



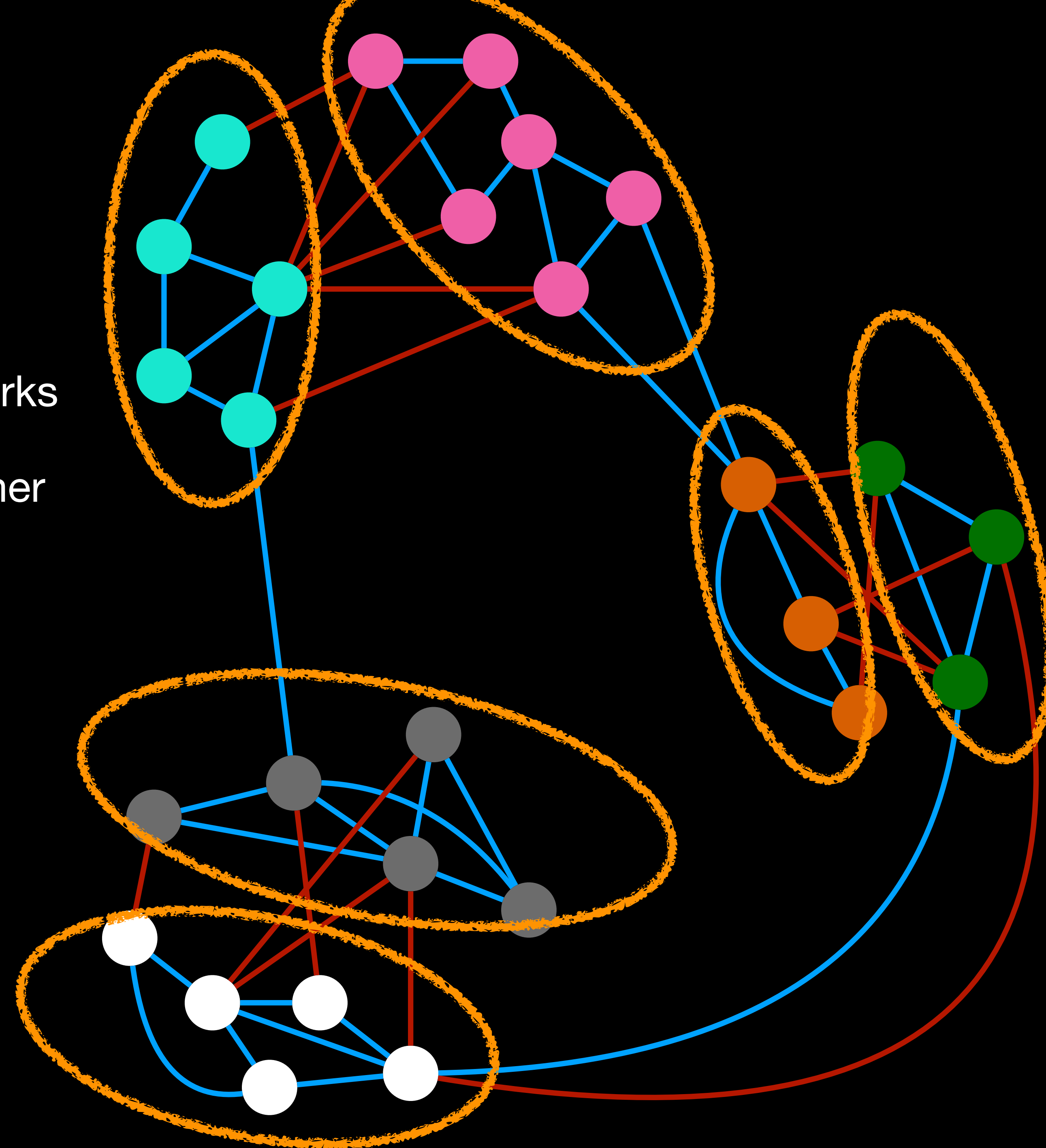
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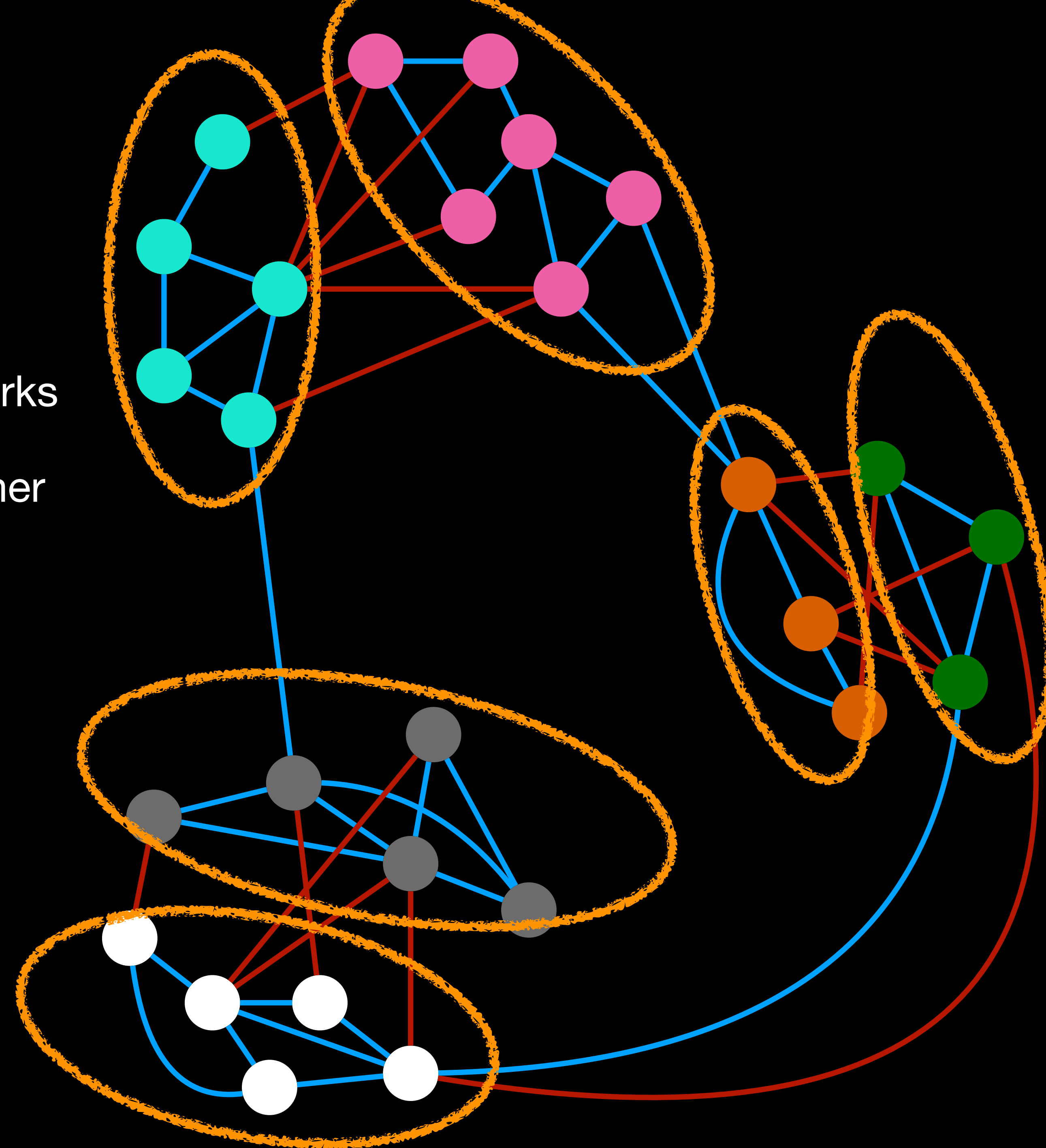
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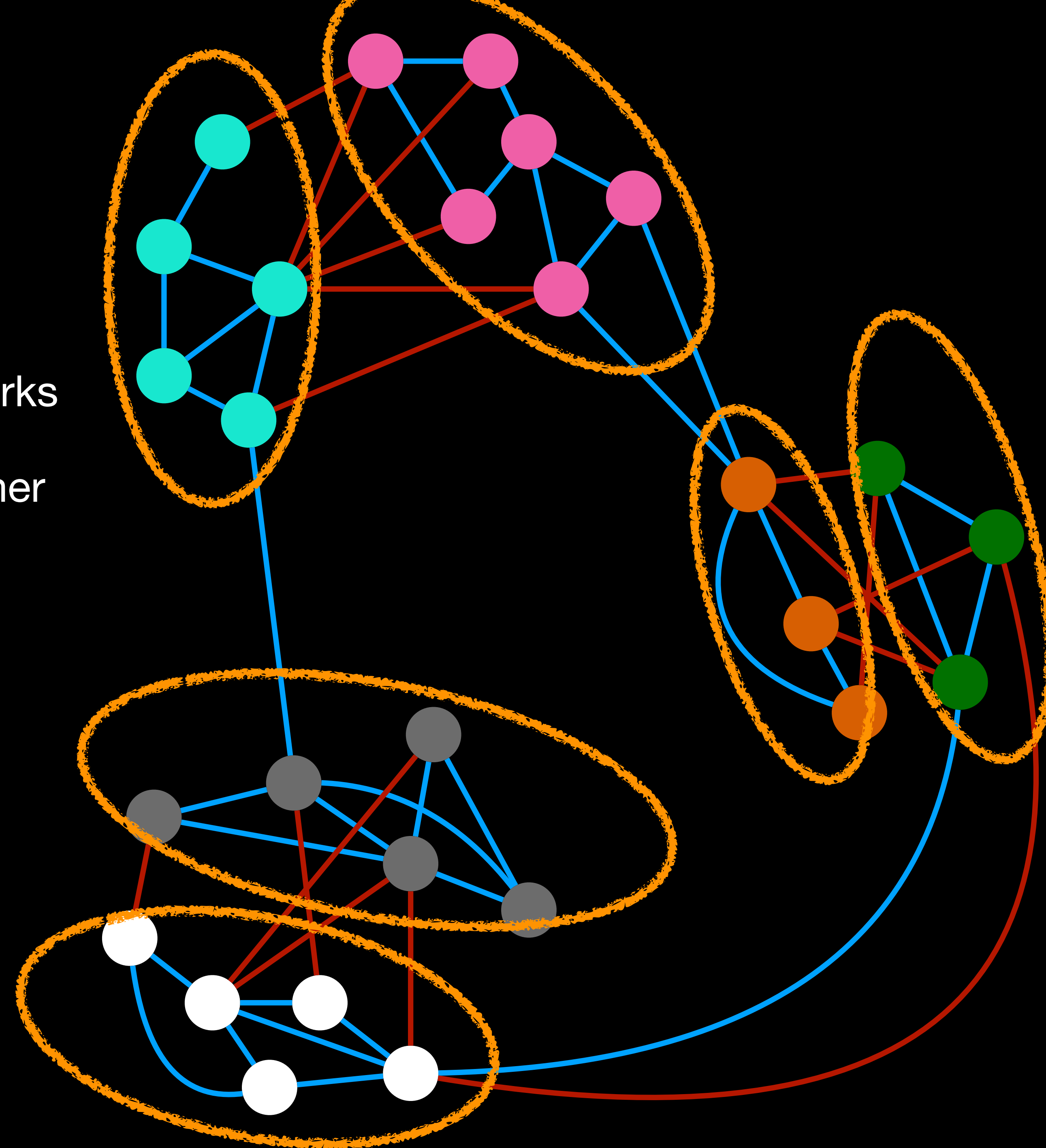
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 - ➔ Democrats vs Republicans



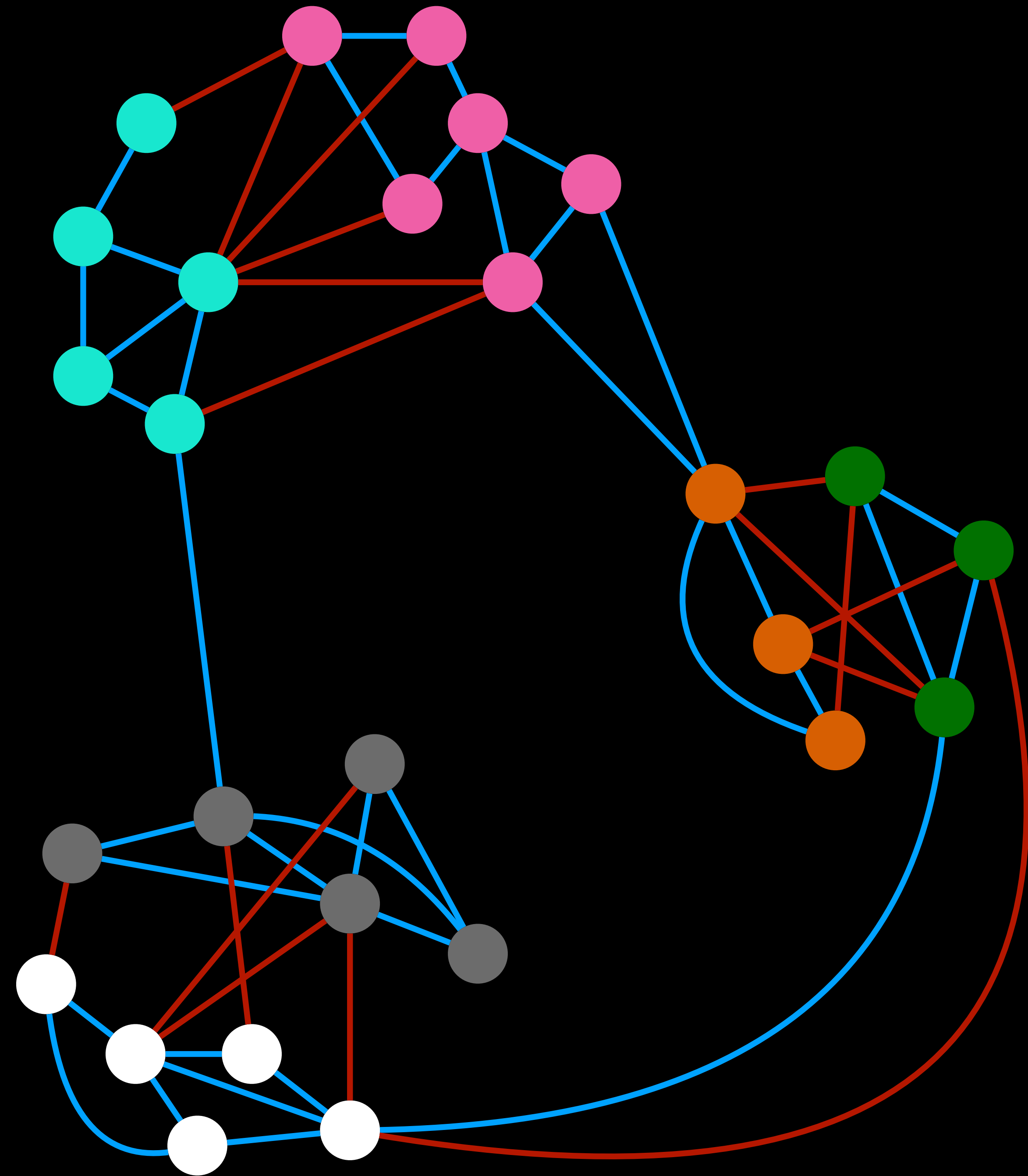
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 - ➔ Democrats vs Republicans
 - ➔ analyze trust in Bitcoin networks



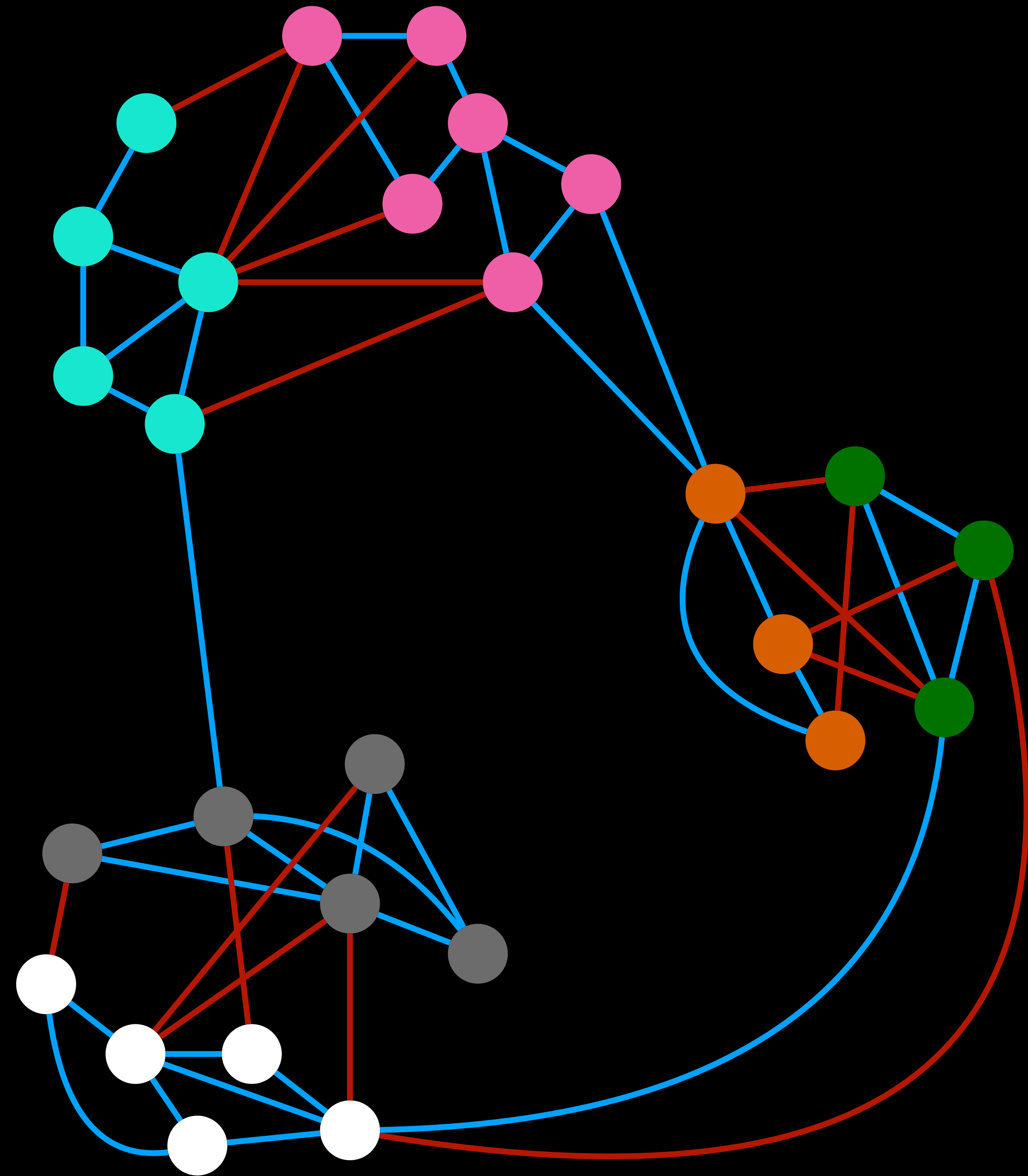
Sublinear-Time Algorithms

- Real-world graphs are huge and clustering the entire graph is **often infeasible**



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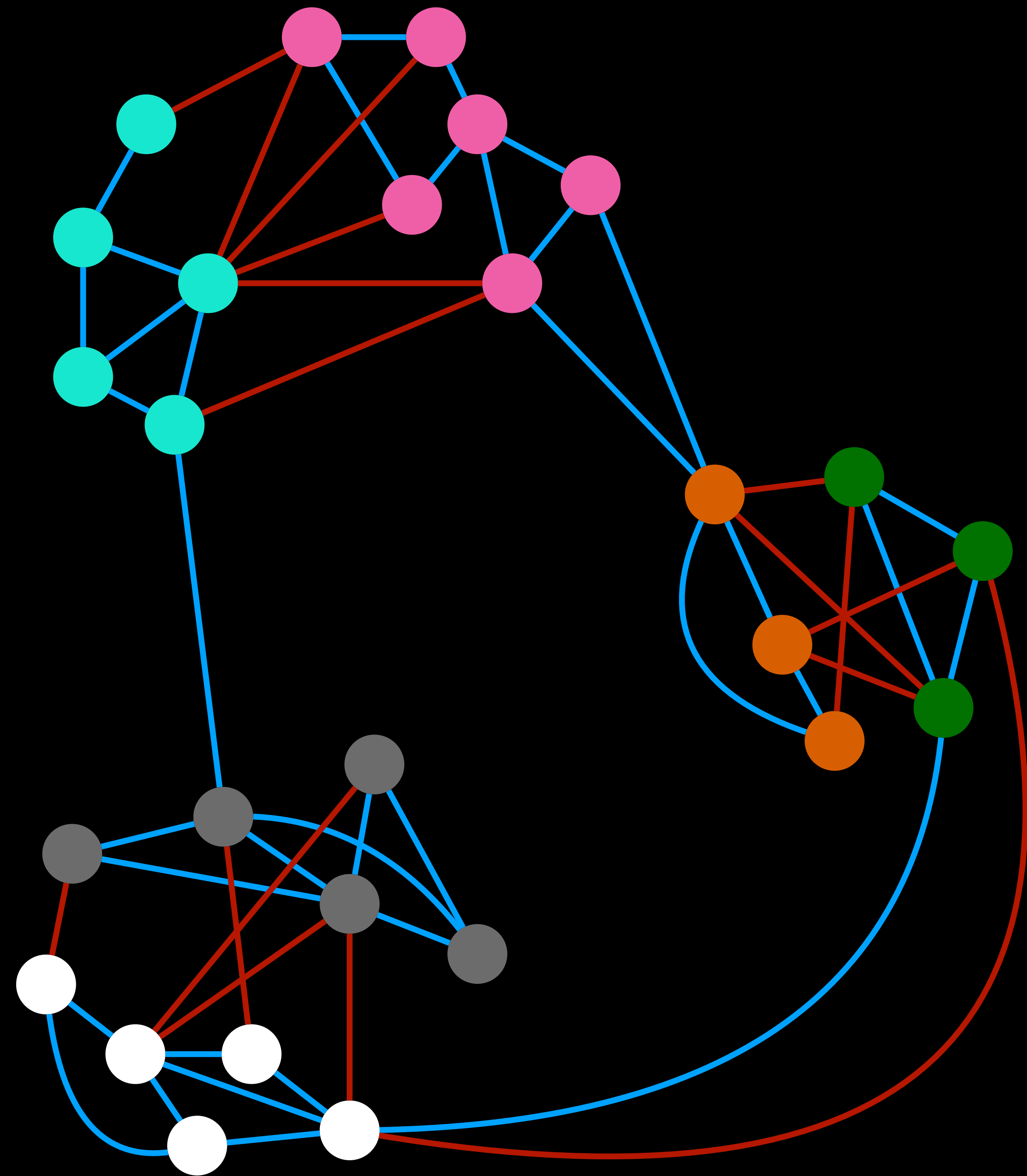
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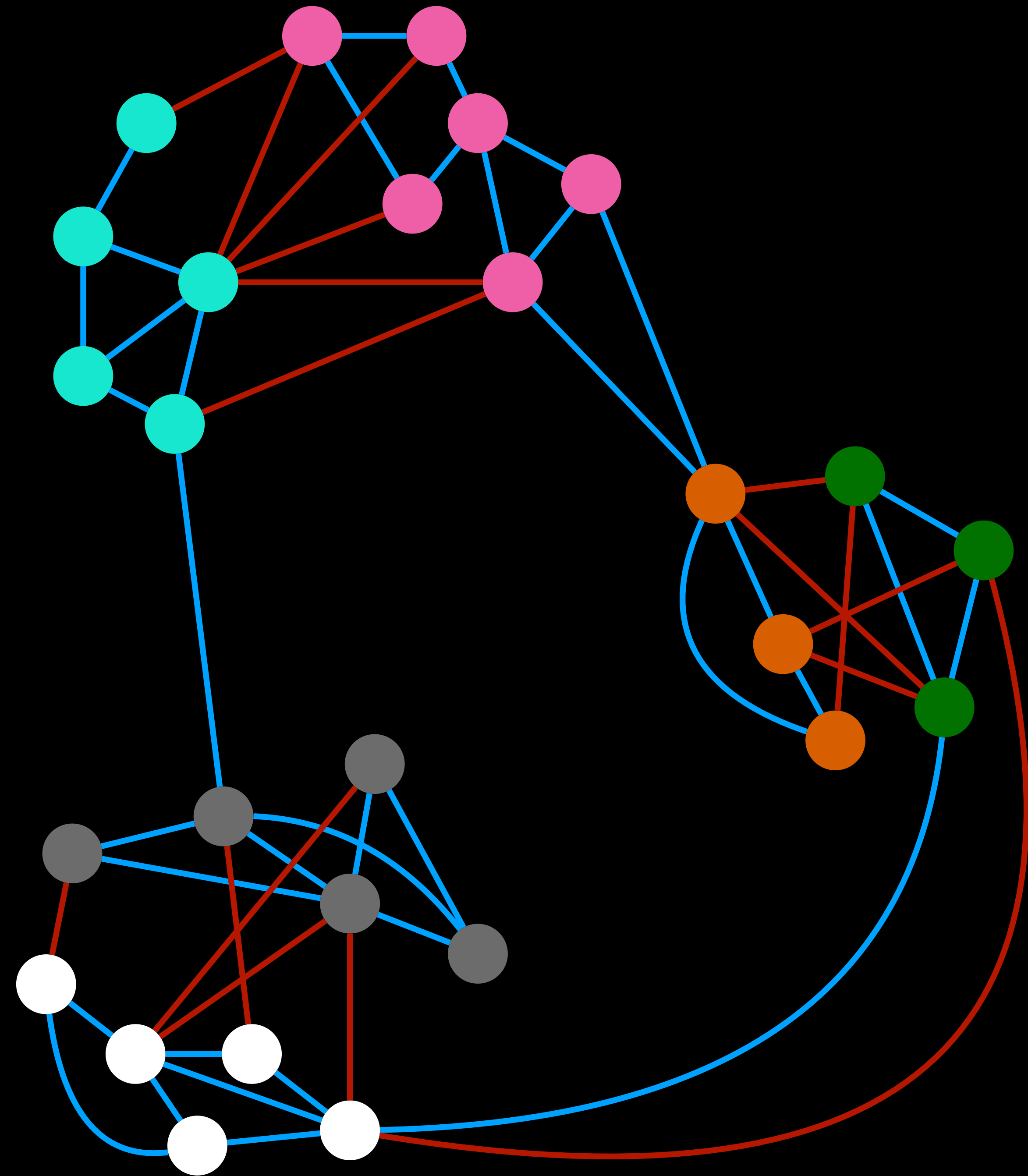
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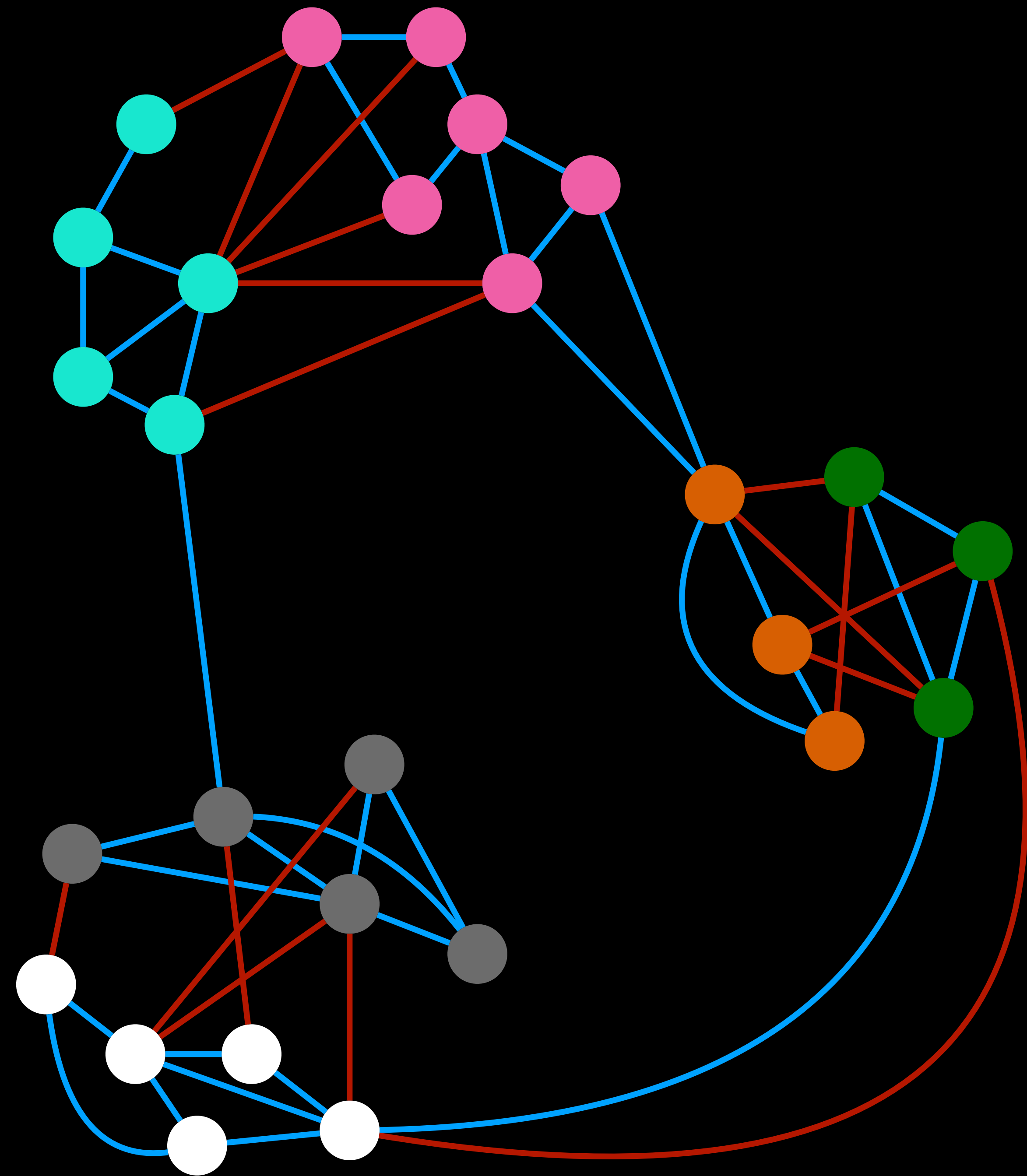
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- In many applications we need the cluster information **only for a few vertices**

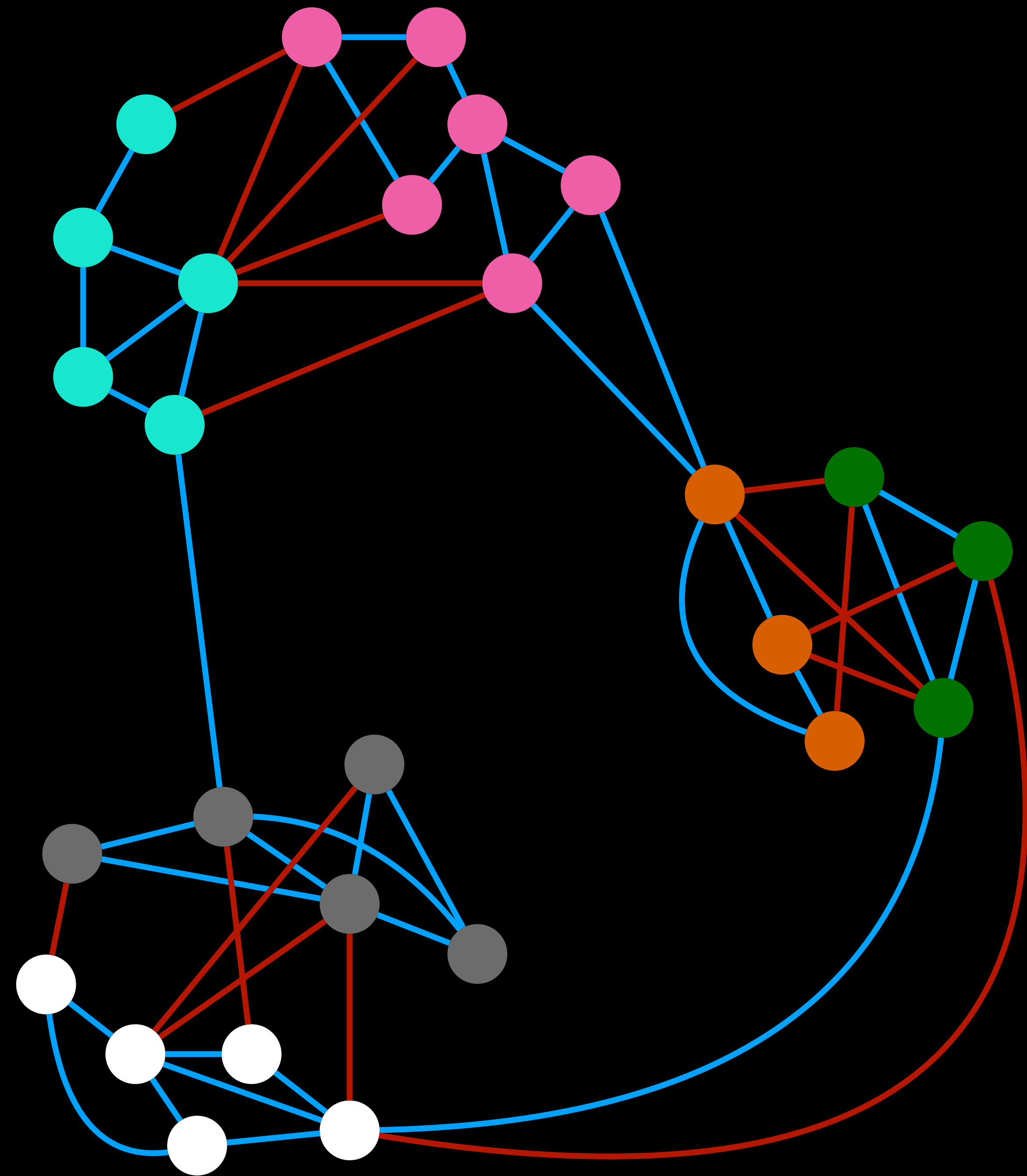


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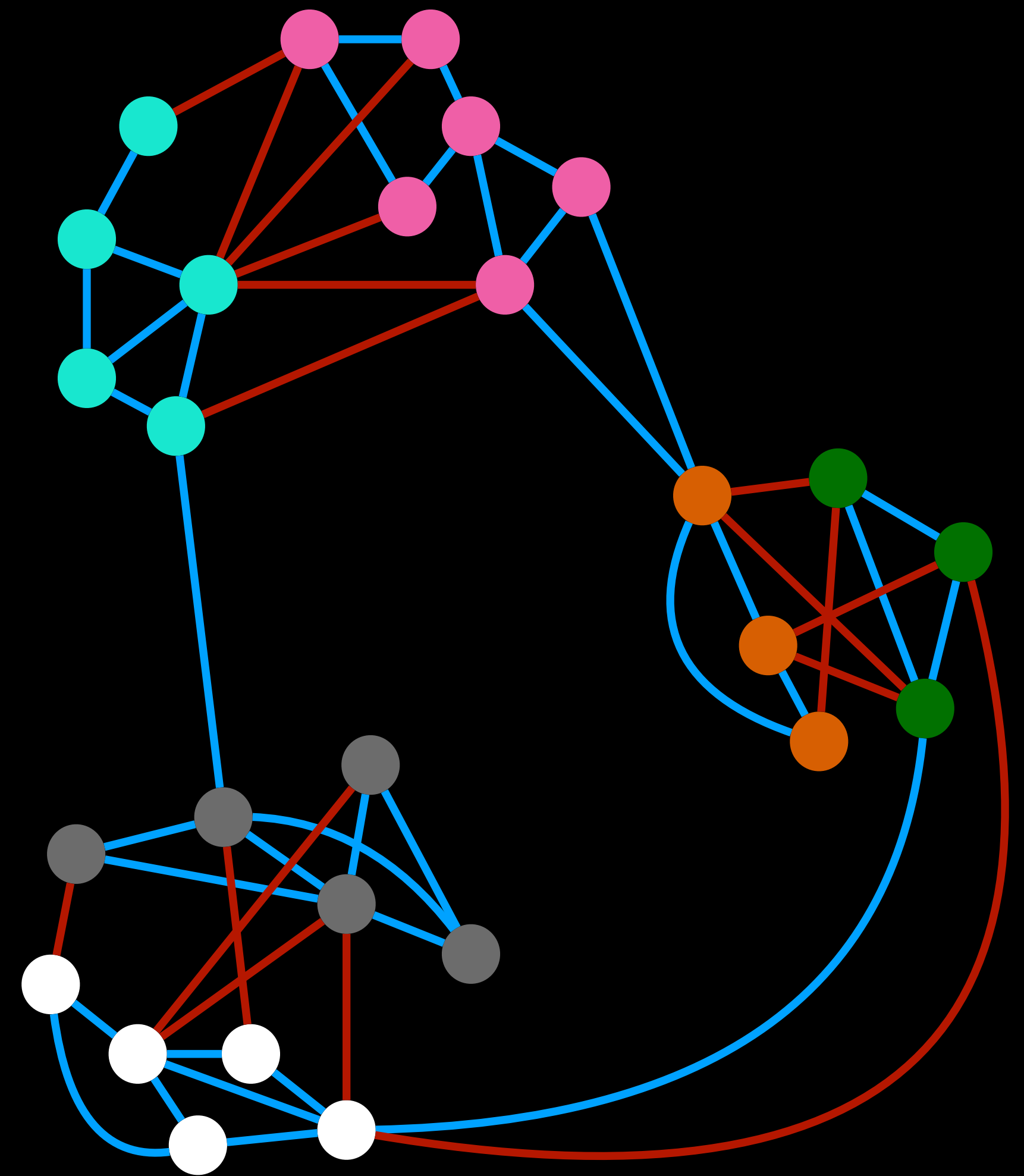
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- Must be very fast and space efficient

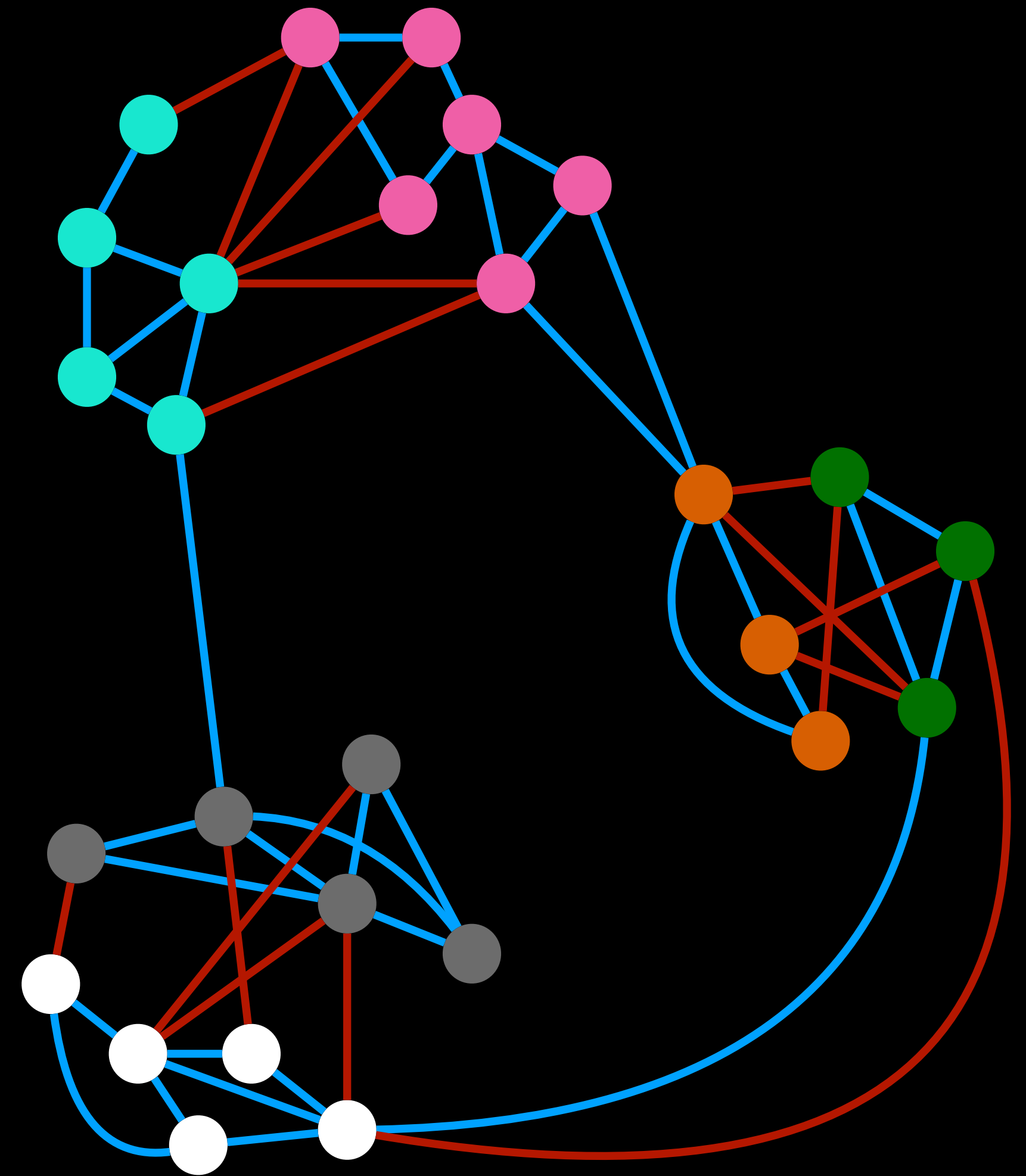


Our Signed Oracle



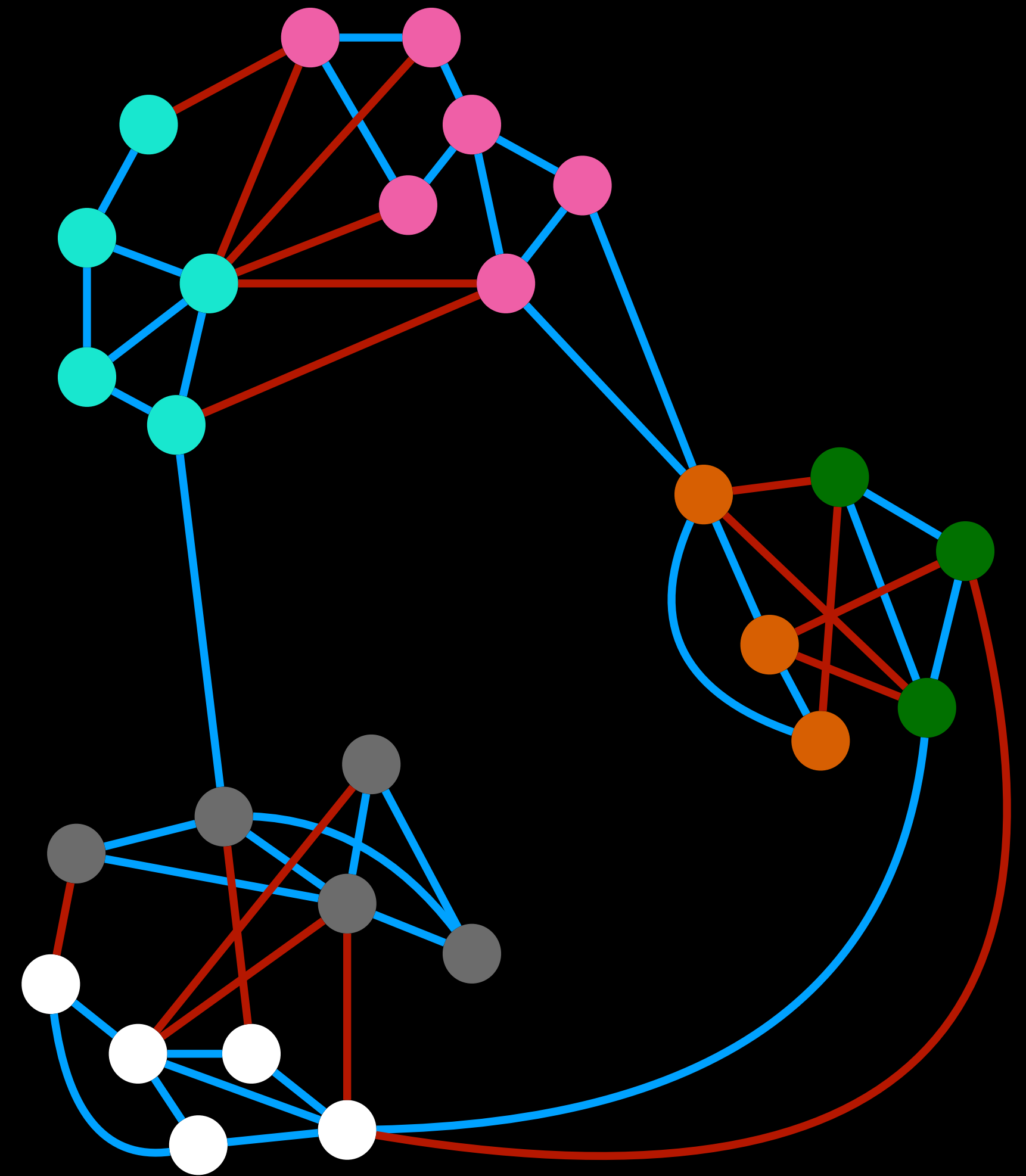
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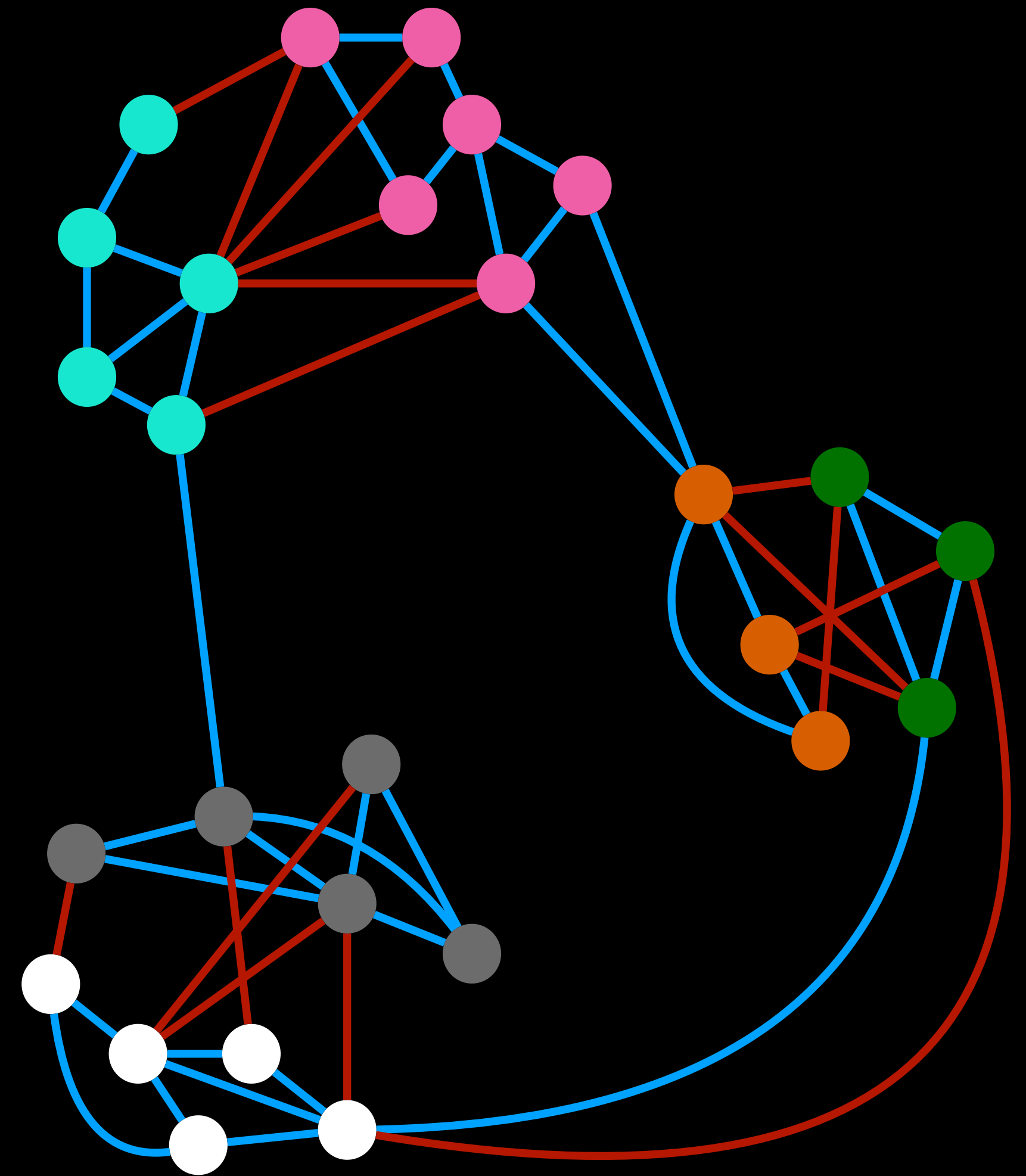
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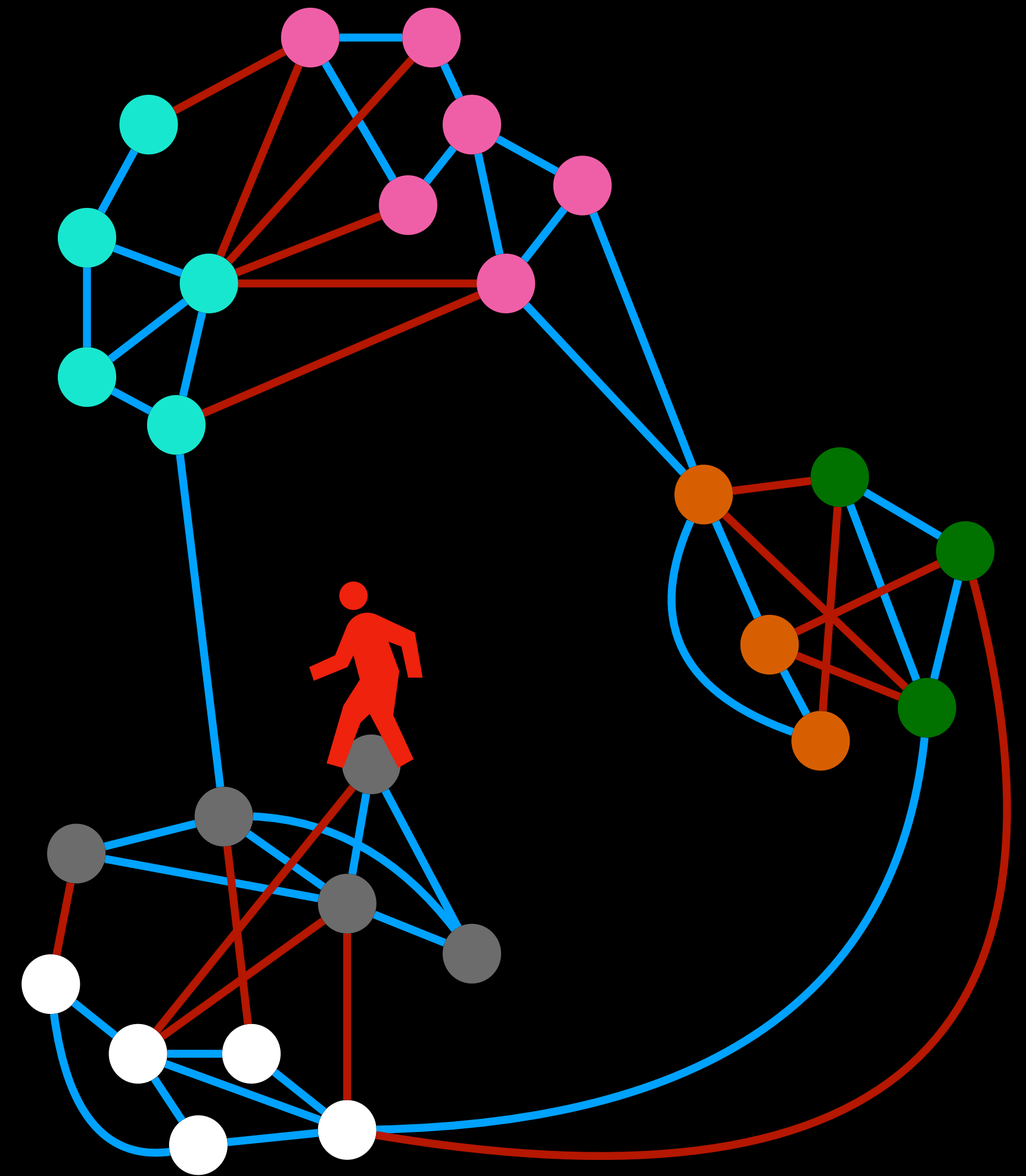
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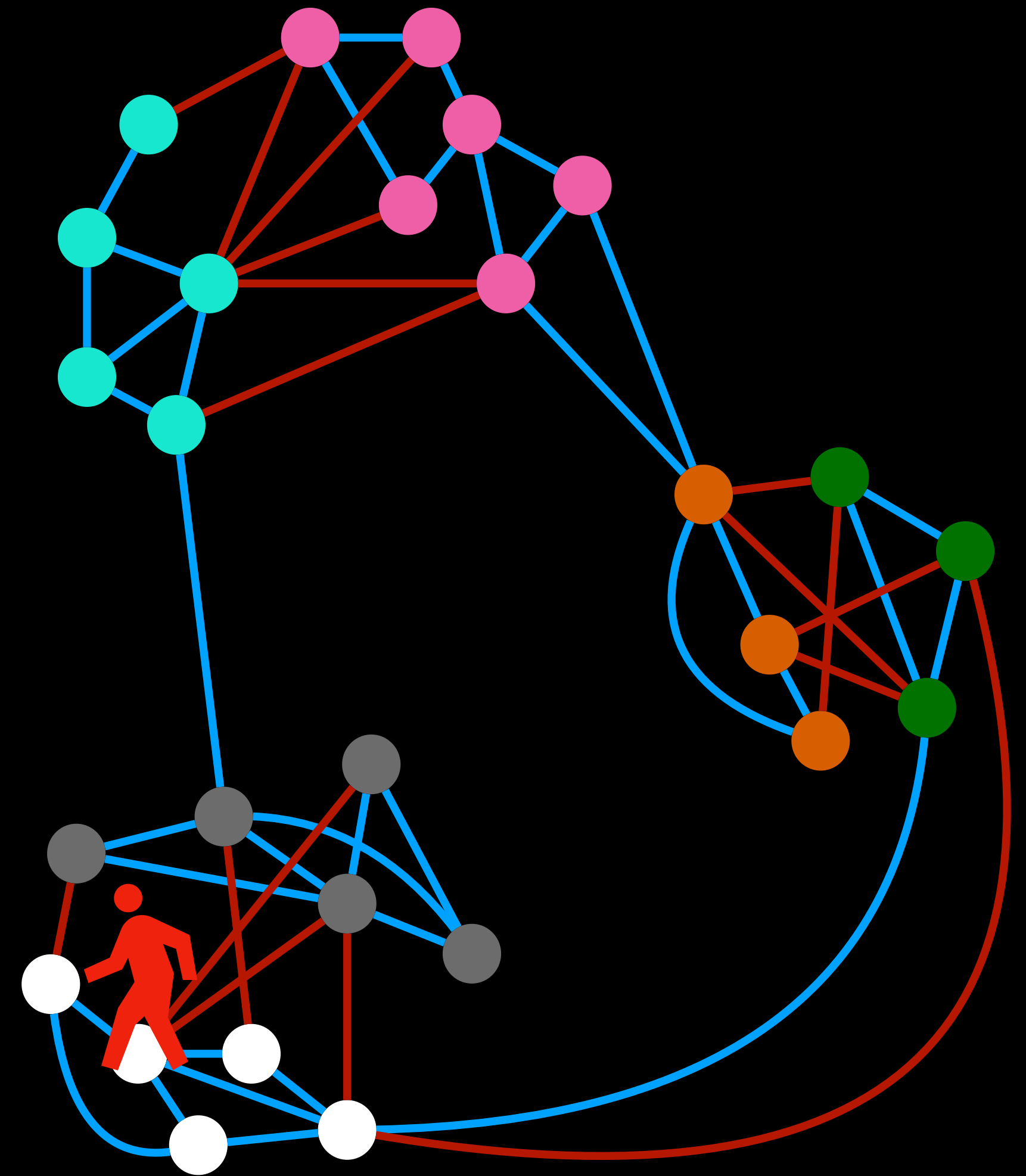
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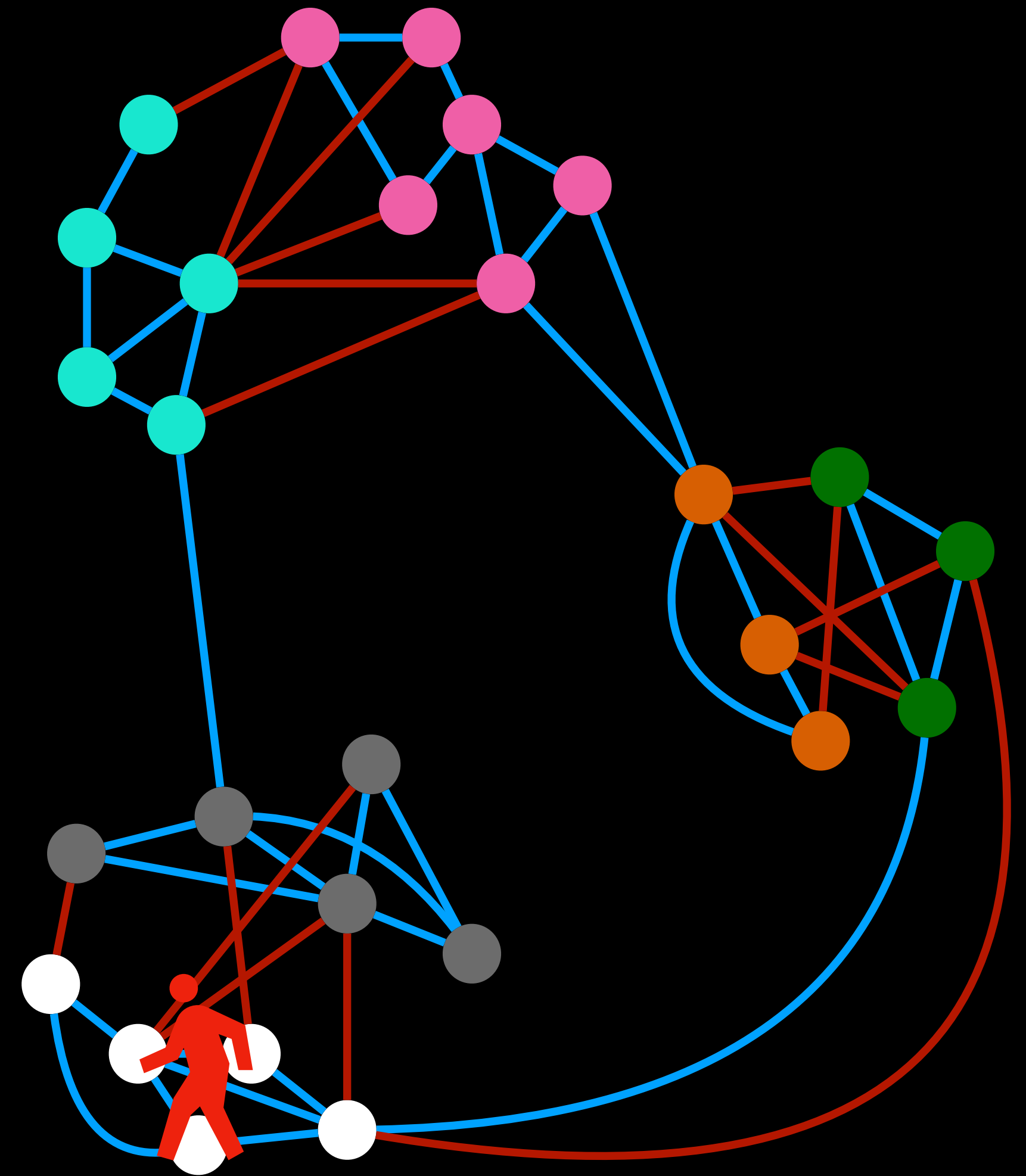
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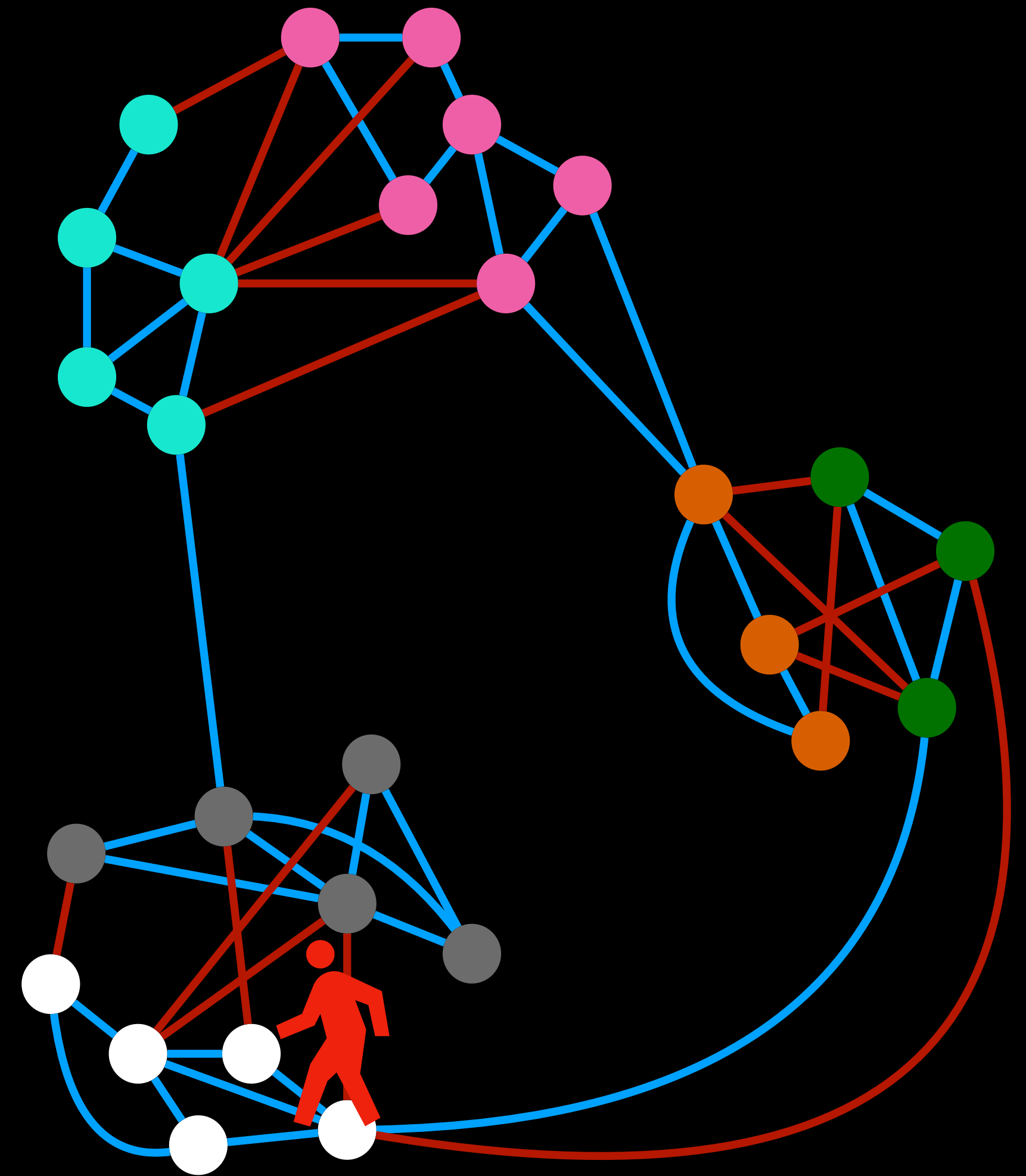
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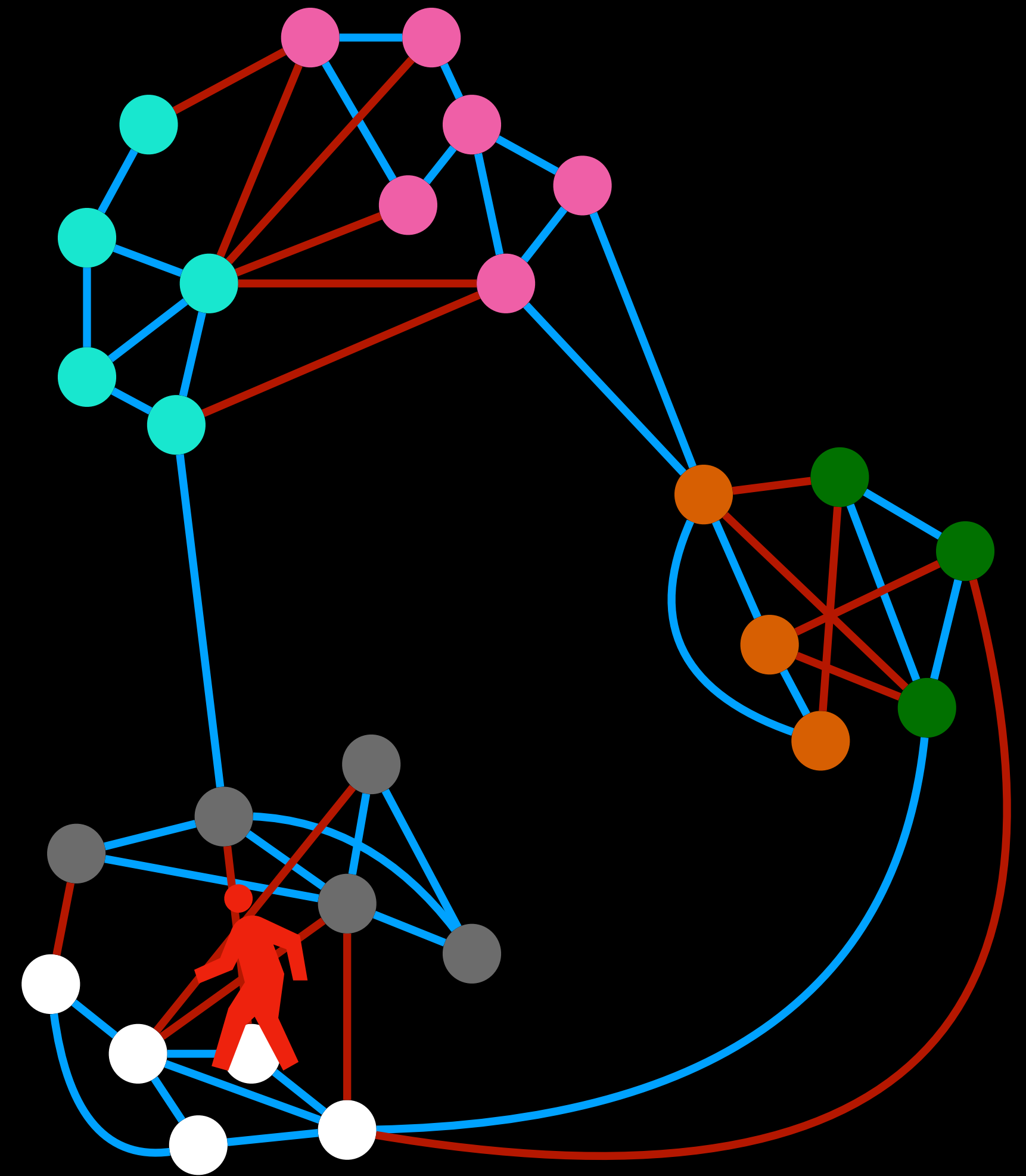
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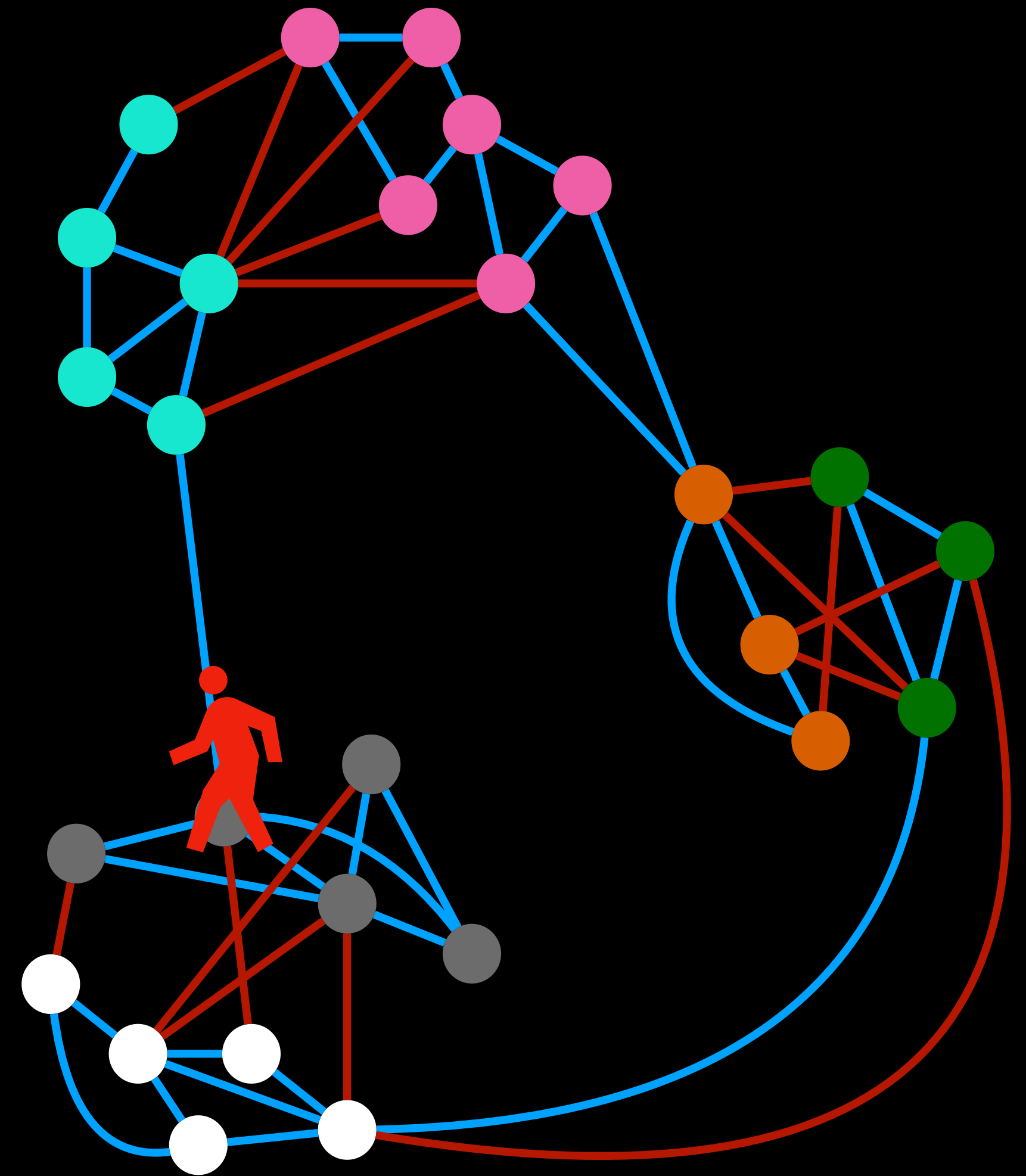
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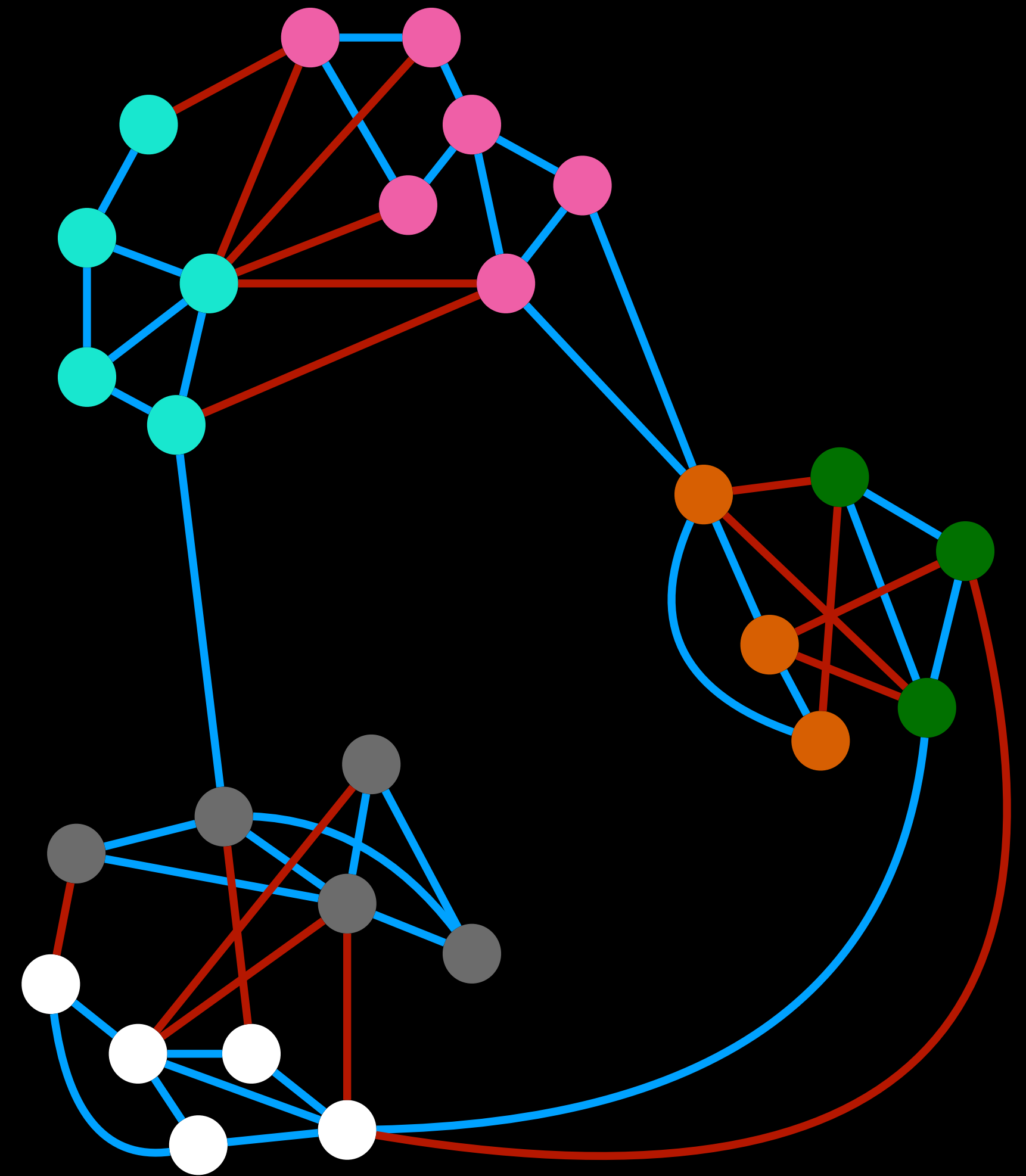
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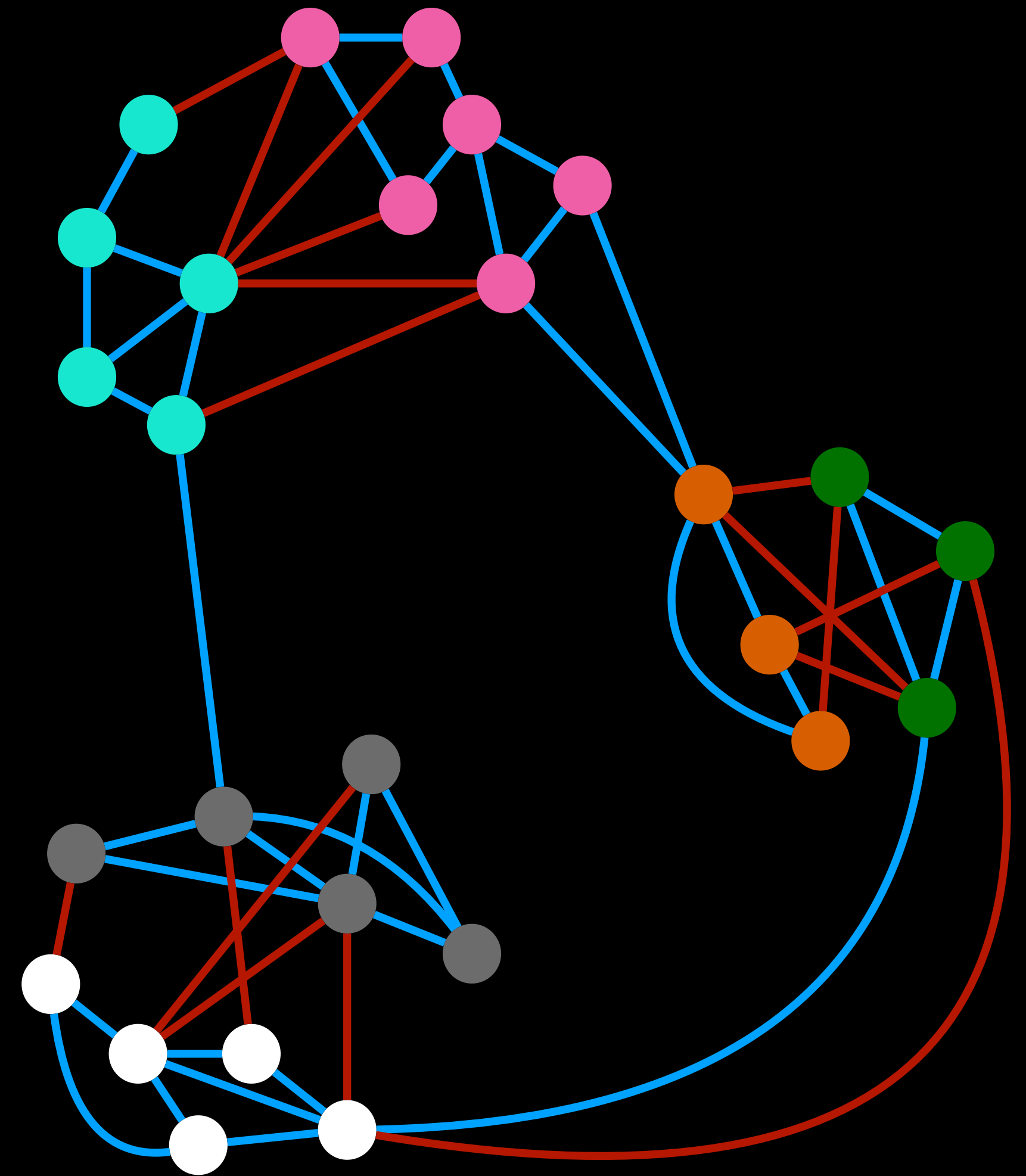
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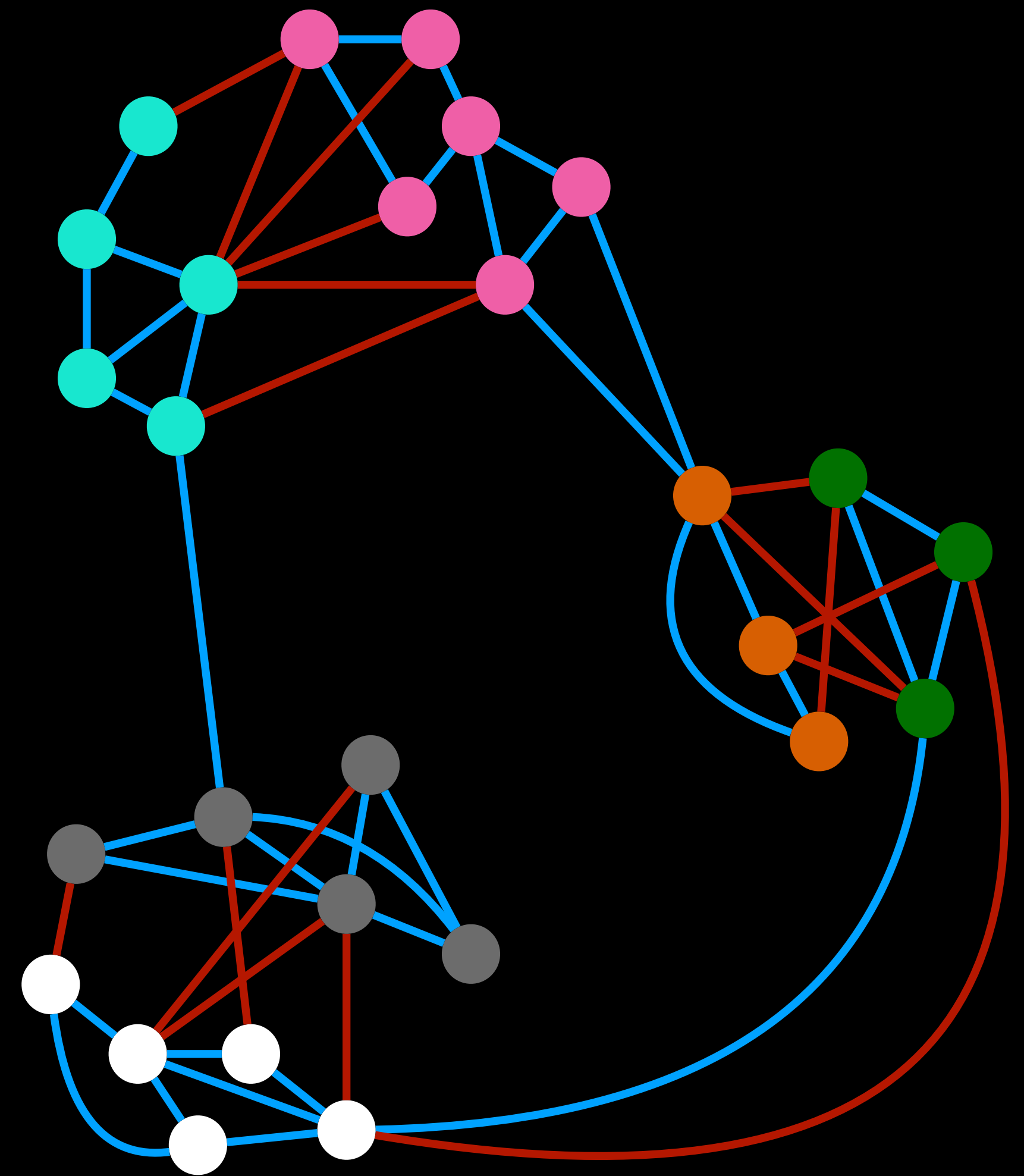
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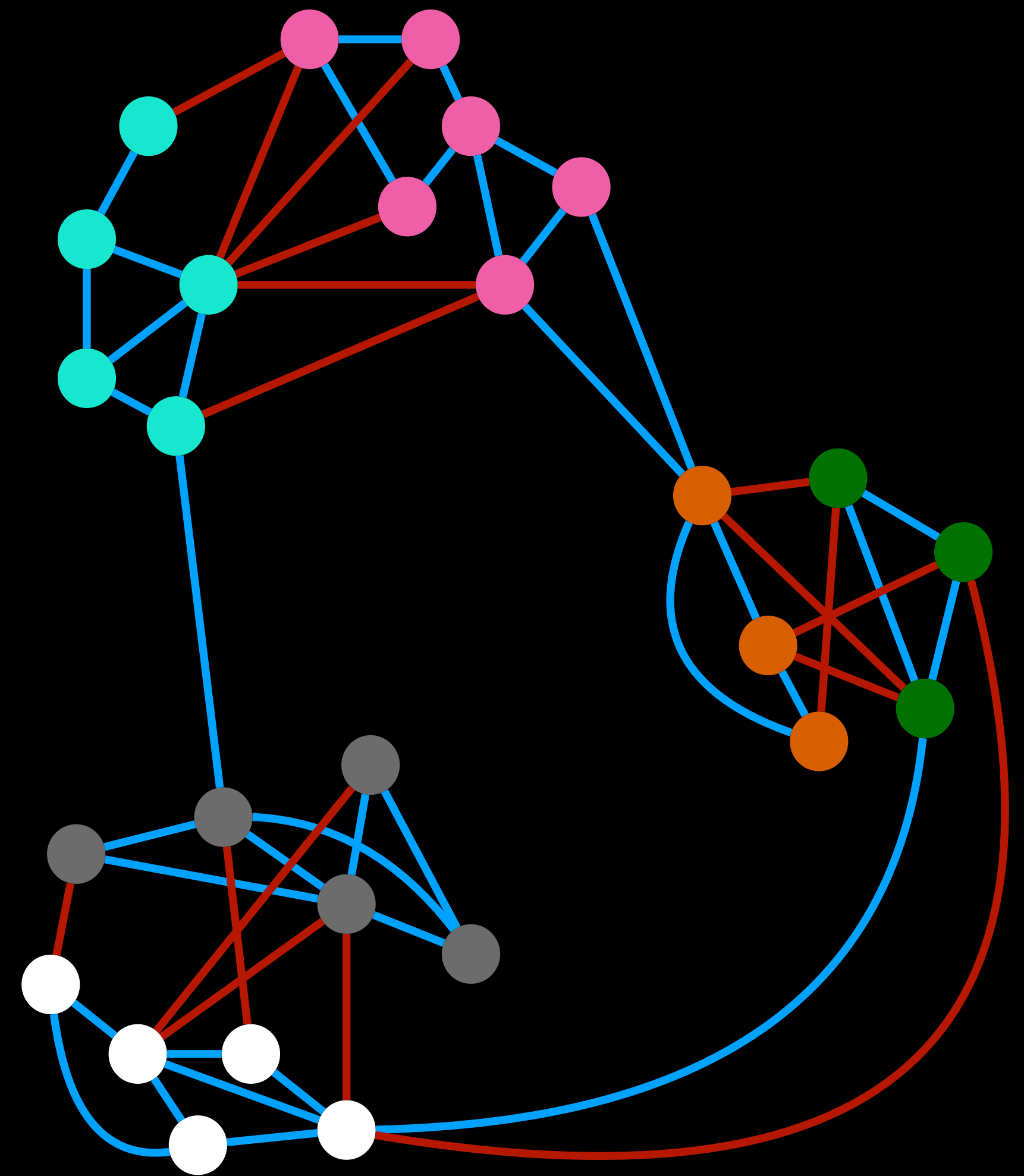


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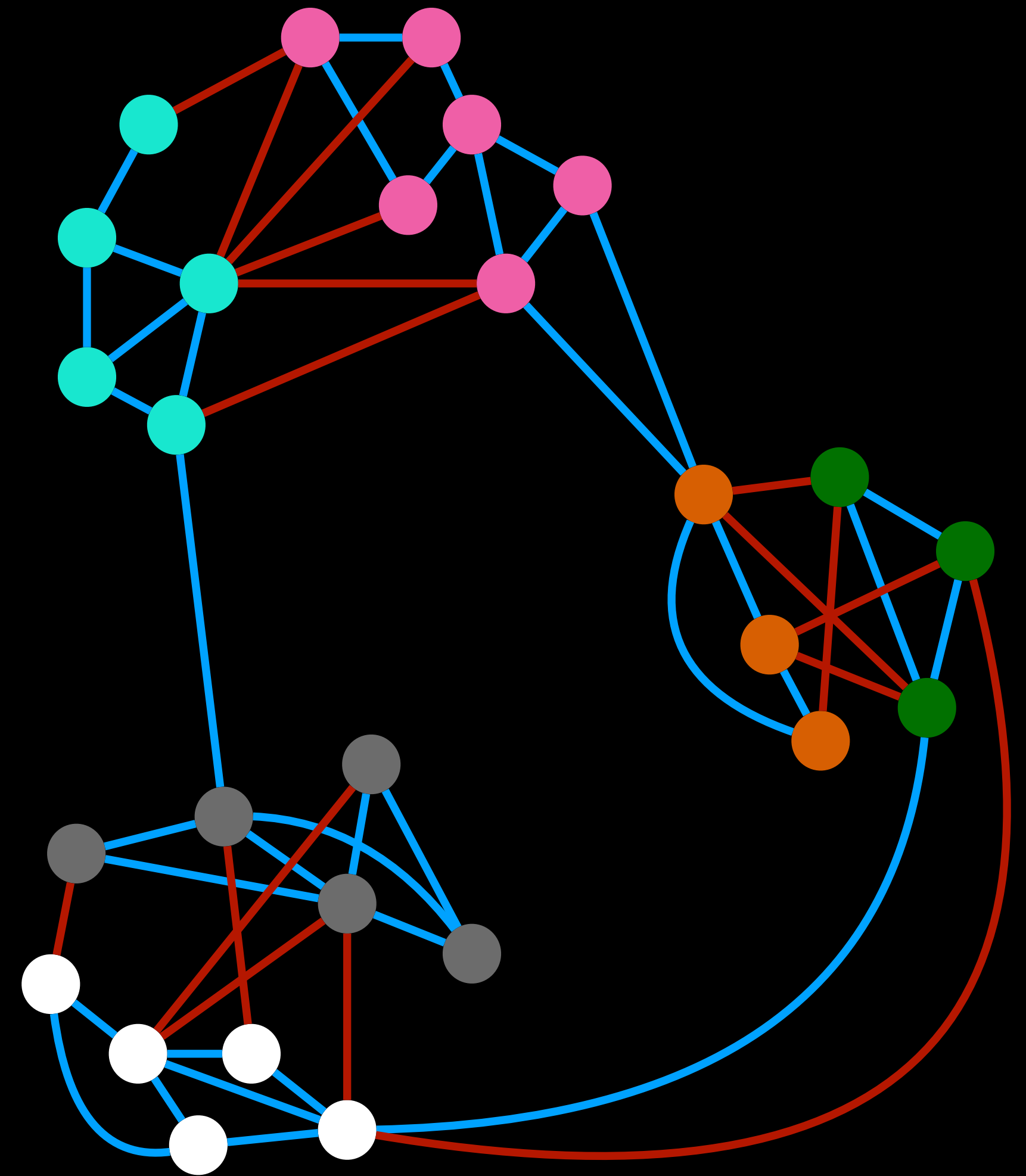


We Can Provably Recover Planted Clusters



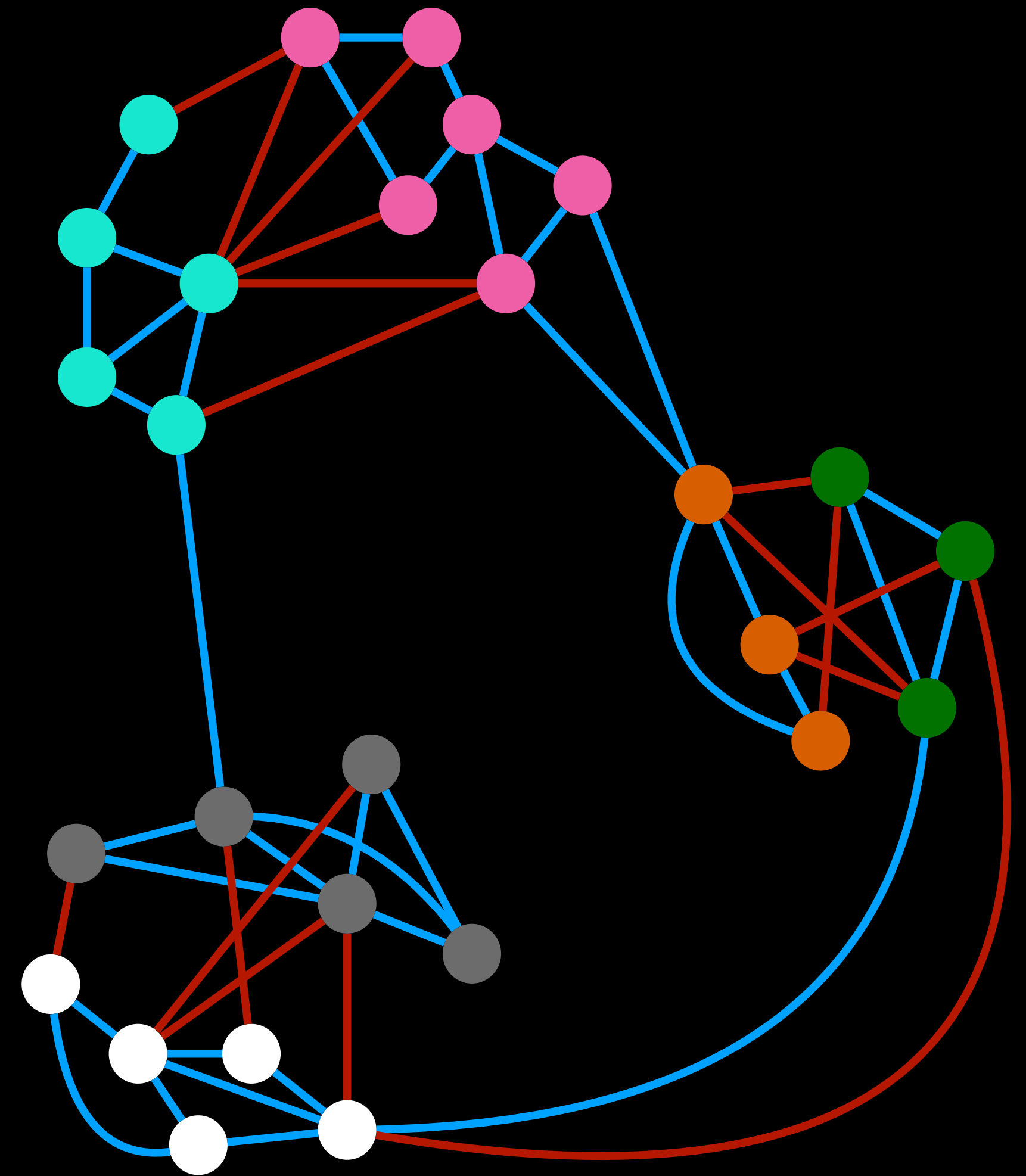
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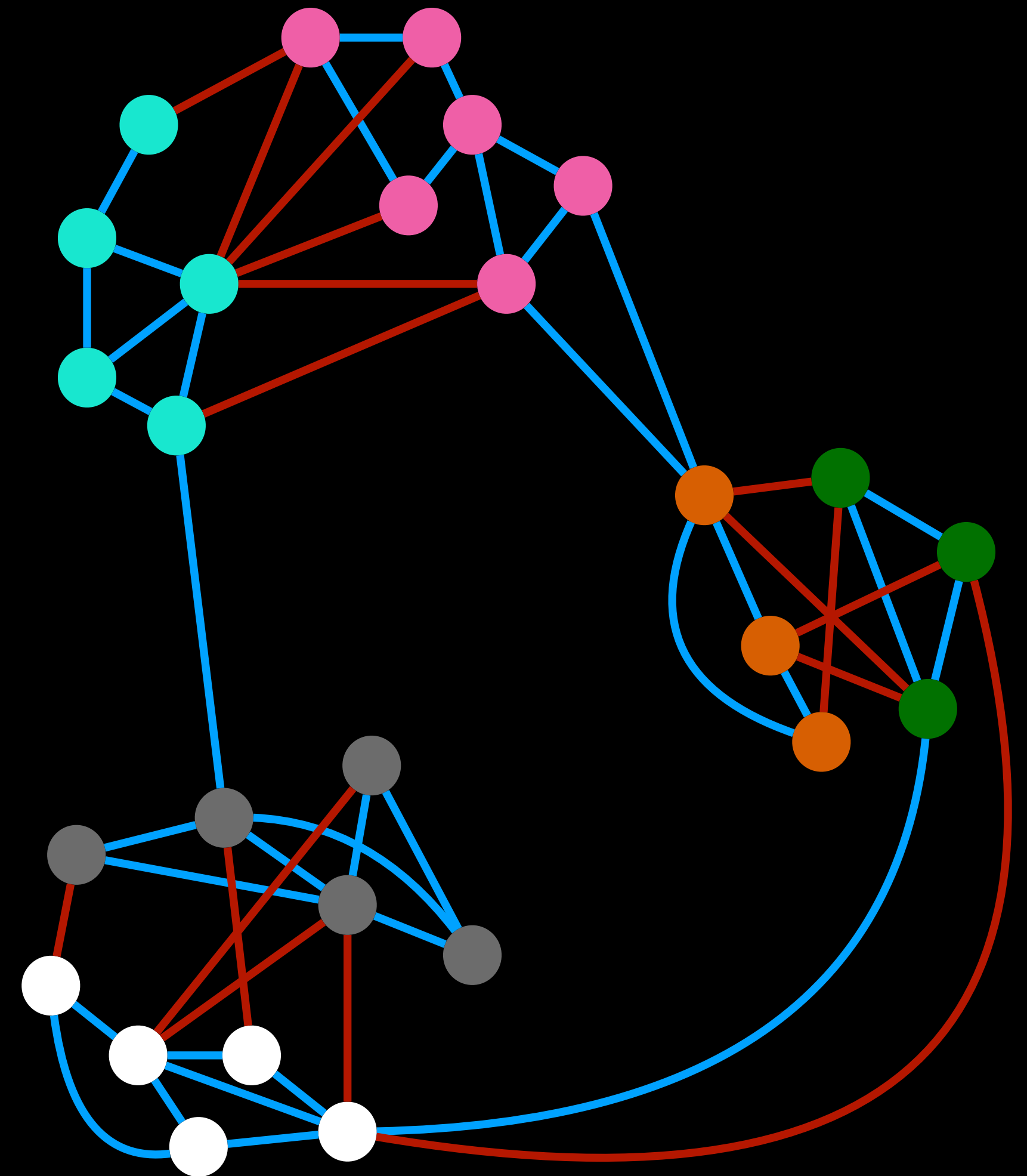
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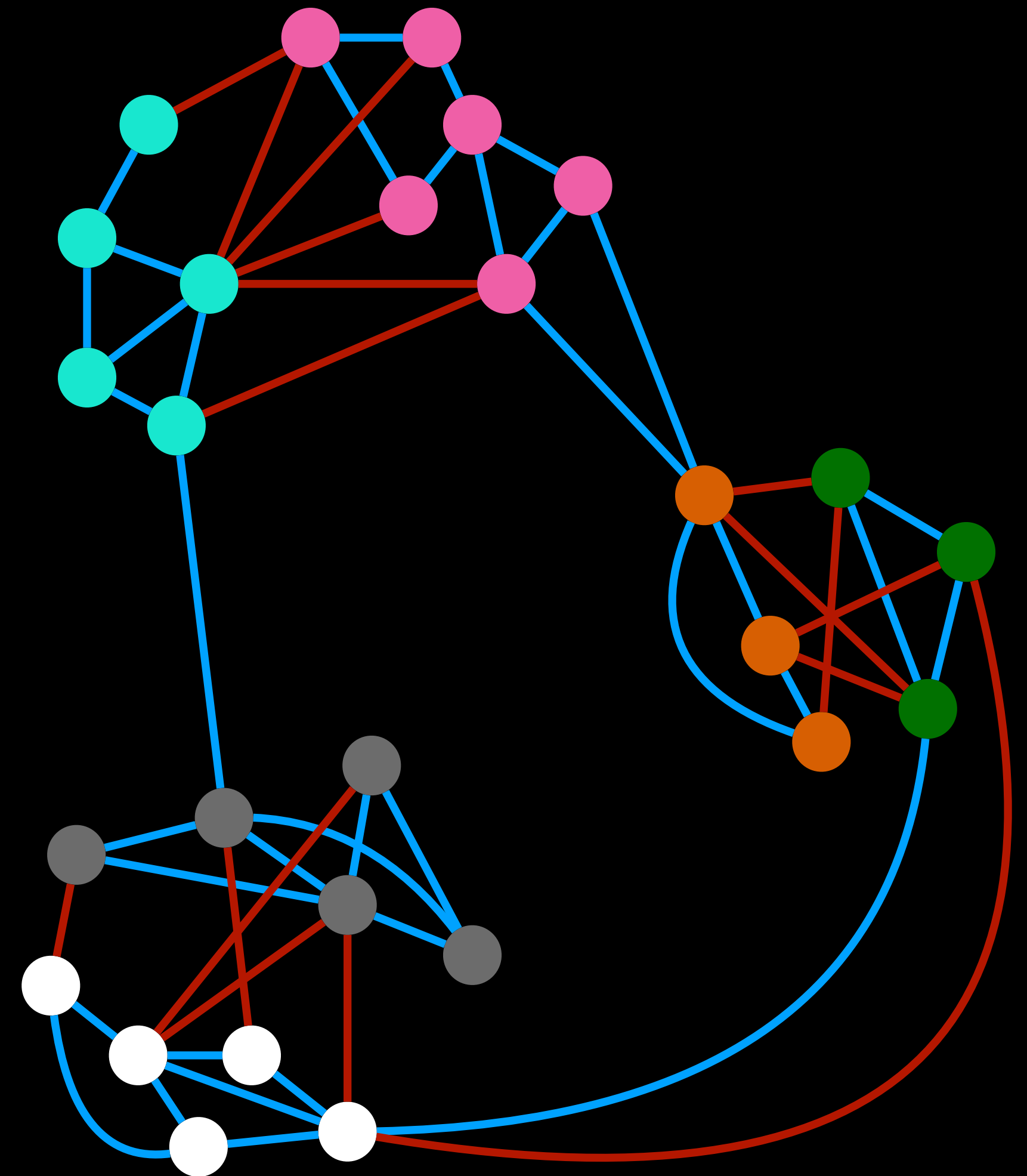
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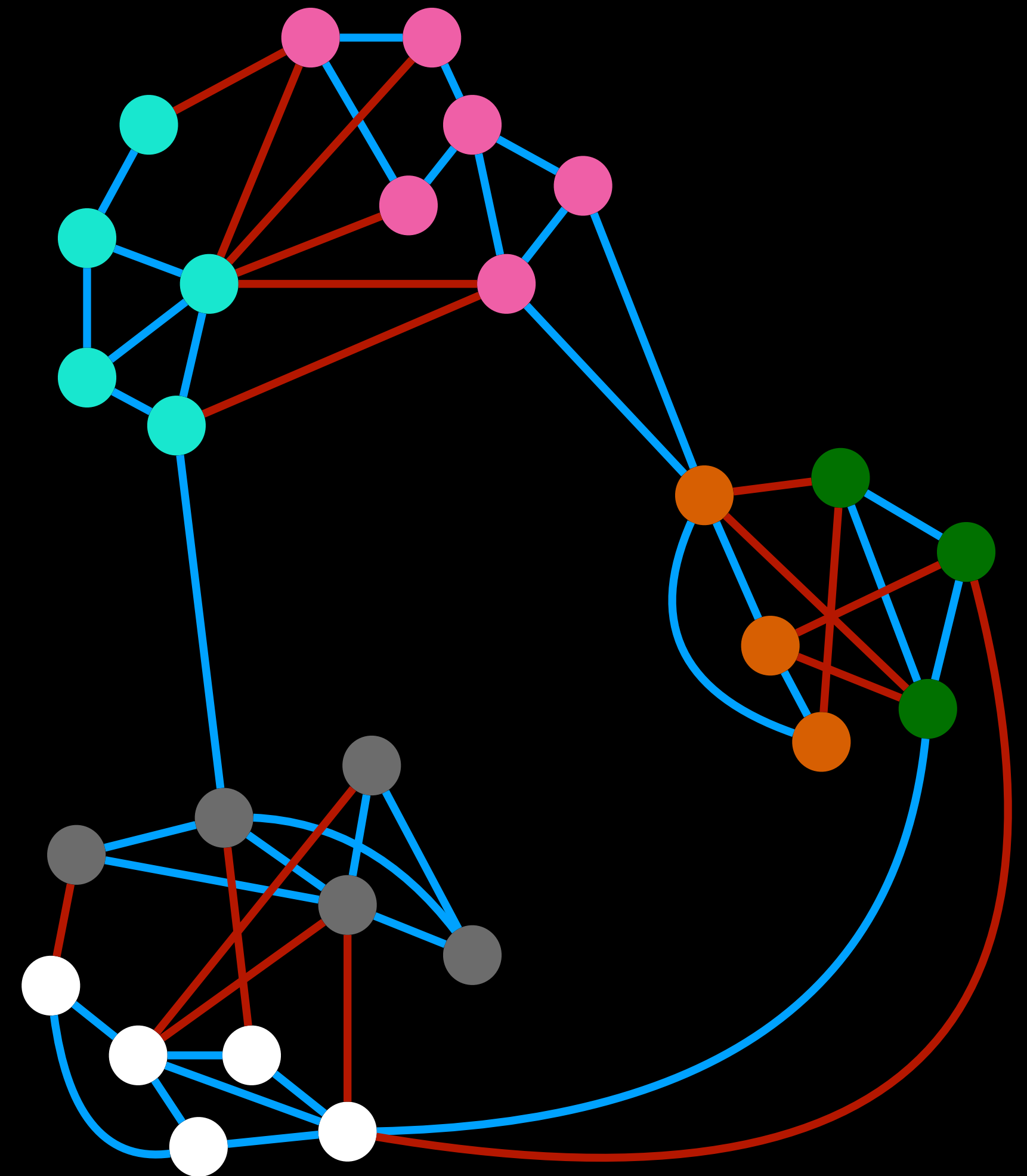
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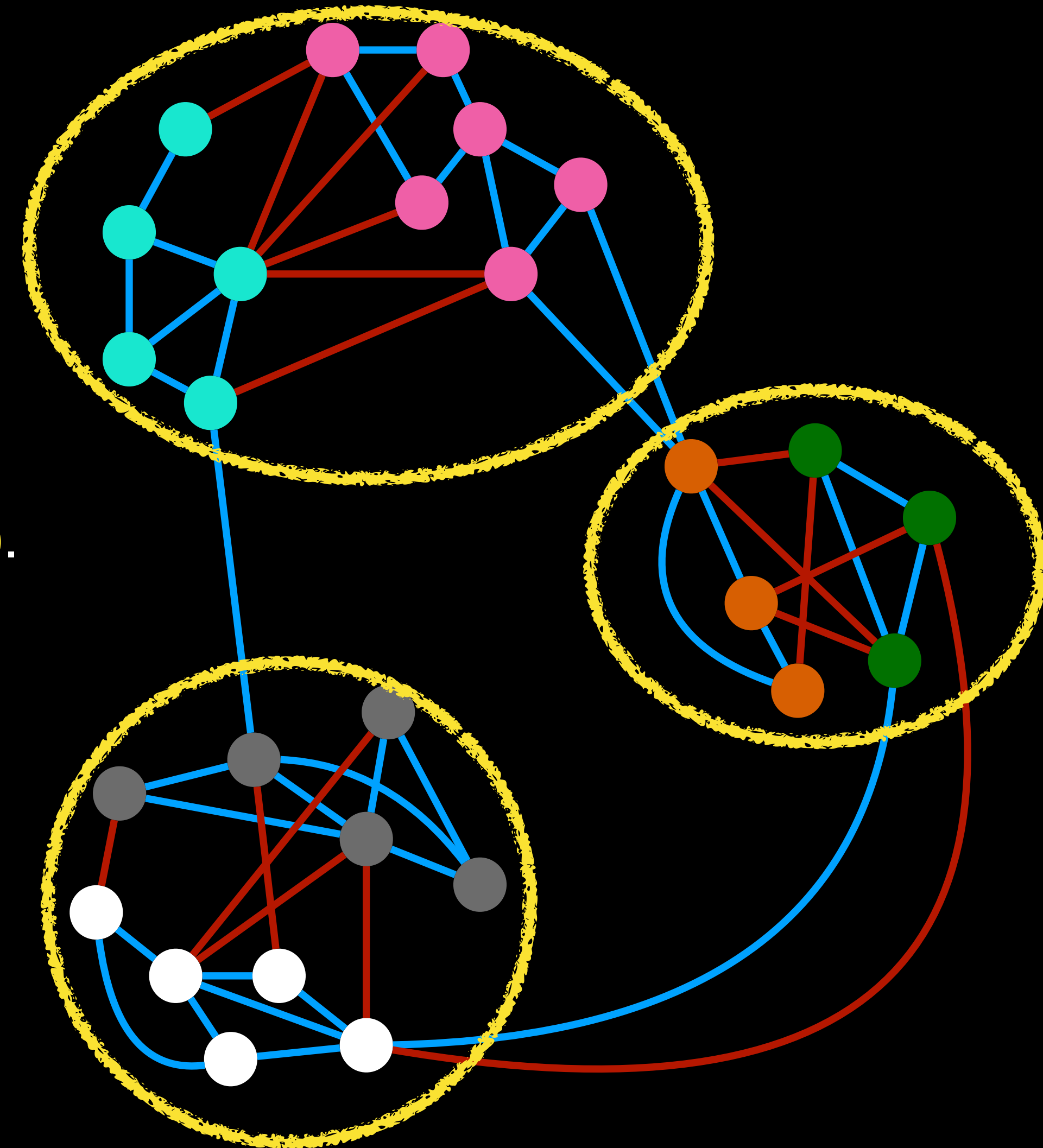
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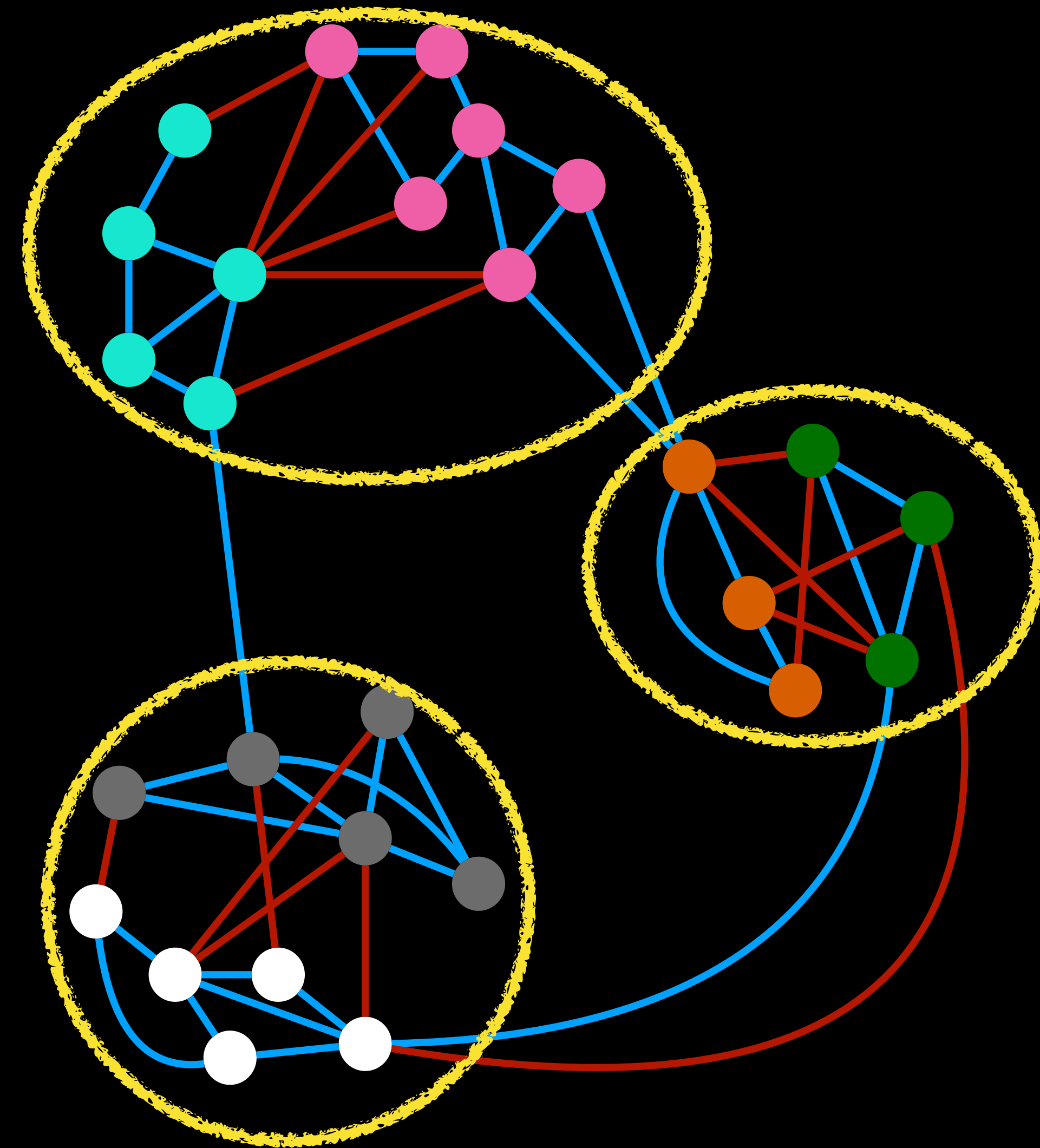


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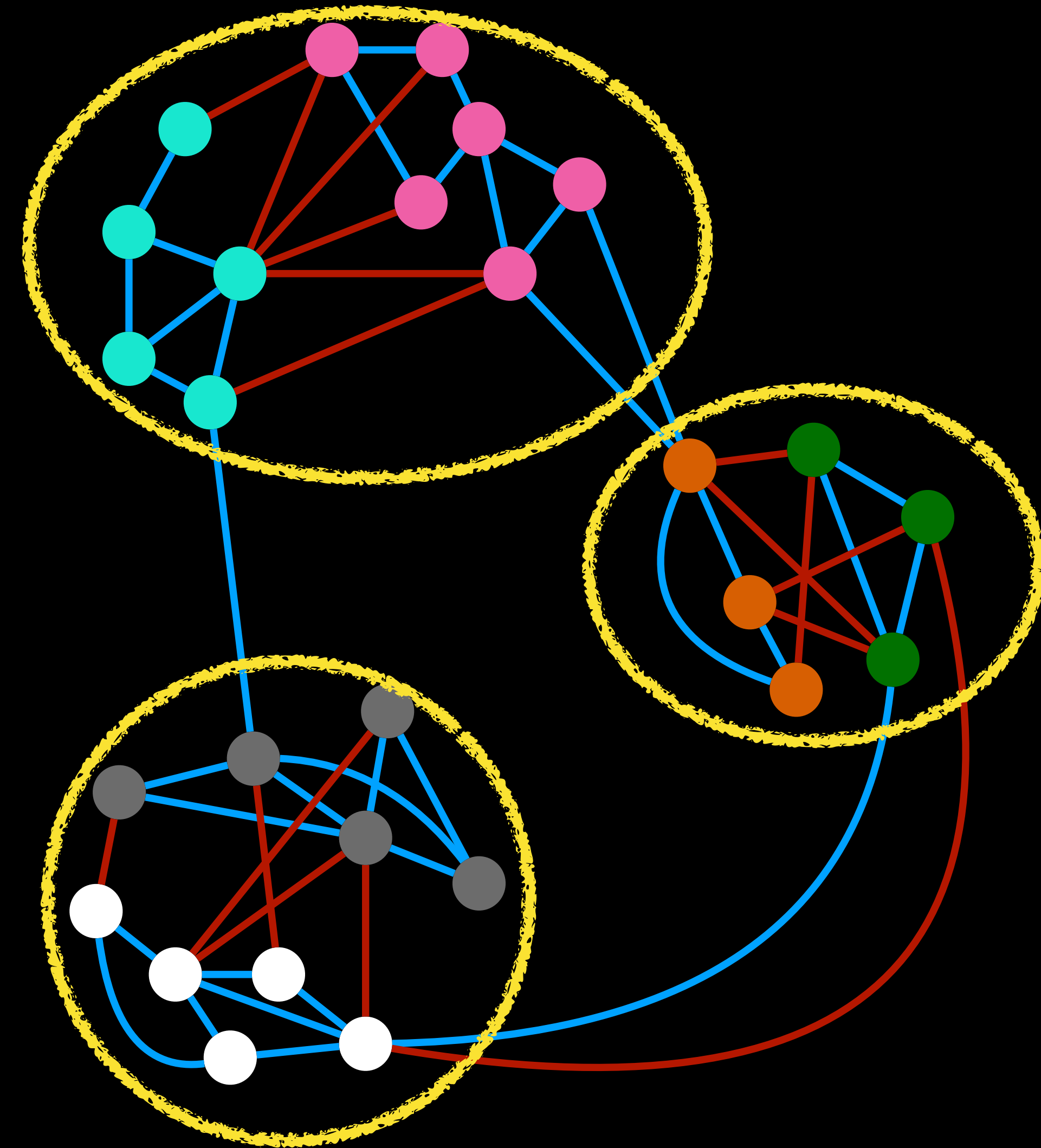


Also Highly Practical



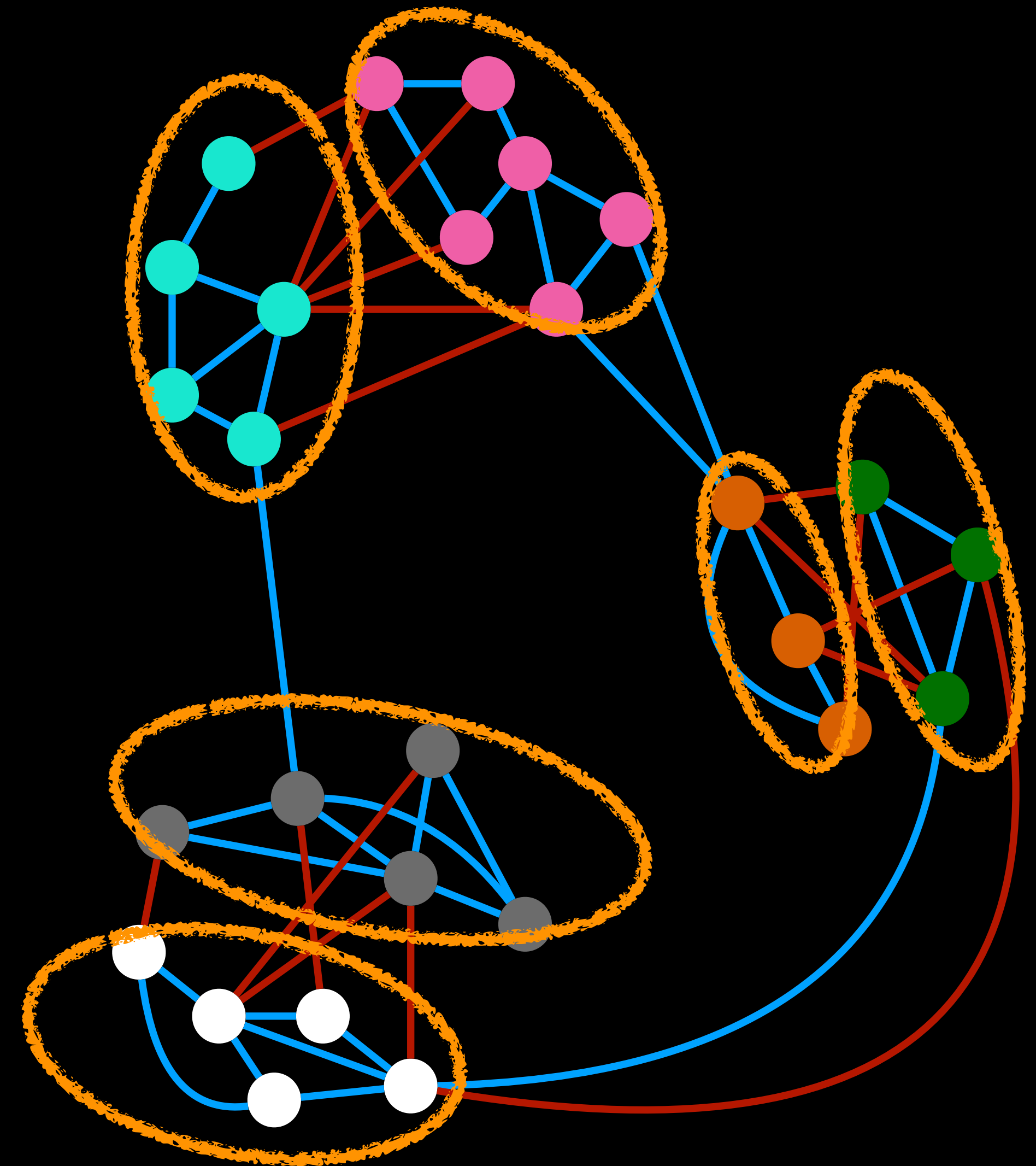
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- In practice, we can identify the conflict groups!



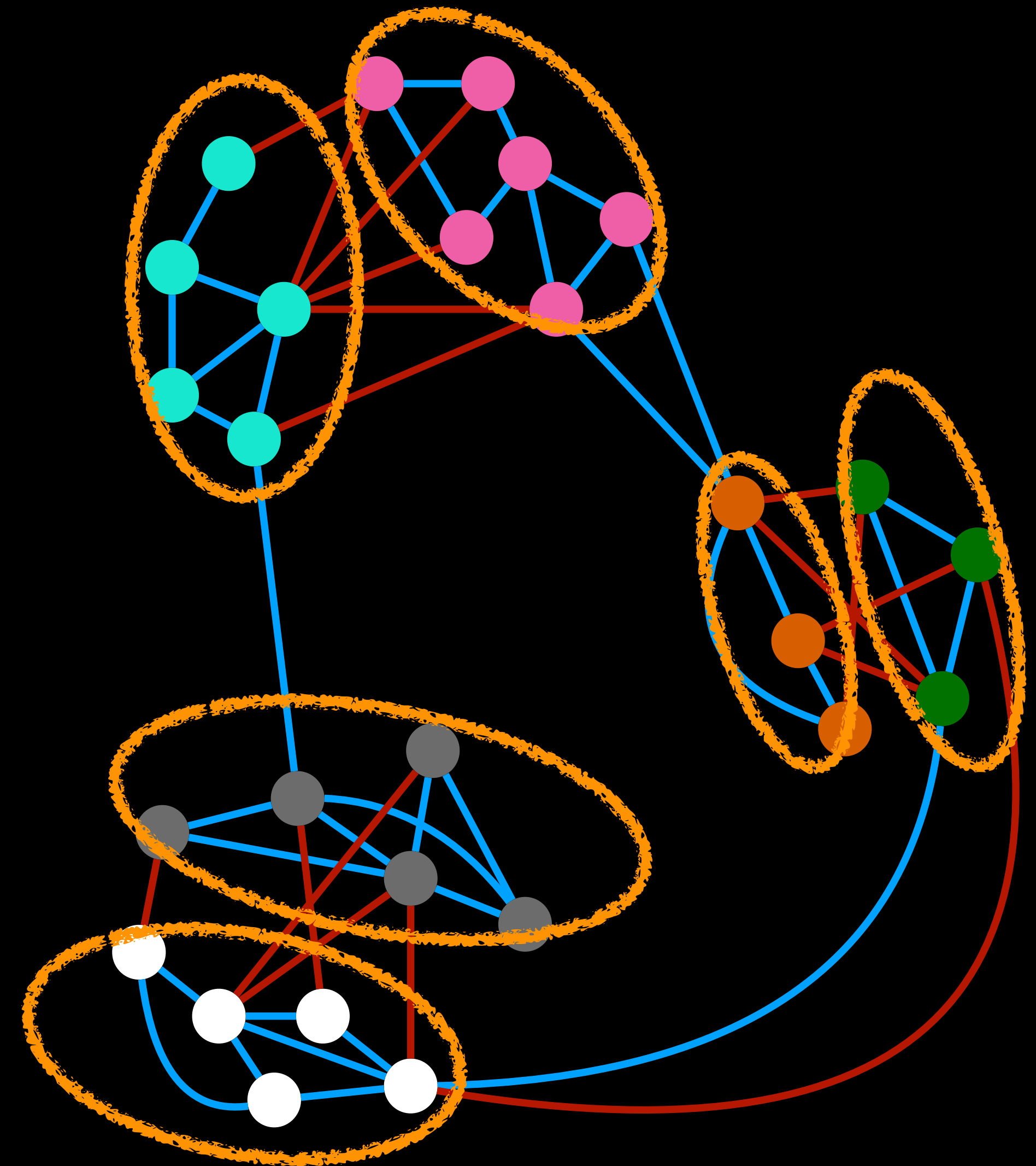
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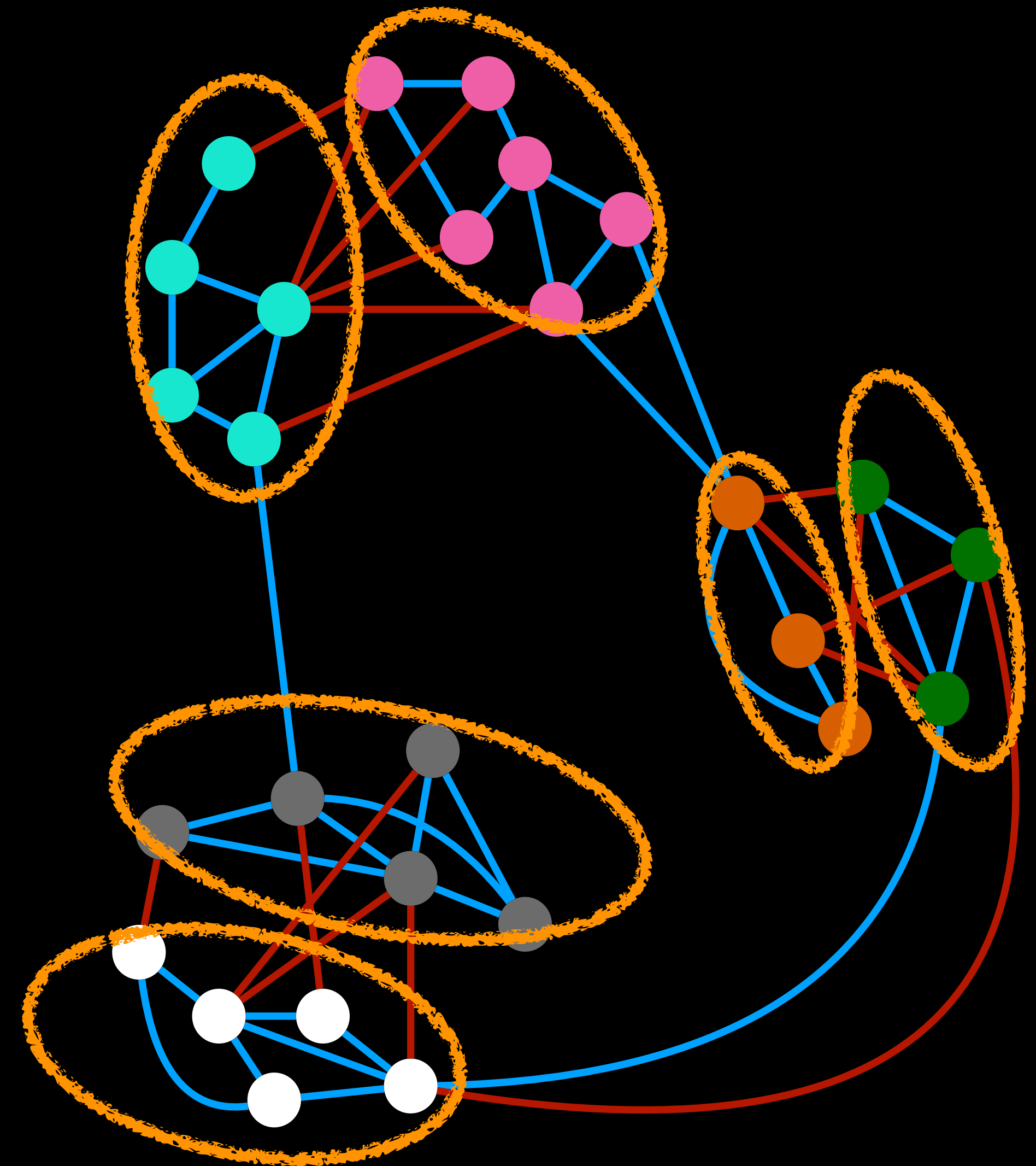
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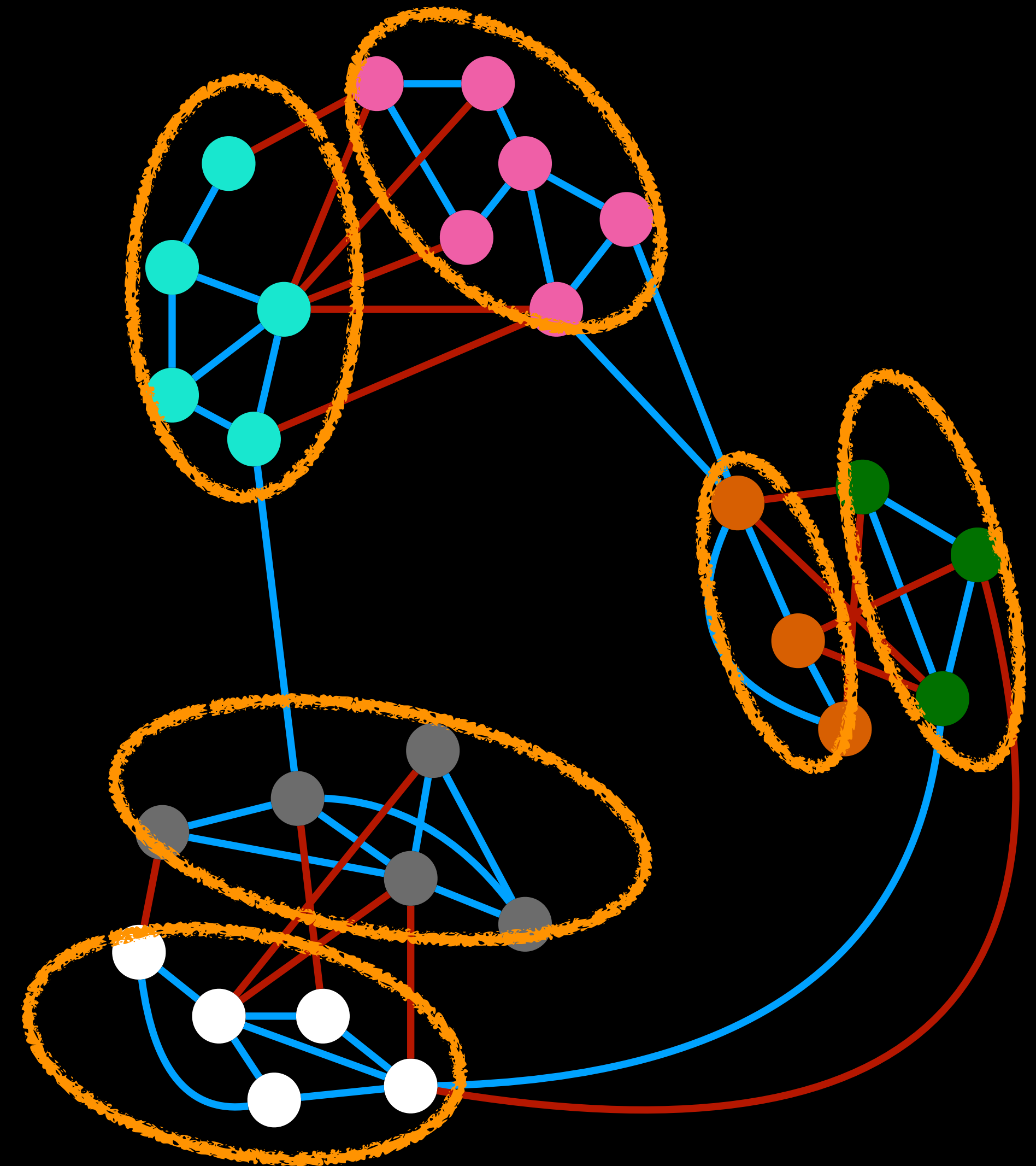
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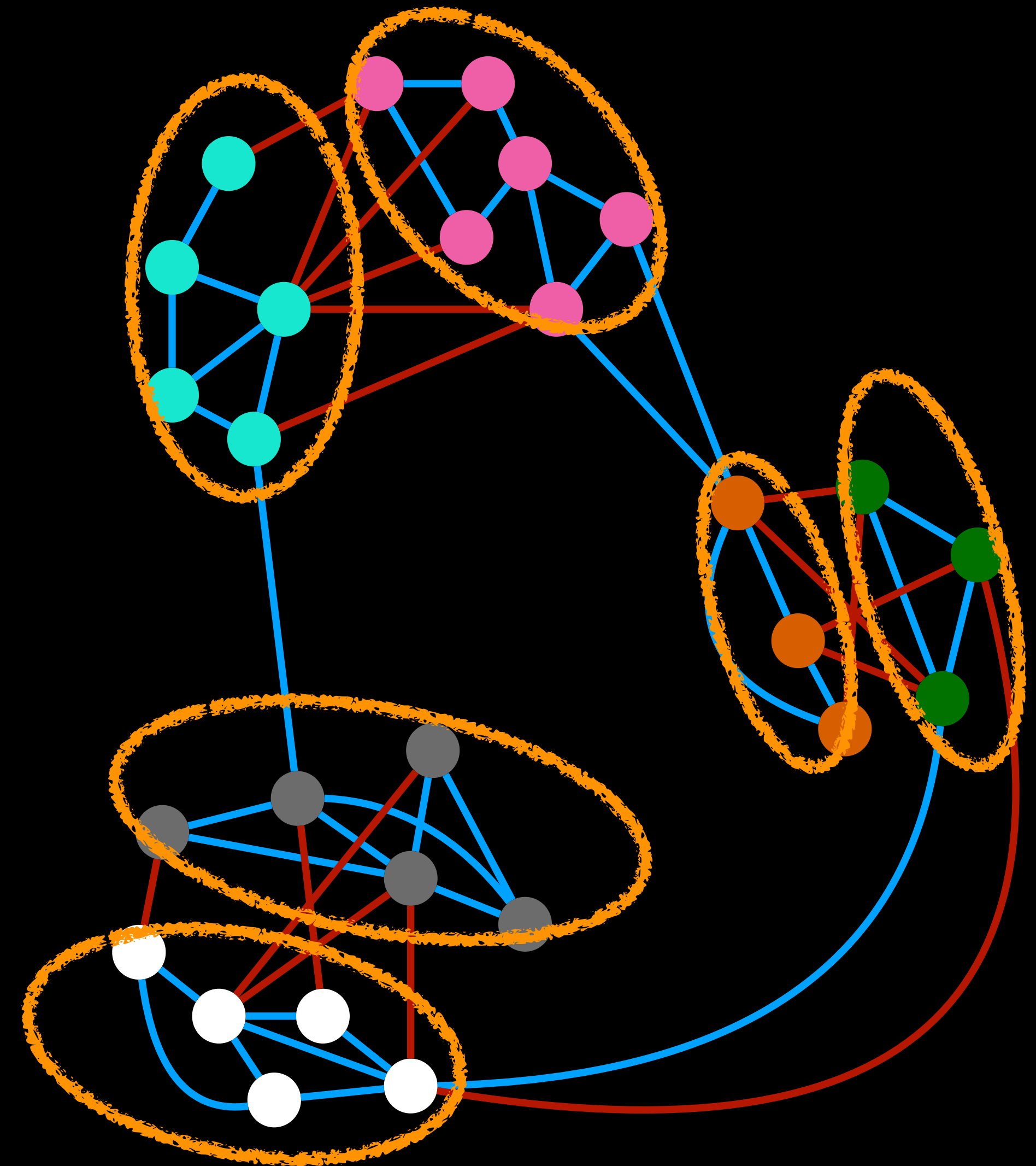
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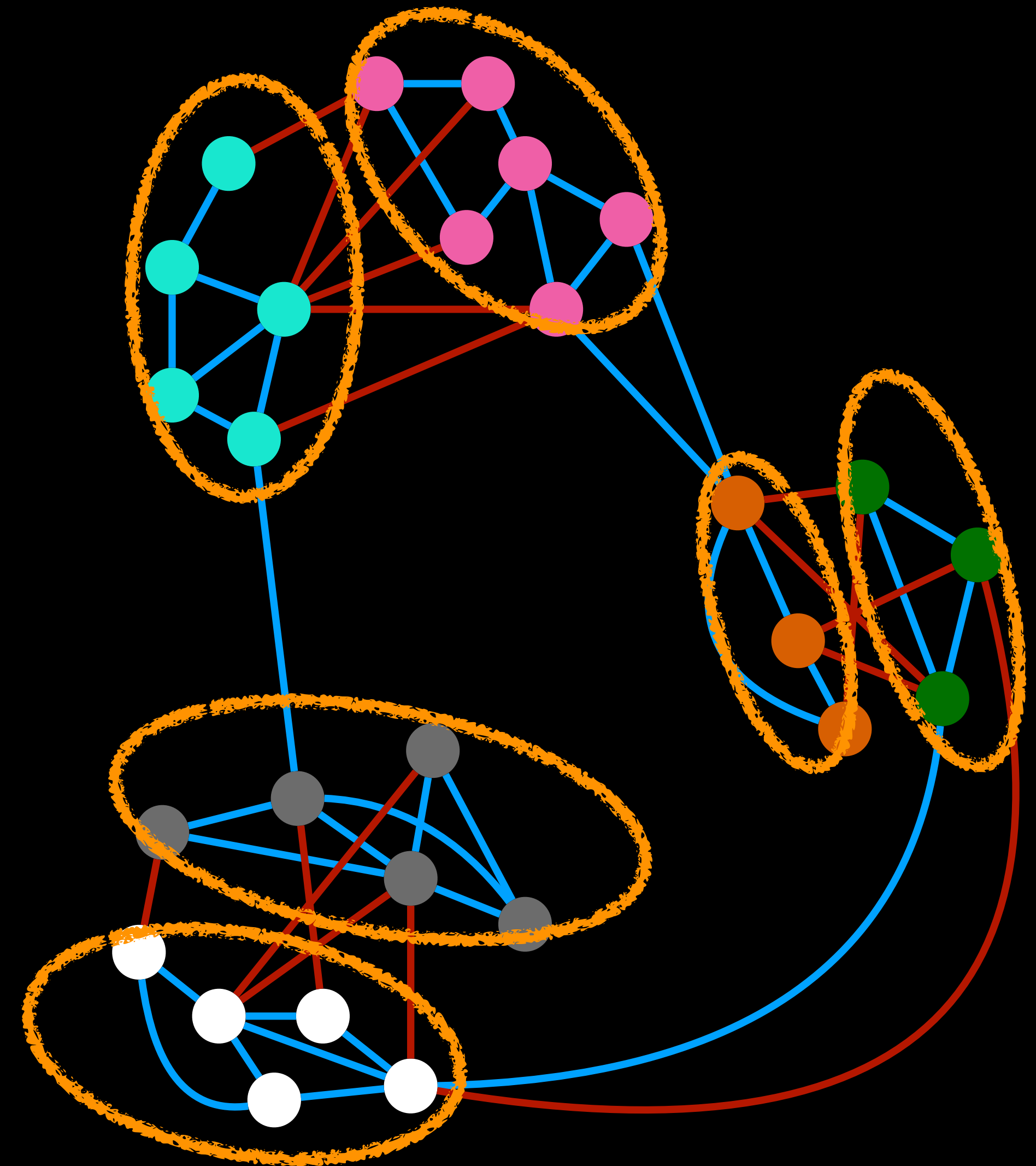
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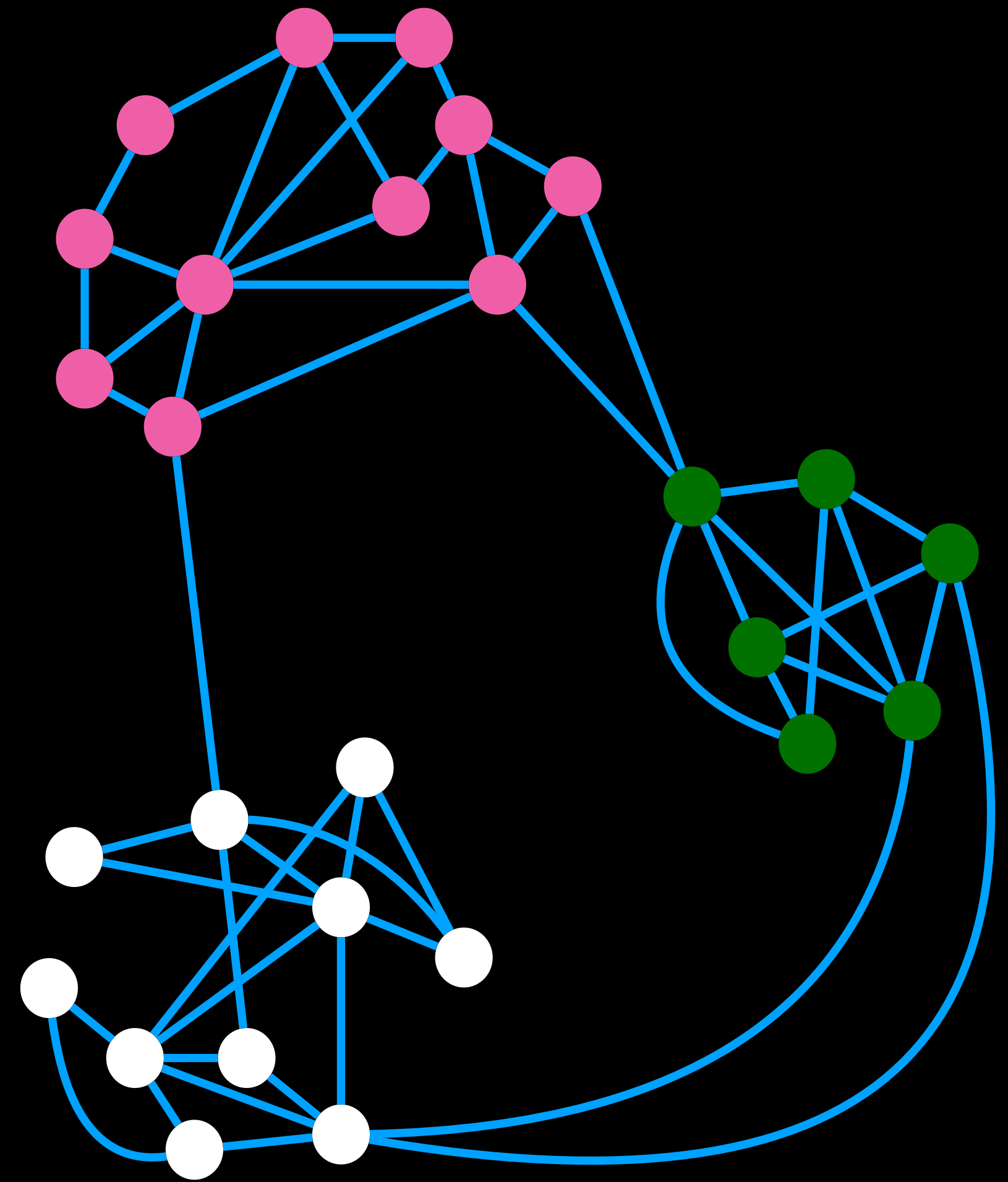
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 - ➔ Easy to parallelize
- We provide a **new signed graph dataset** with large ground-truth communities

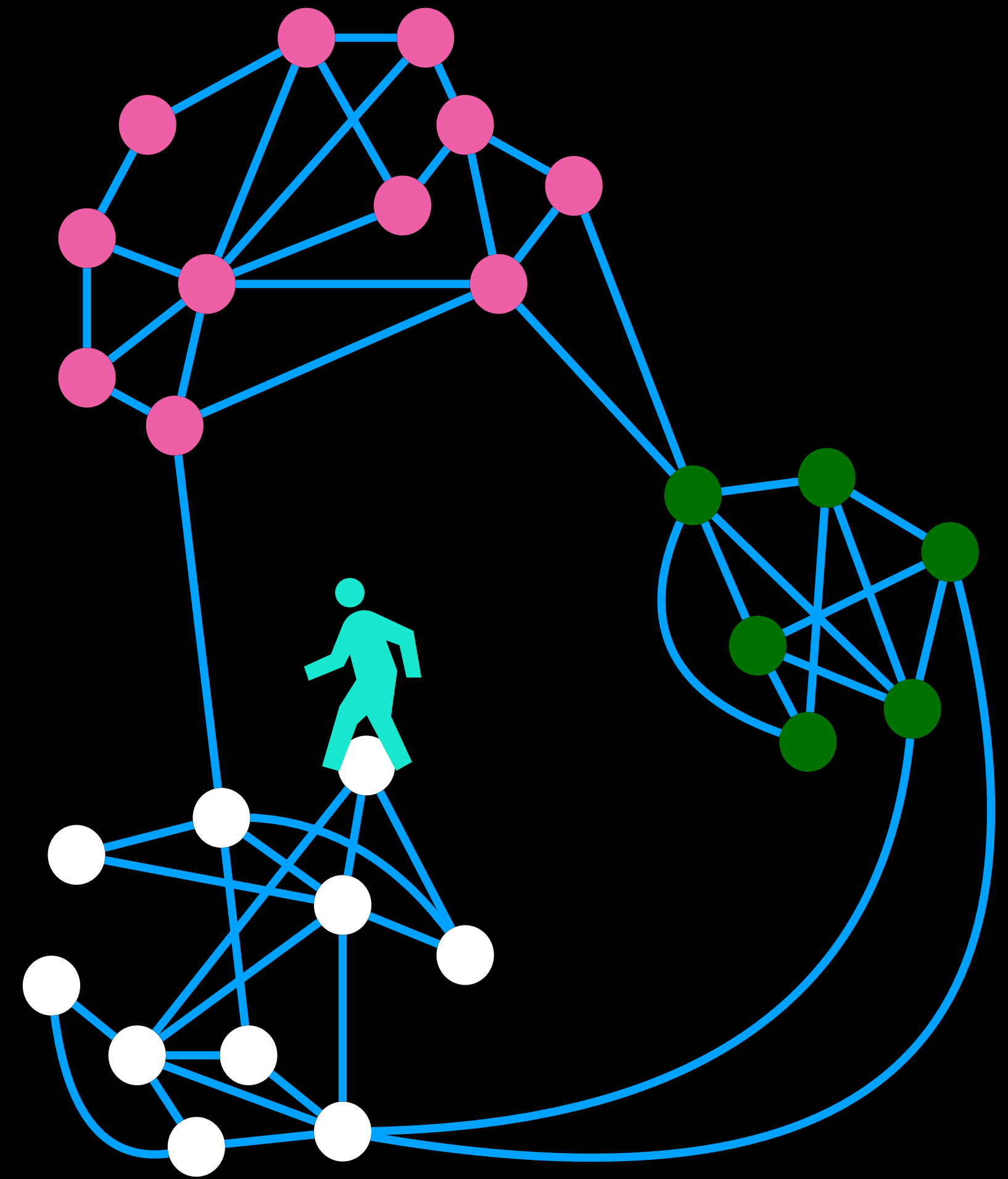


How Does the Oracle Work?

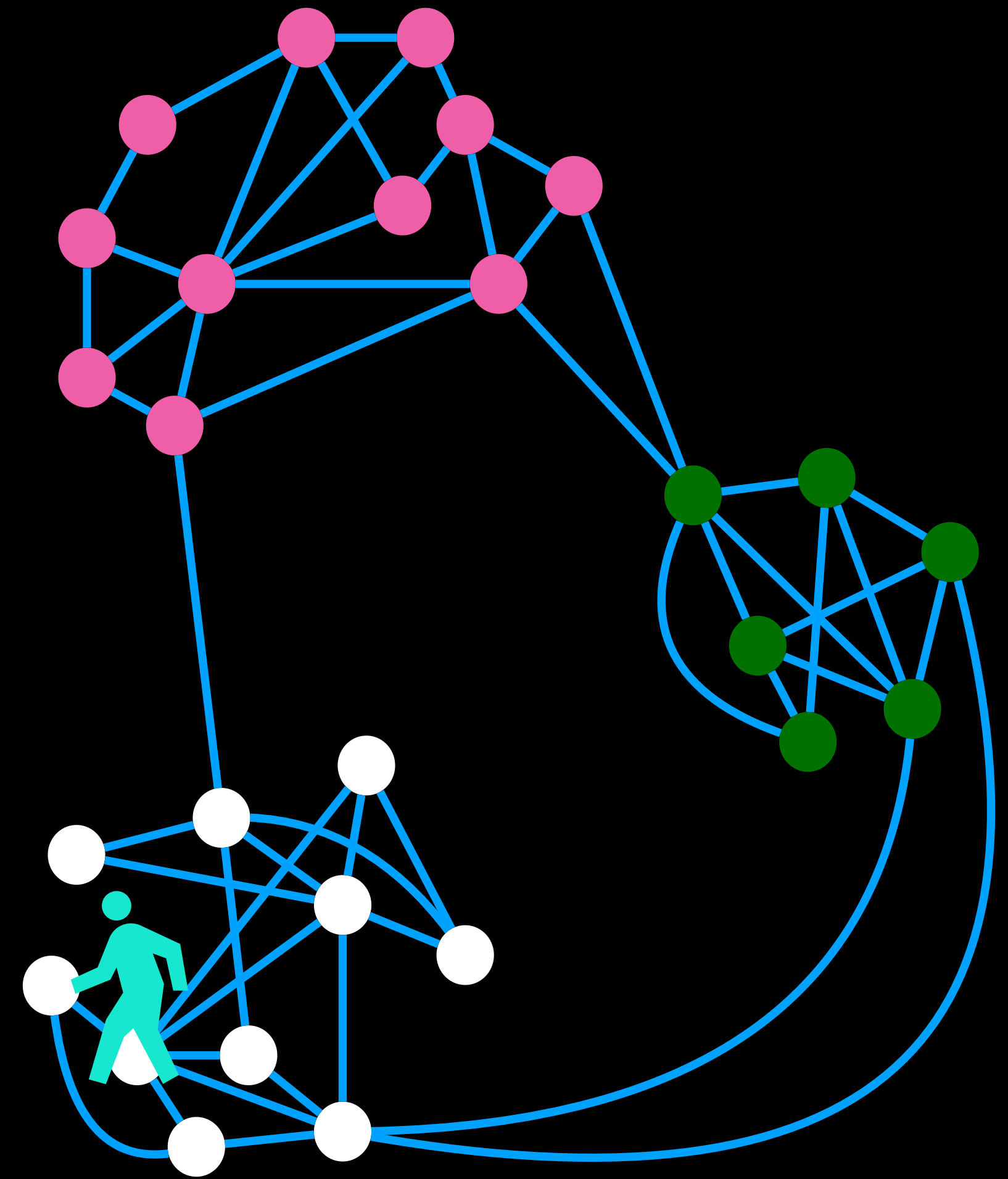
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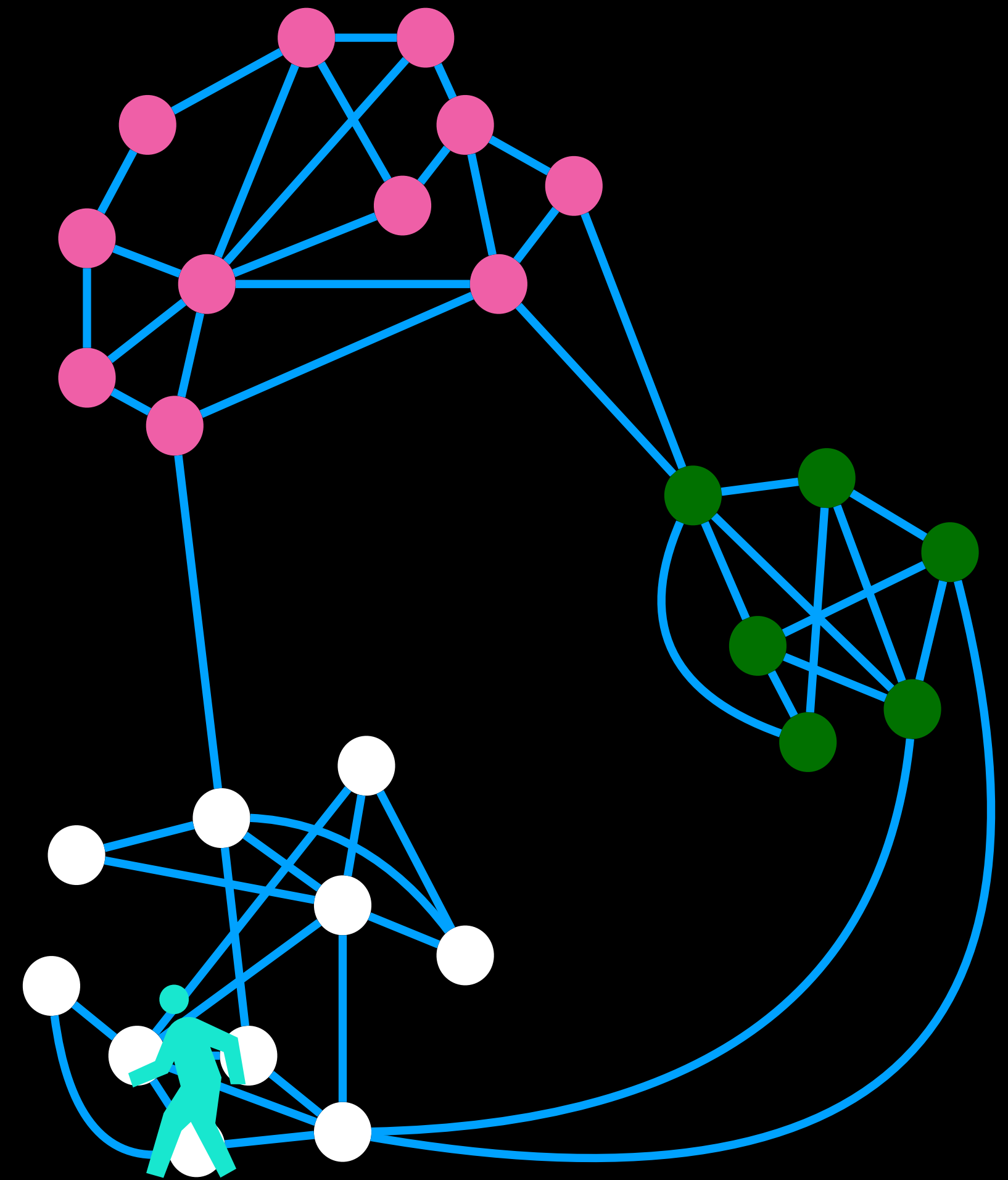
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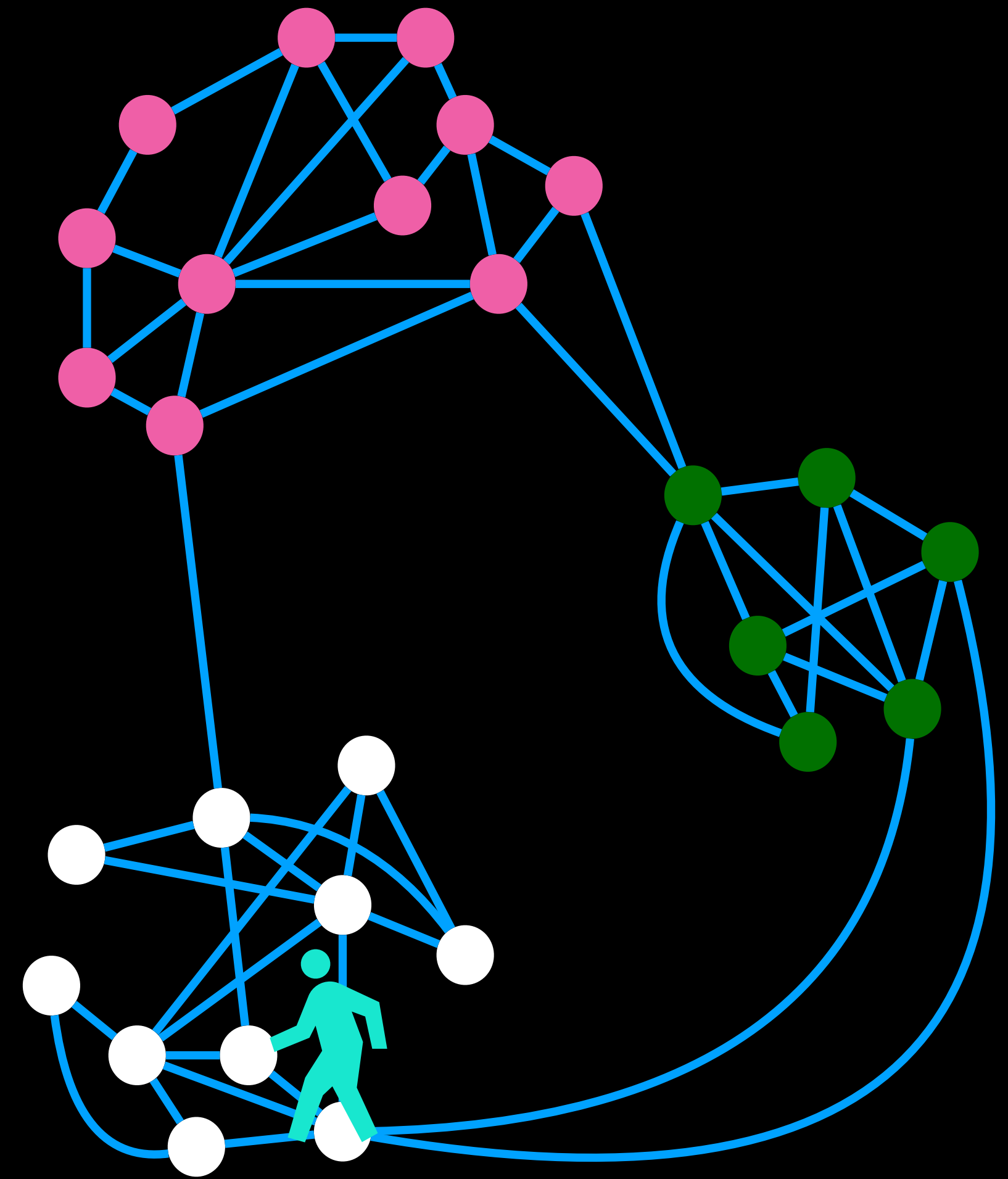
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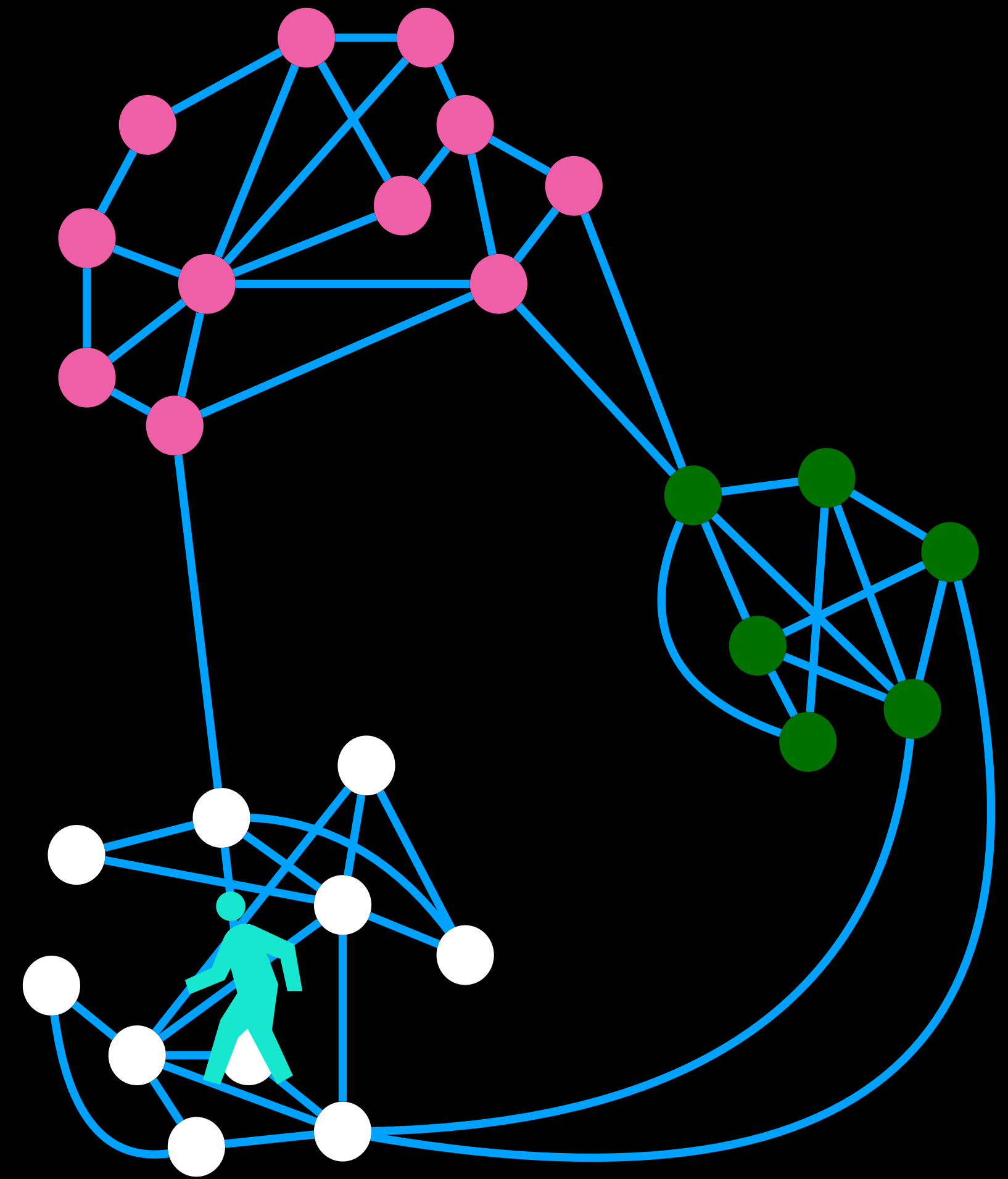
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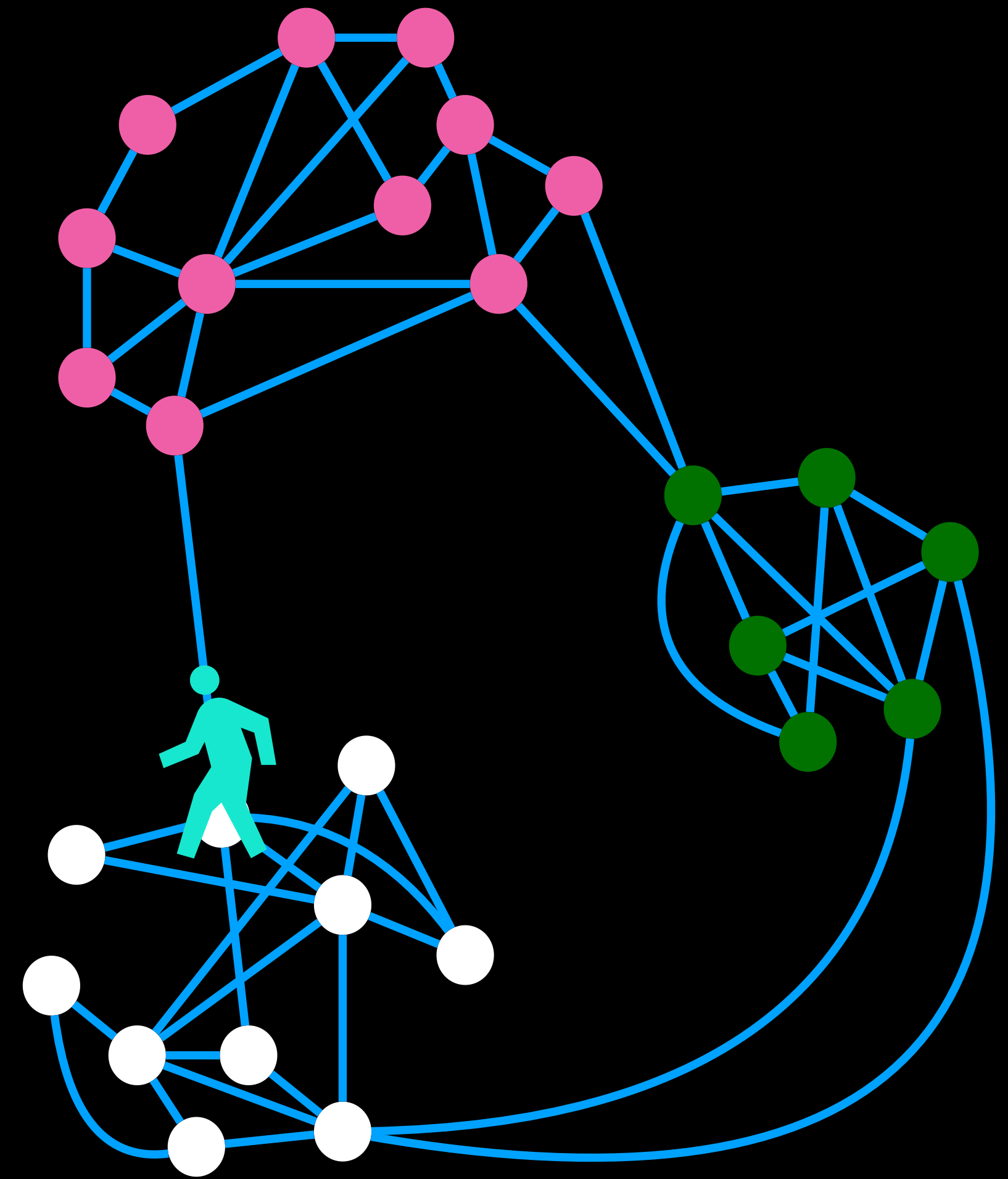
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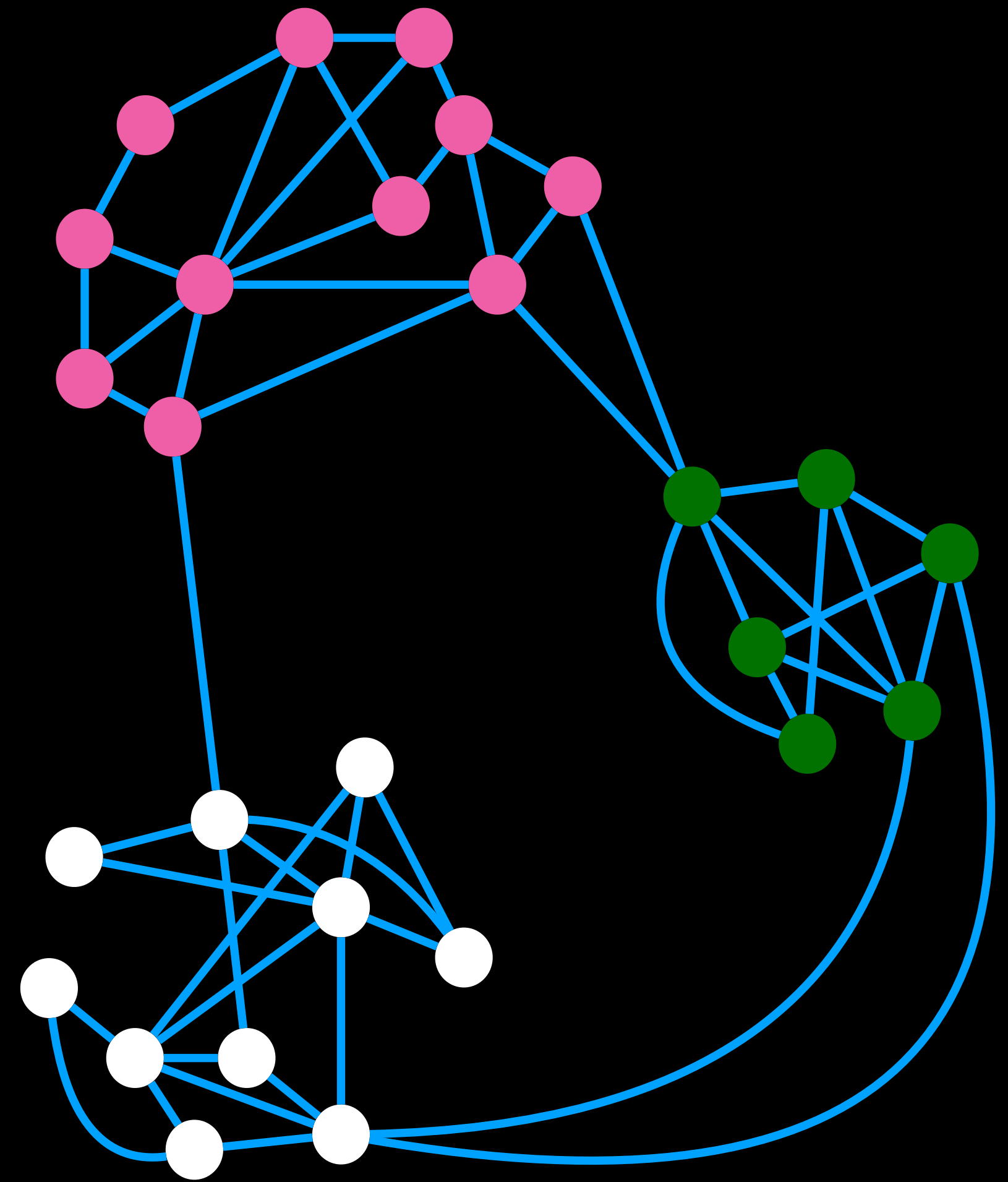
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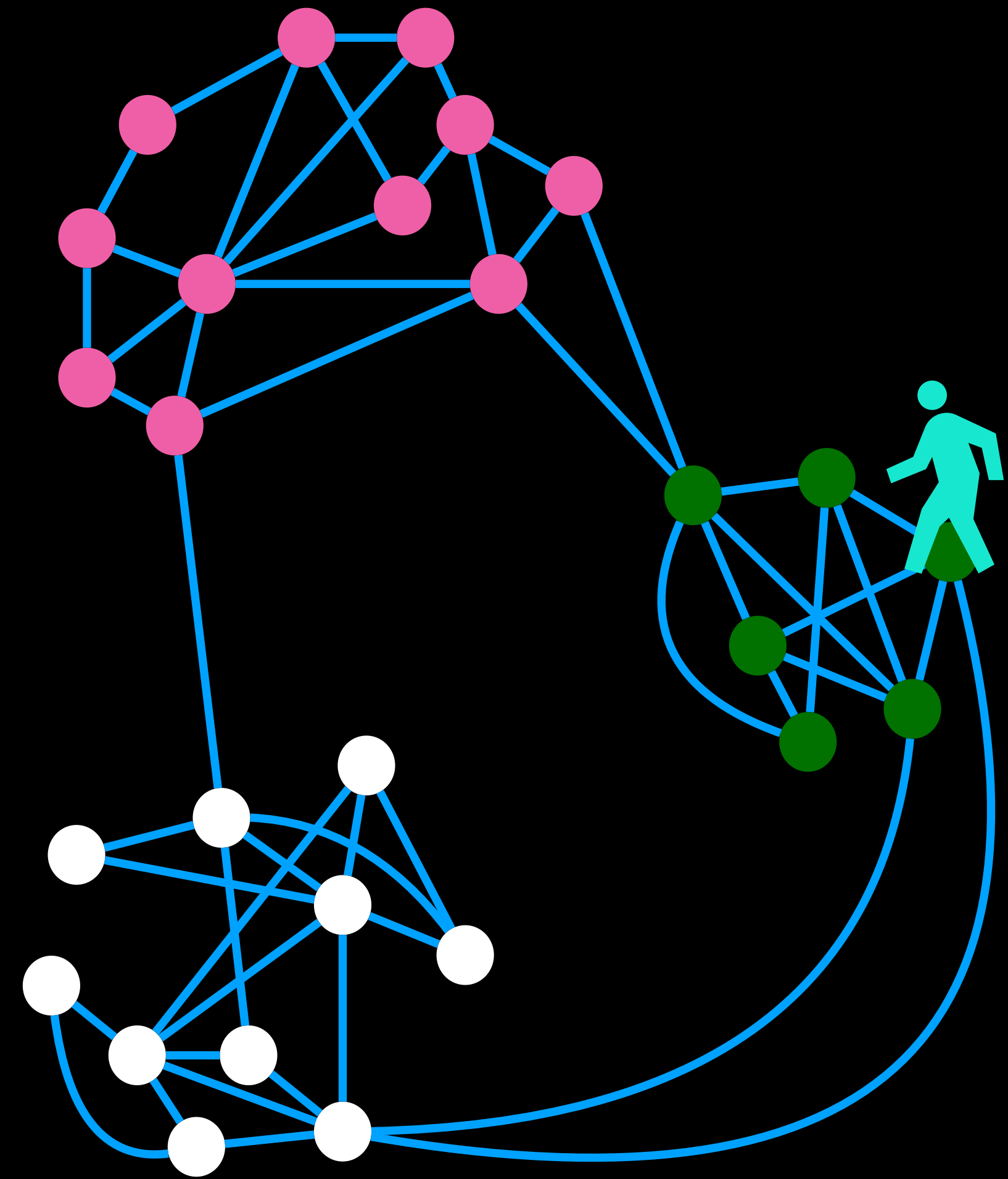
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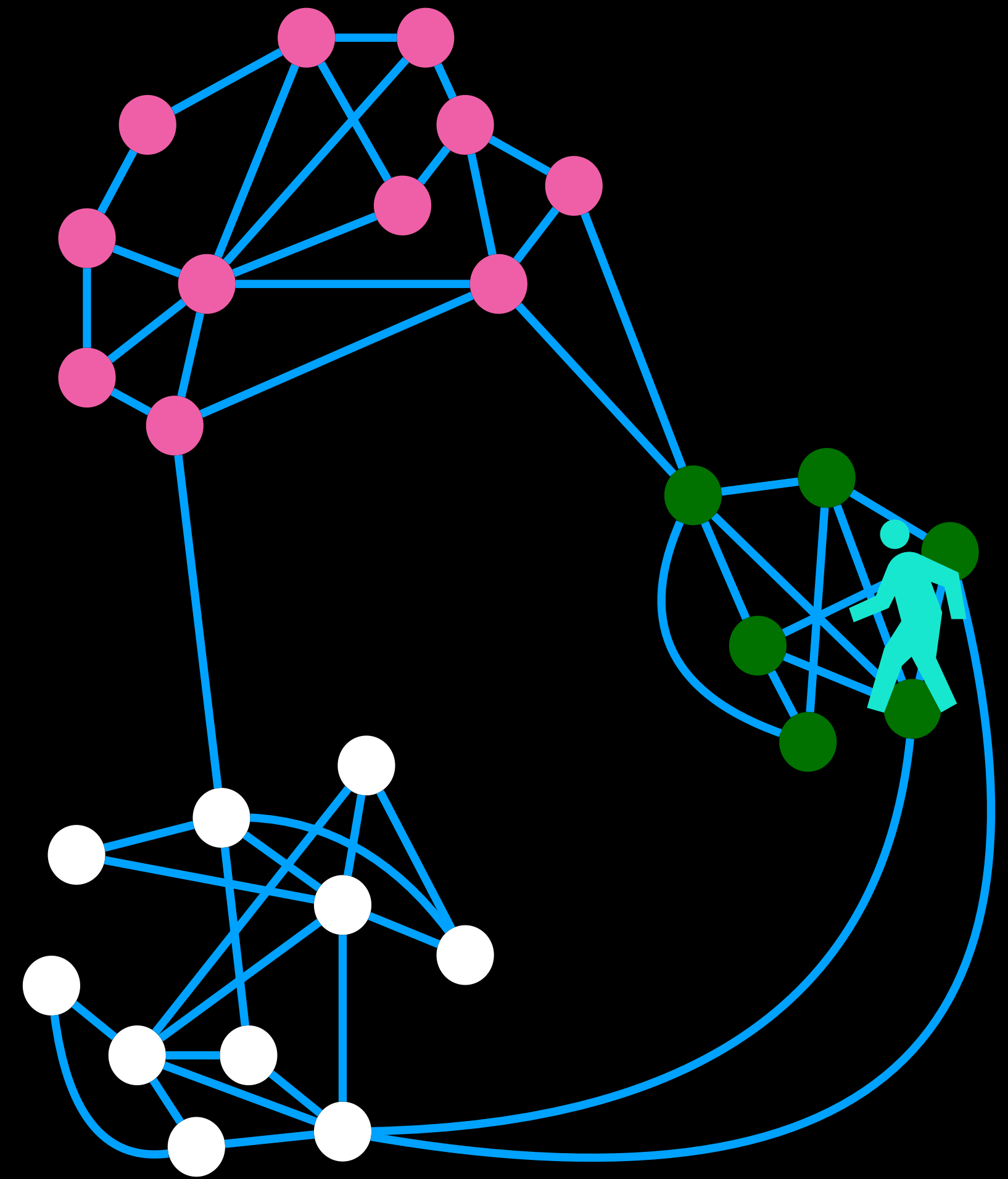
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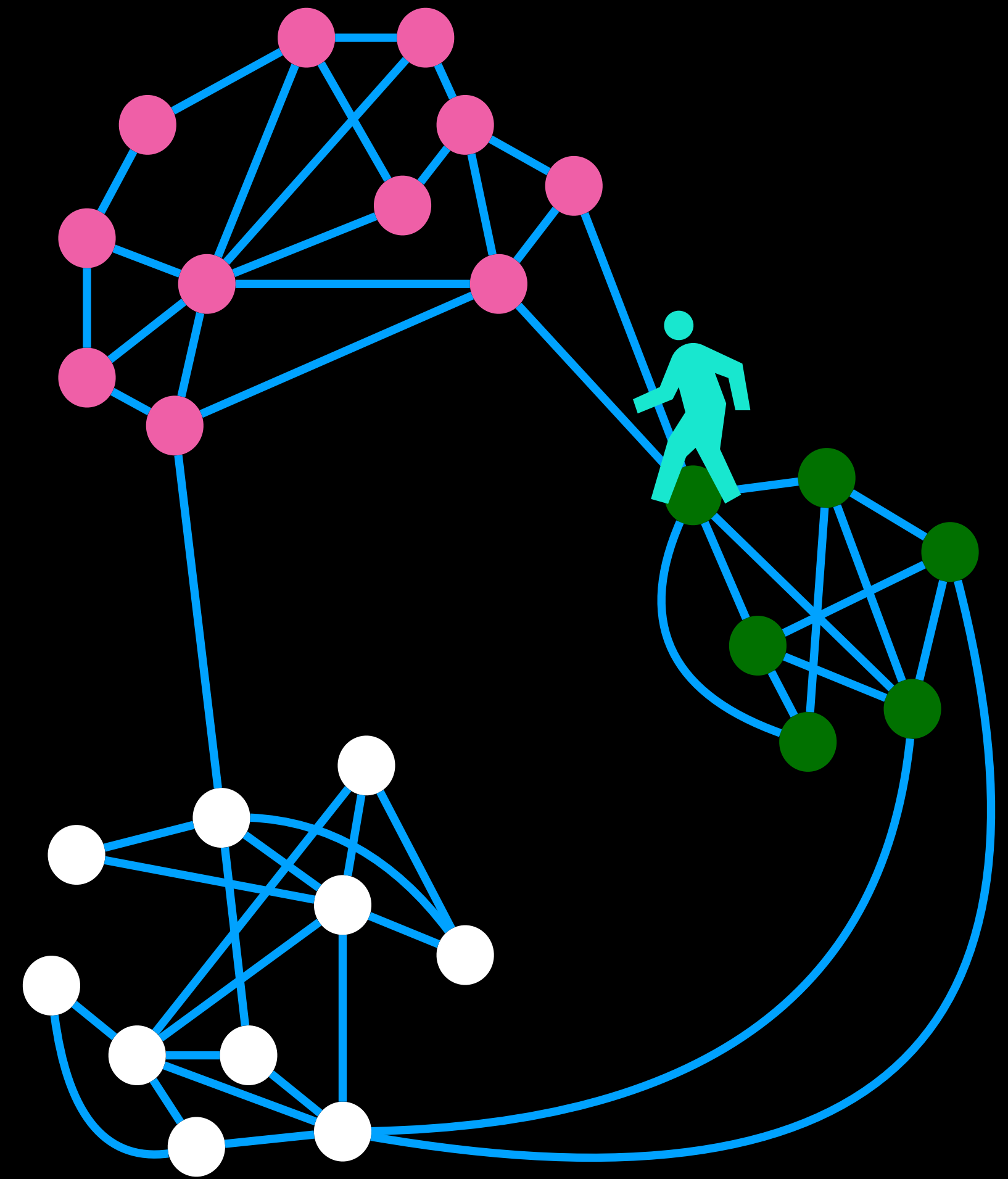
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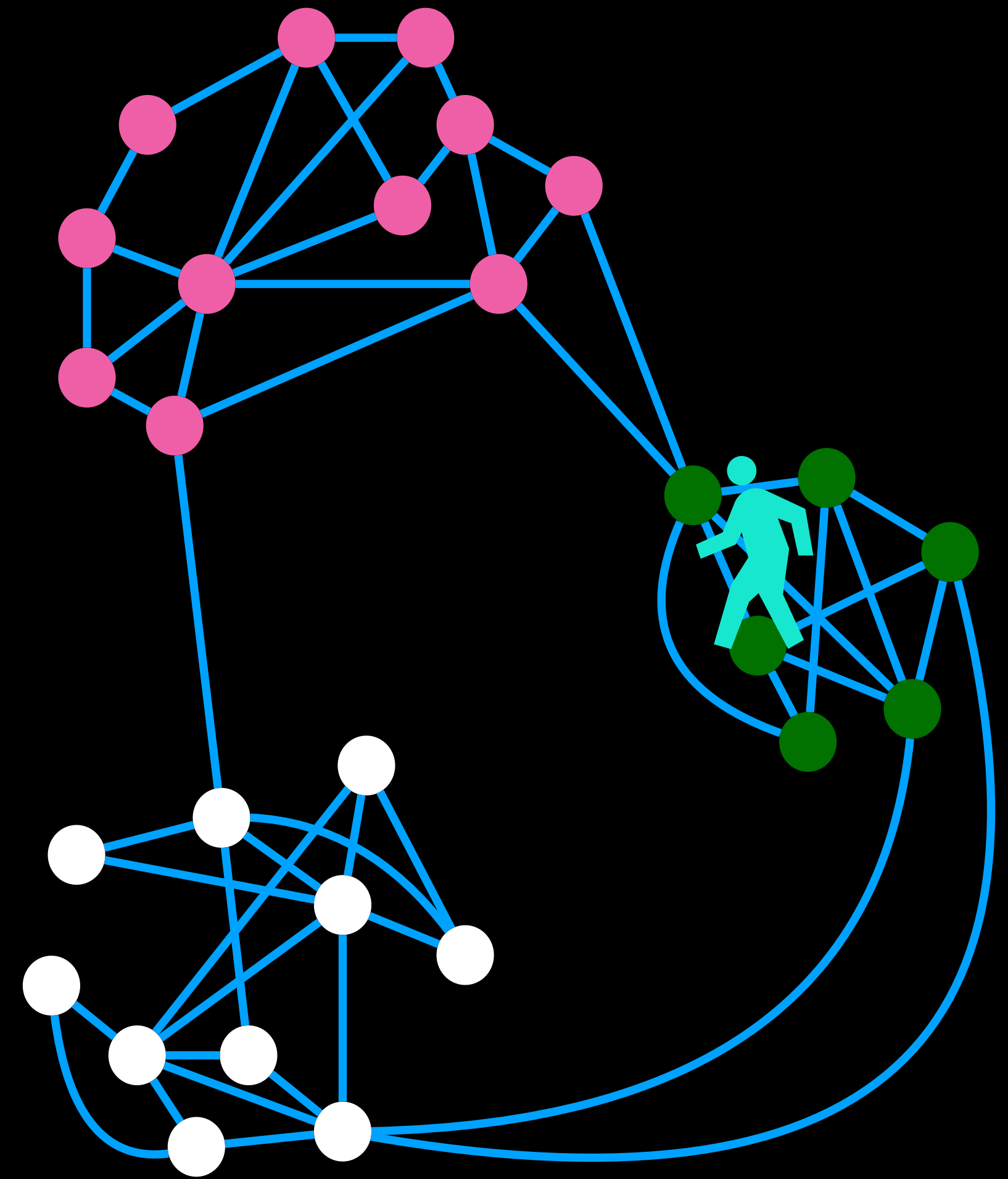
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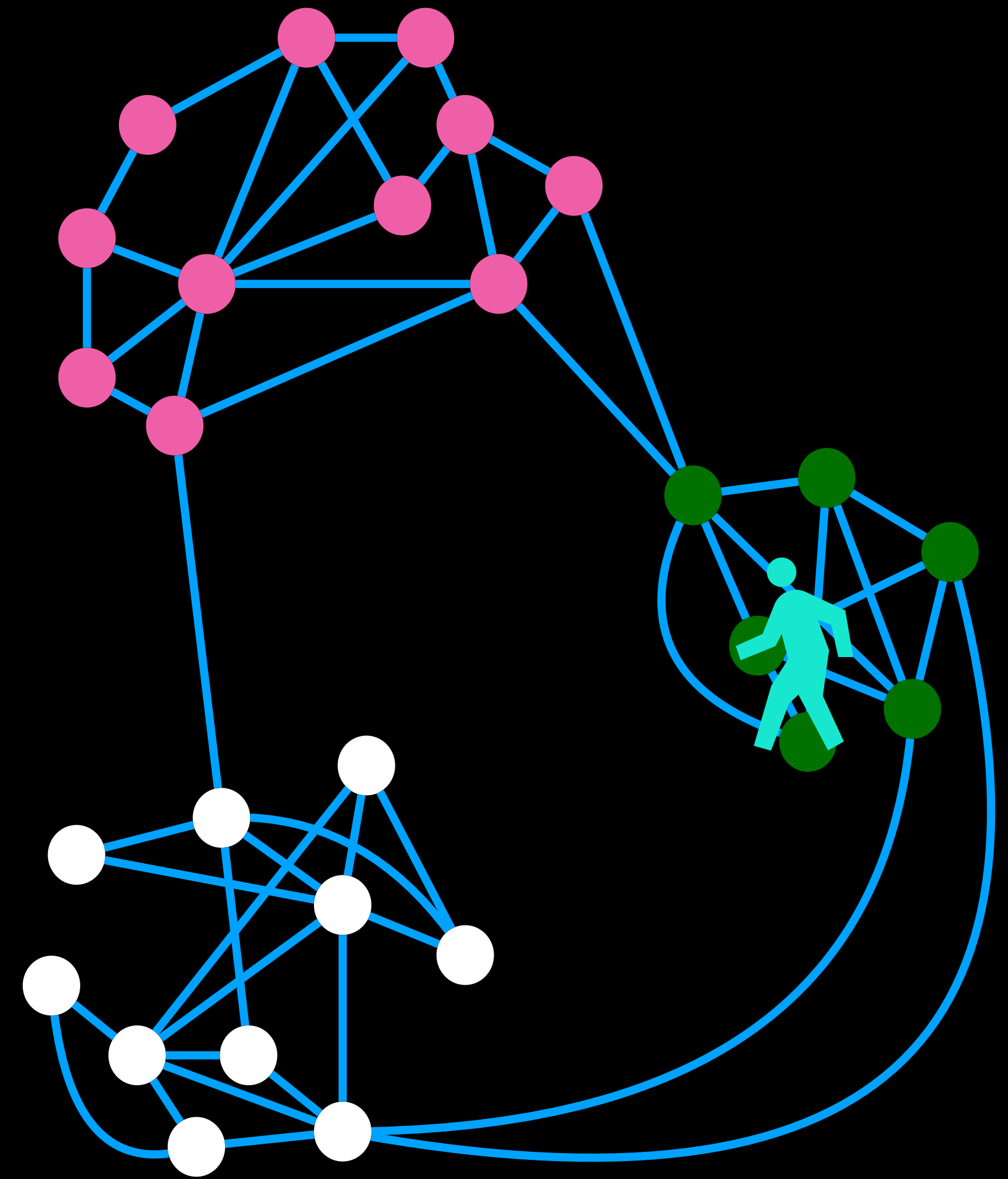
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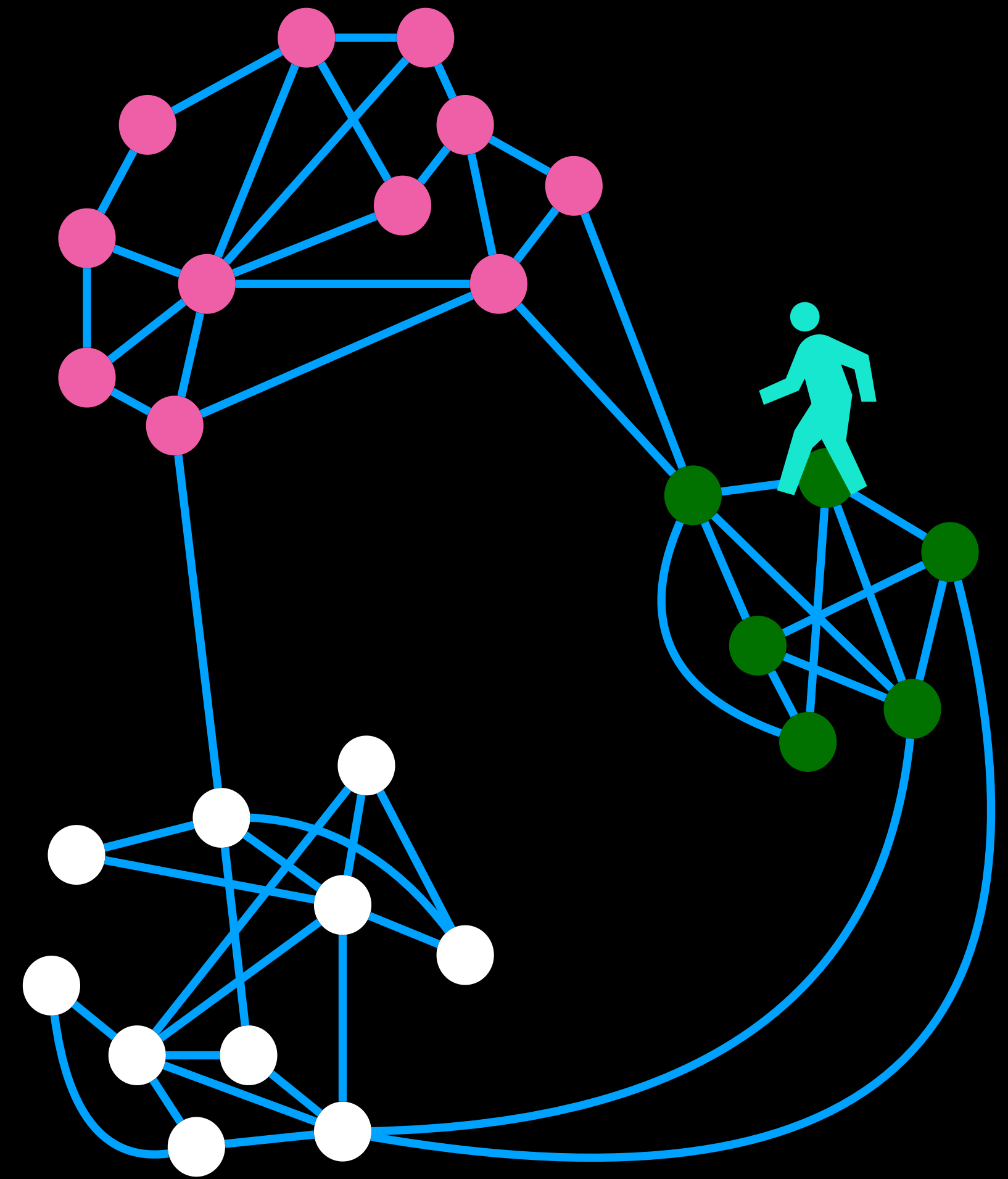
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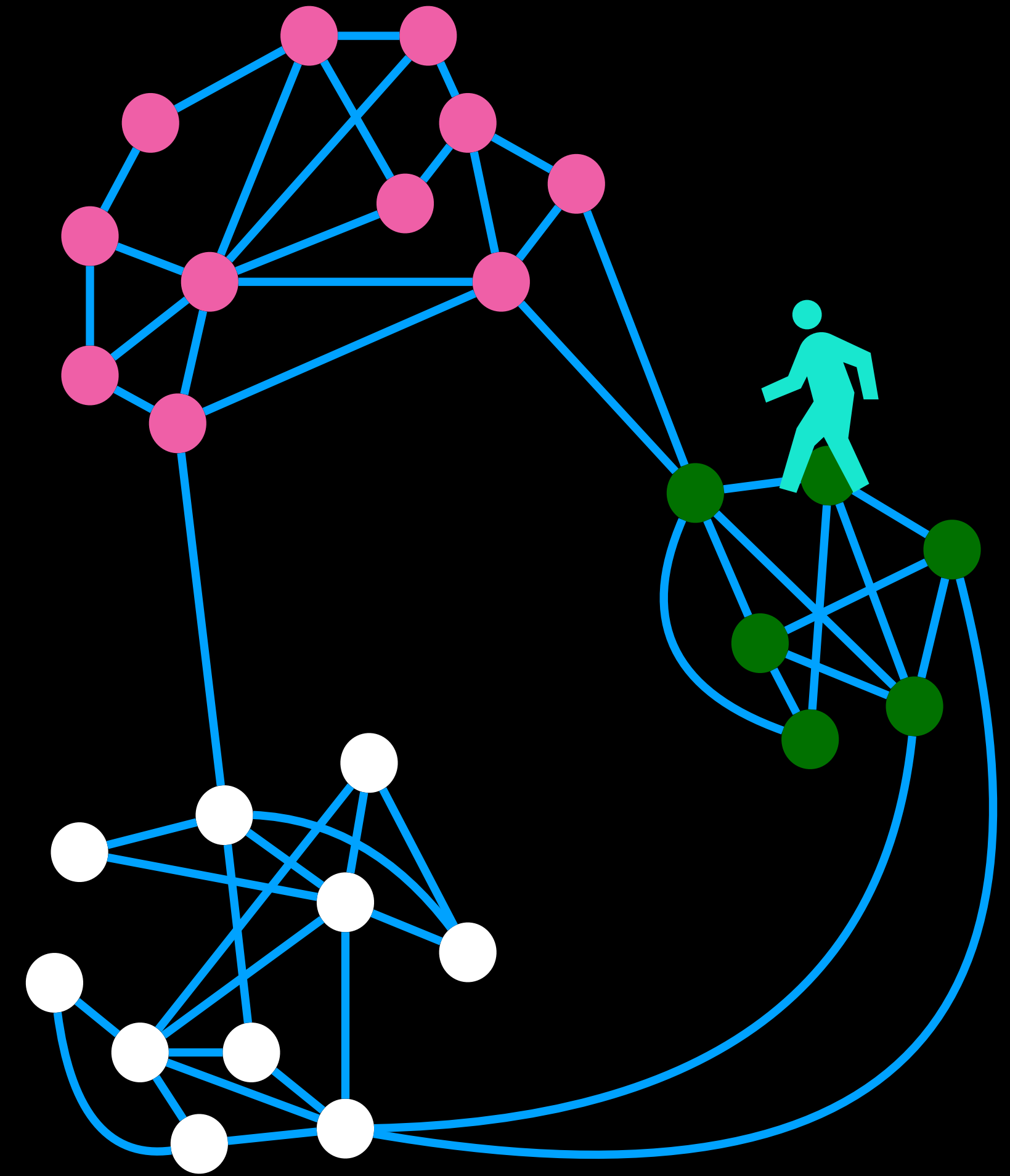
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Computing communities in large networks using random walks

[P Pons](#), [M Latapy](#) - [International symposium on computer and information ...](#), 2005 - Springer

Dense subgraphs of sparse graphs (communities), which appear in most real-world complex networks, play an important role in many contexts. Computing them however is generally ...

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Unsigned Graphs

Computing communities in large networks using random walks

[P Pons](#), [M Latapy](#) - [International symposium on computer and information ...](#), 2005 - Springer

Dense subgraphs of sparse graphs (communities), which appear in most real-world complex networks, play an important role in many contexts. Computing them however is generally ...

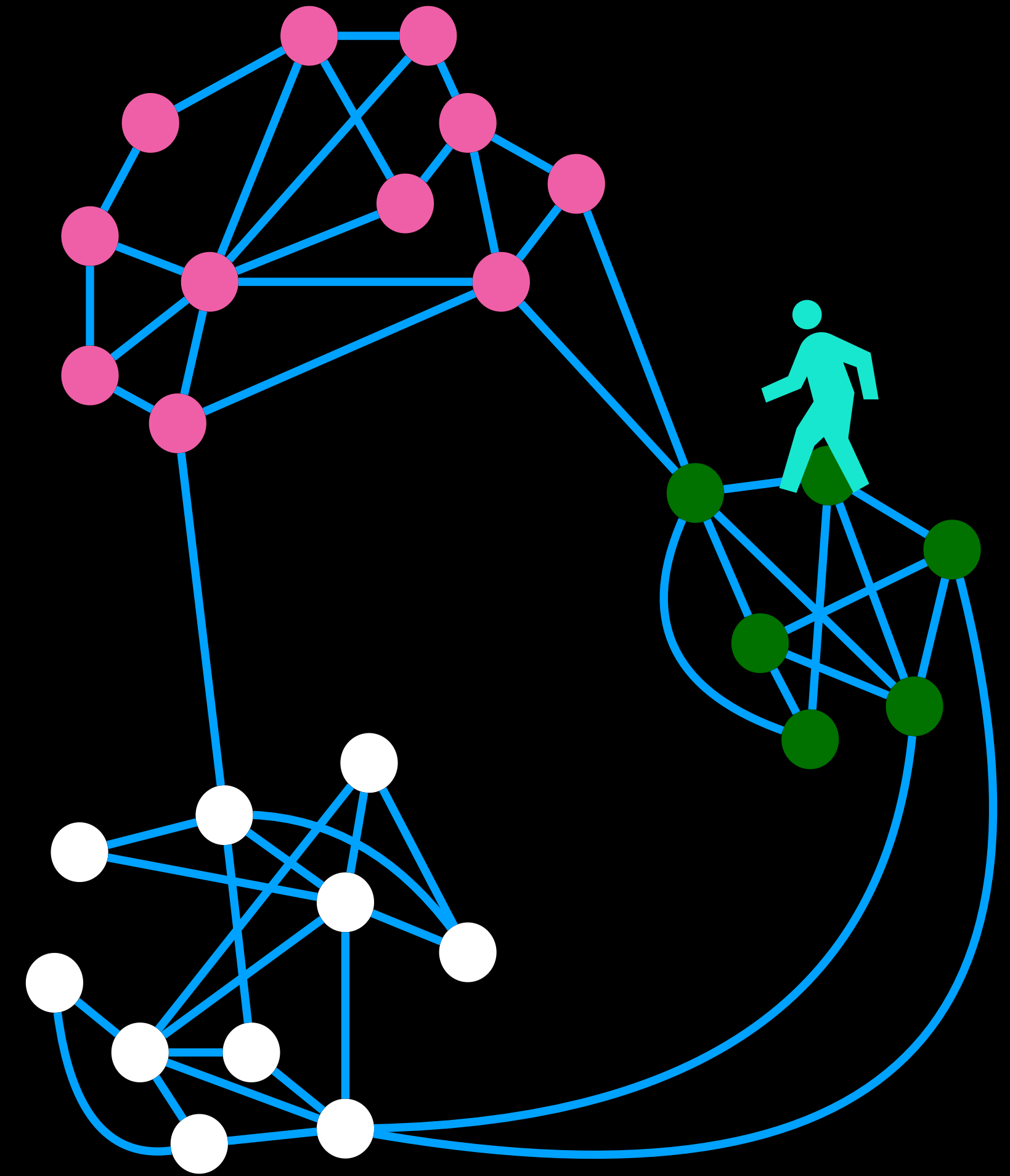
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Testing cluster structure of graphs

[A Czumaj](#), [P Peng](#), [C Sohler](#) - [Proceedings of the forty-seventh annual ...](#), 2015 - dl.acm.org

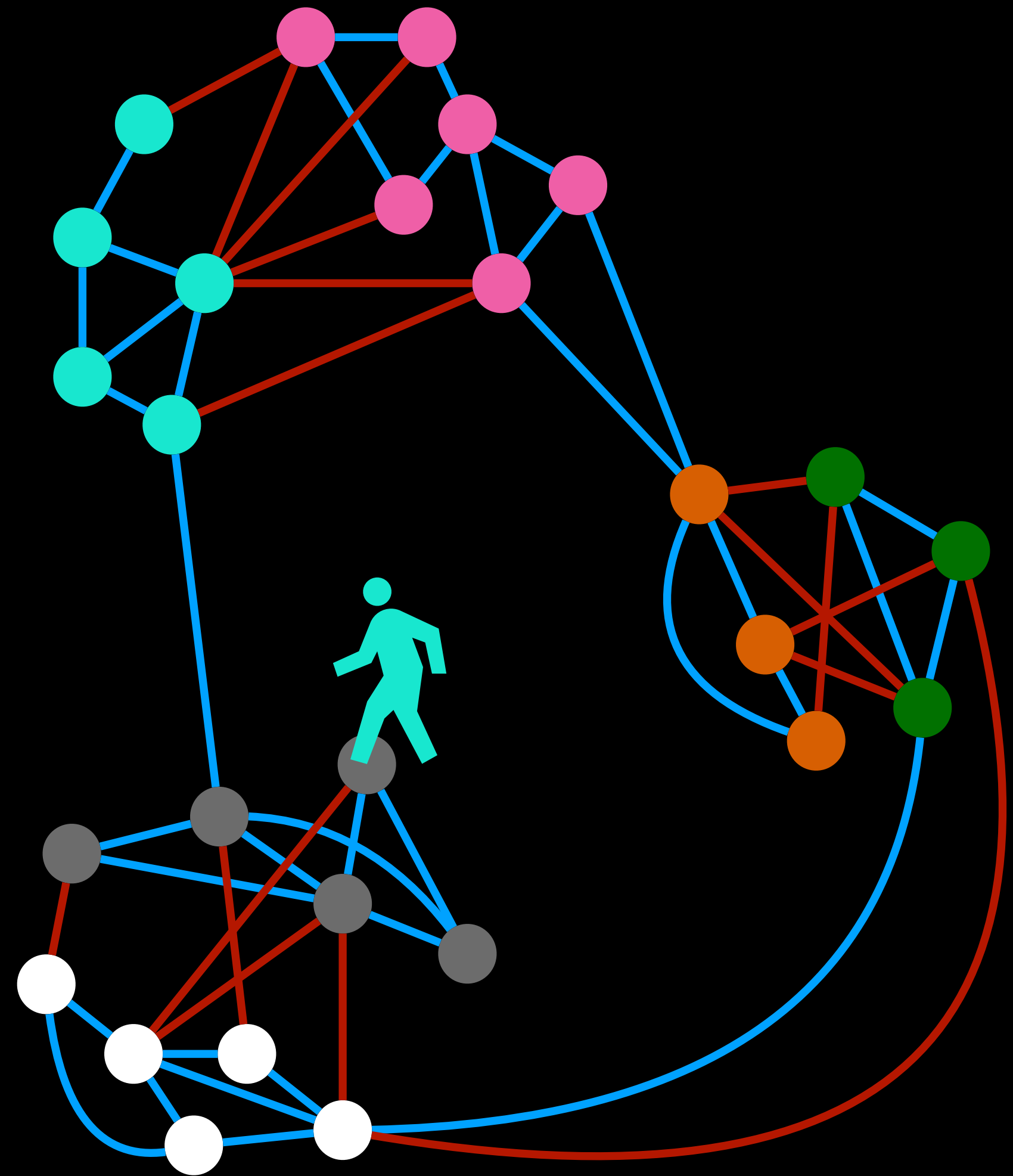
We study the problem of recognizing the cluster structure of a graph in the framework of property testing in the bounded degree model. Given a parameter ϵ , a d -bounded degree graph is defined to be (k, ϕ) -clusterable, if it can be partitioned into no more than k parts, such that the (inner) conductance of the induced subgraph on each part is at least ϕ and the (outer) conductance of each part is at most cd , $k \leq 4/\phi^2$, where cd , k depends only on d , k . Our main result is a sublinear algorithm with the running time $\sim O(\sqrt{n} \cdot \text{poly}(\phi, k, 1/\epsilon))$ that ...

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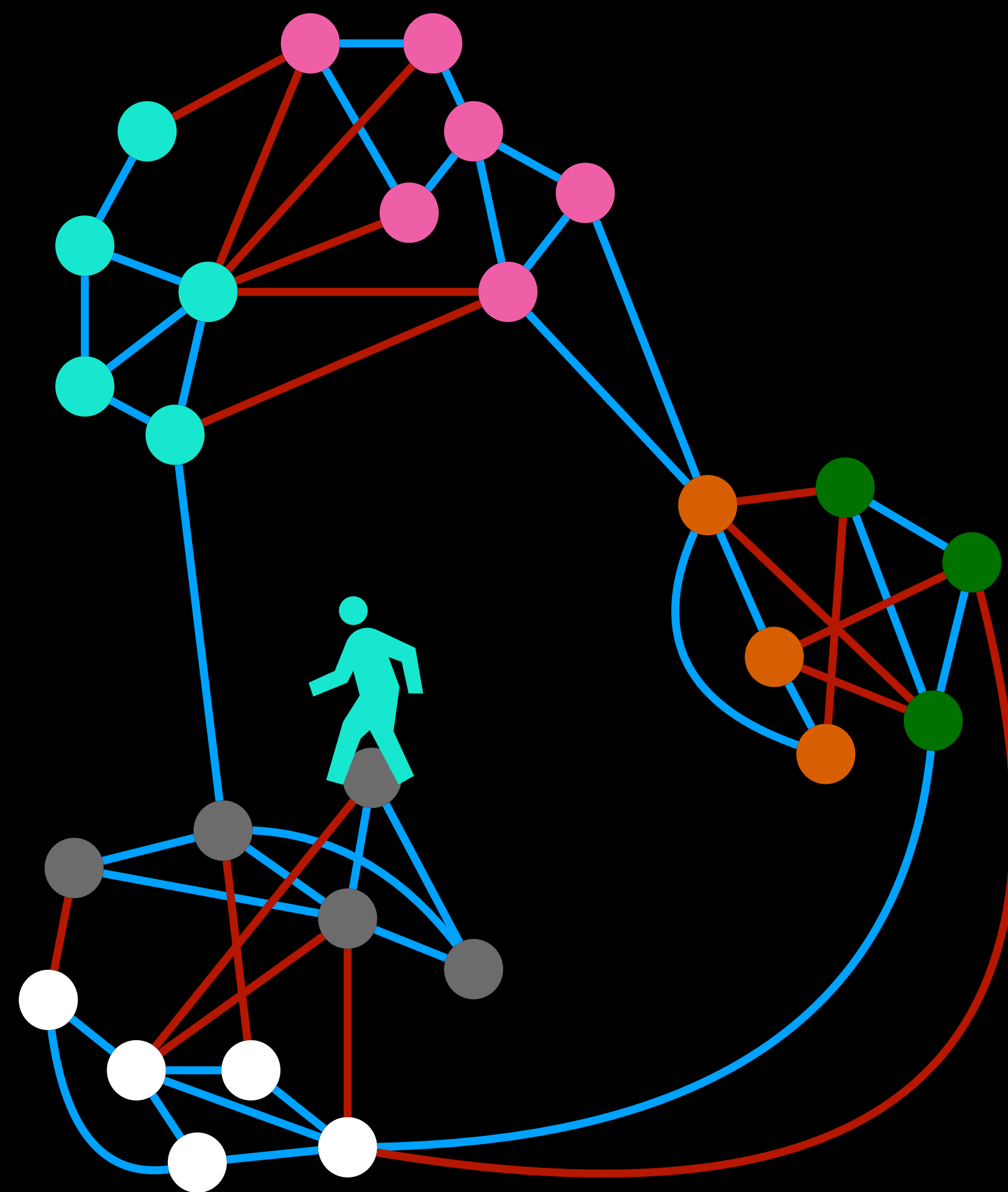


Signed Graphs

What changes when we have signs?

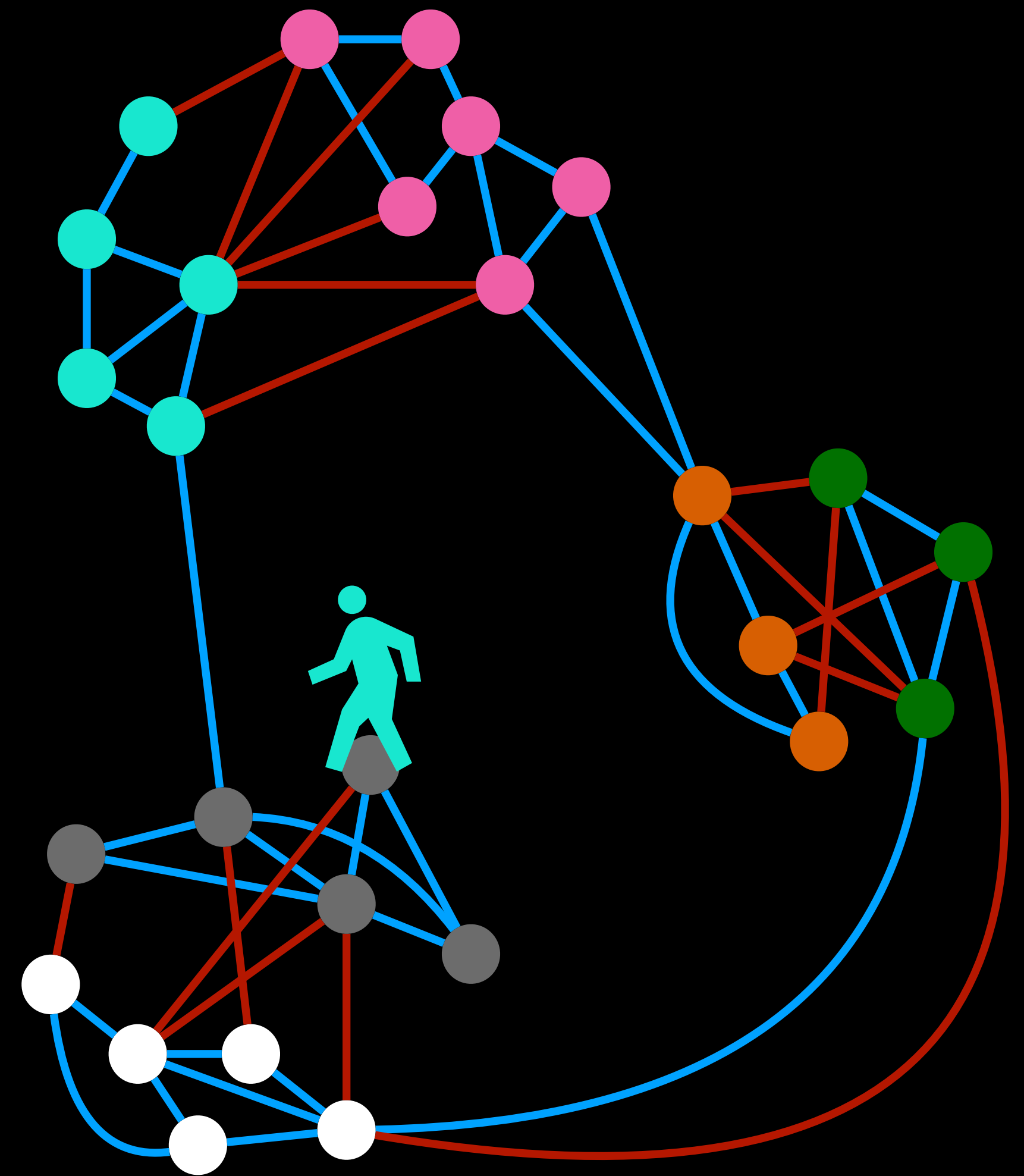


Random Walks With Signs



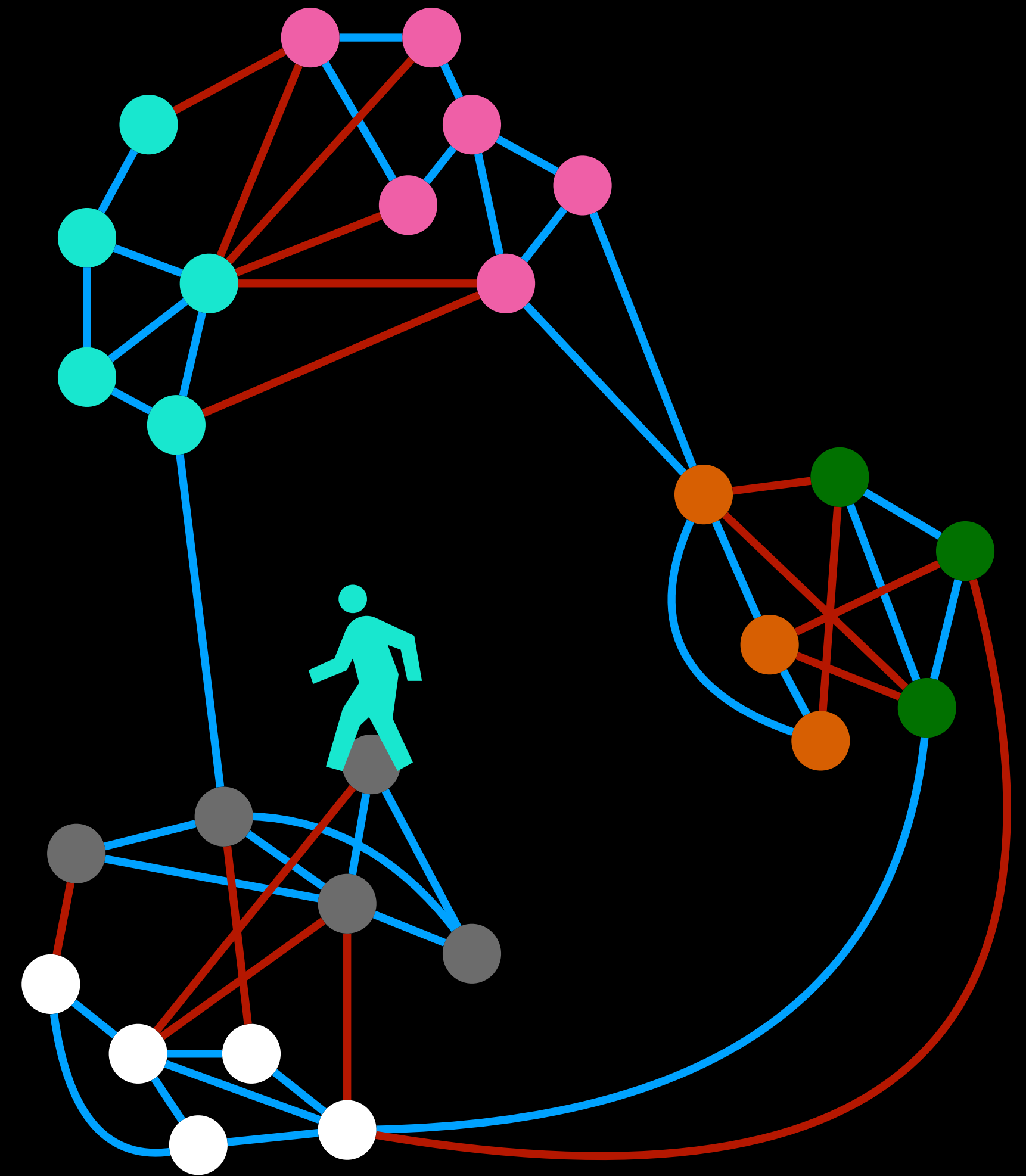
Random Walks With Signs

- Initialize sign $s = +$



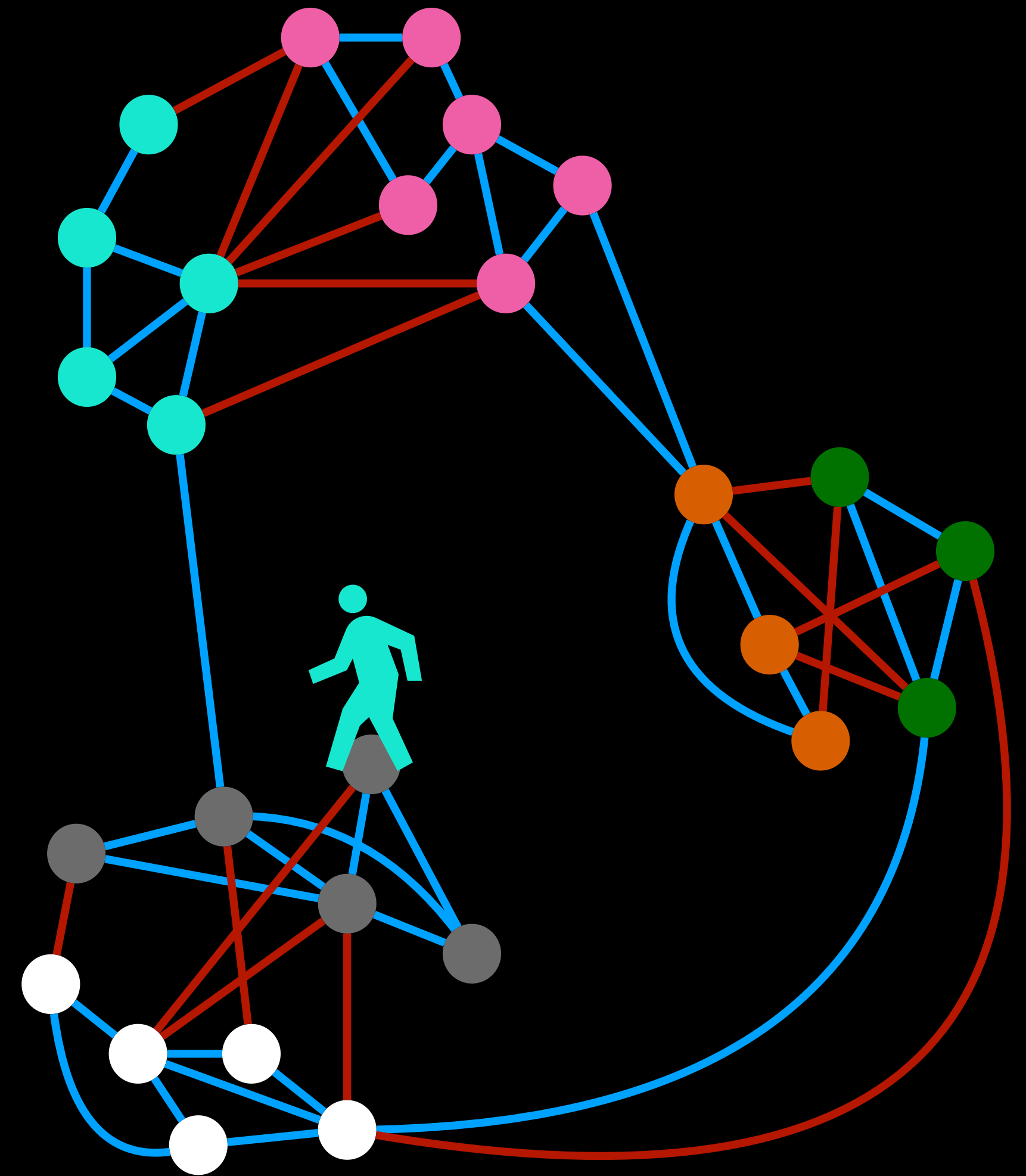
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- Initialize **sign** $s = +$
- When traversing an edge, multiply s with the edge sign



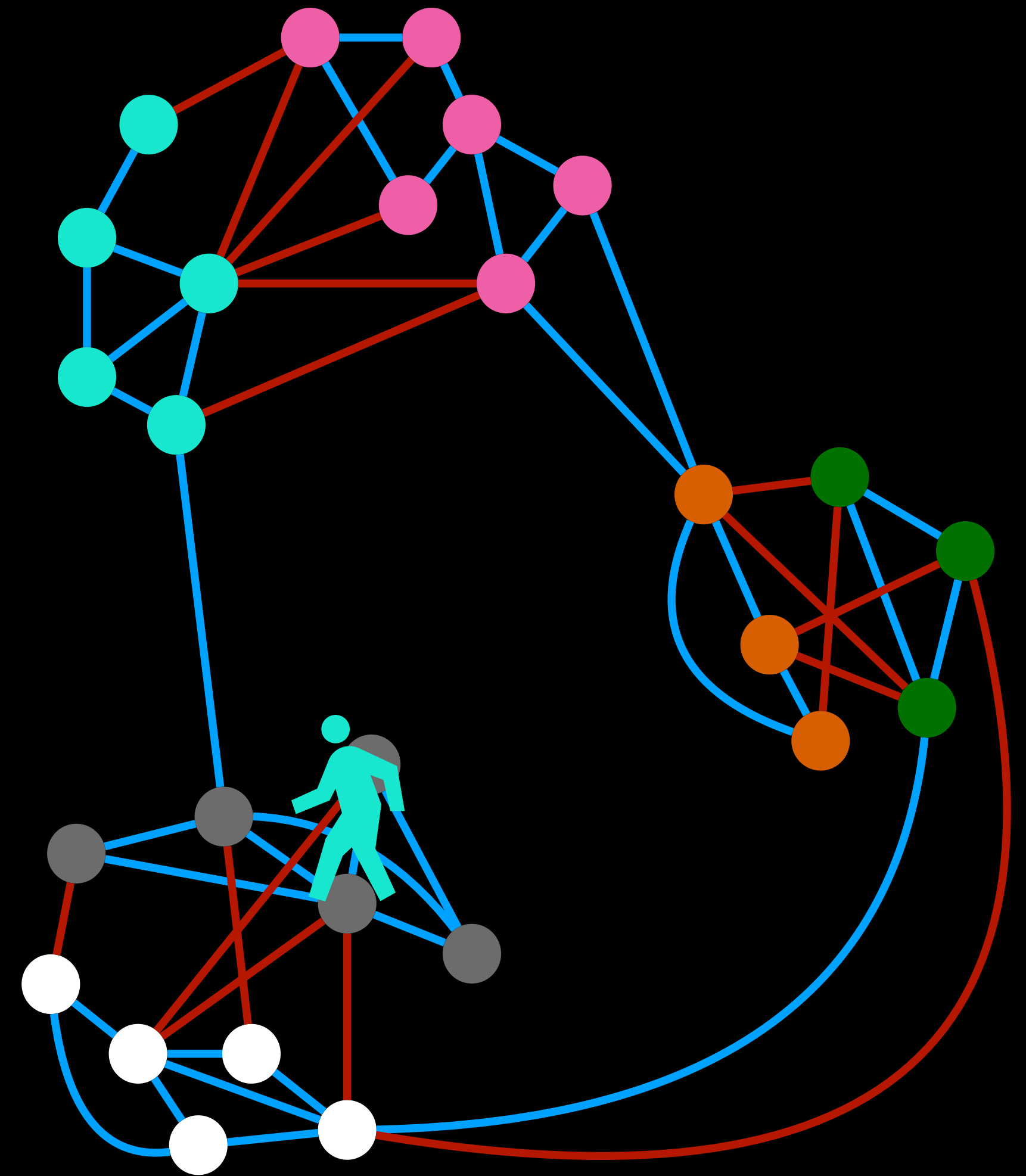
Random Walks With Signs

- Initialize **sign** $s = +$
- When traversing an edge, multiply s with the edge sign
 - ➔ the friend of my friend is my friend
 - if $s = +$ and we traverse a **positive** edge, new sign $s = +$



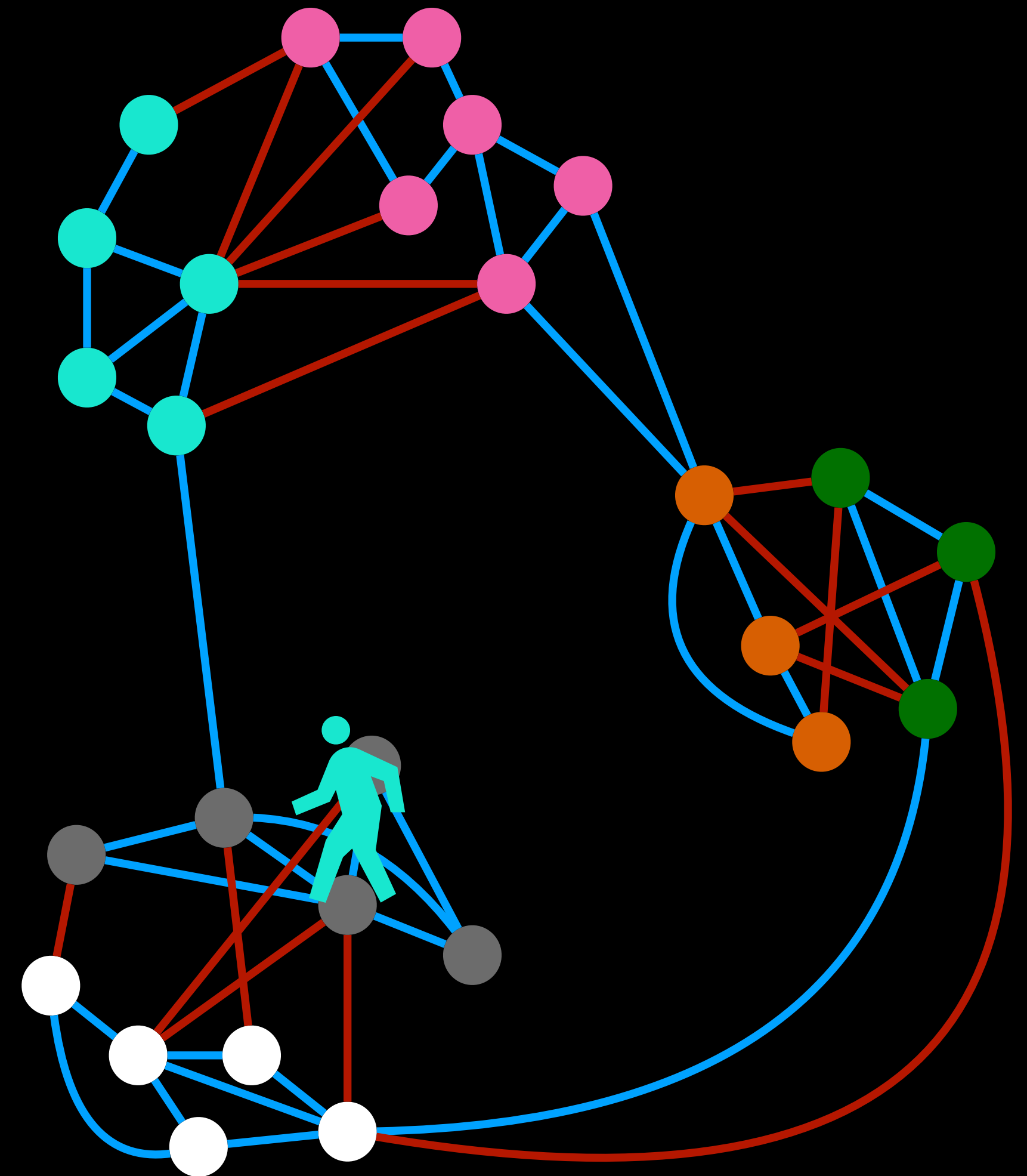
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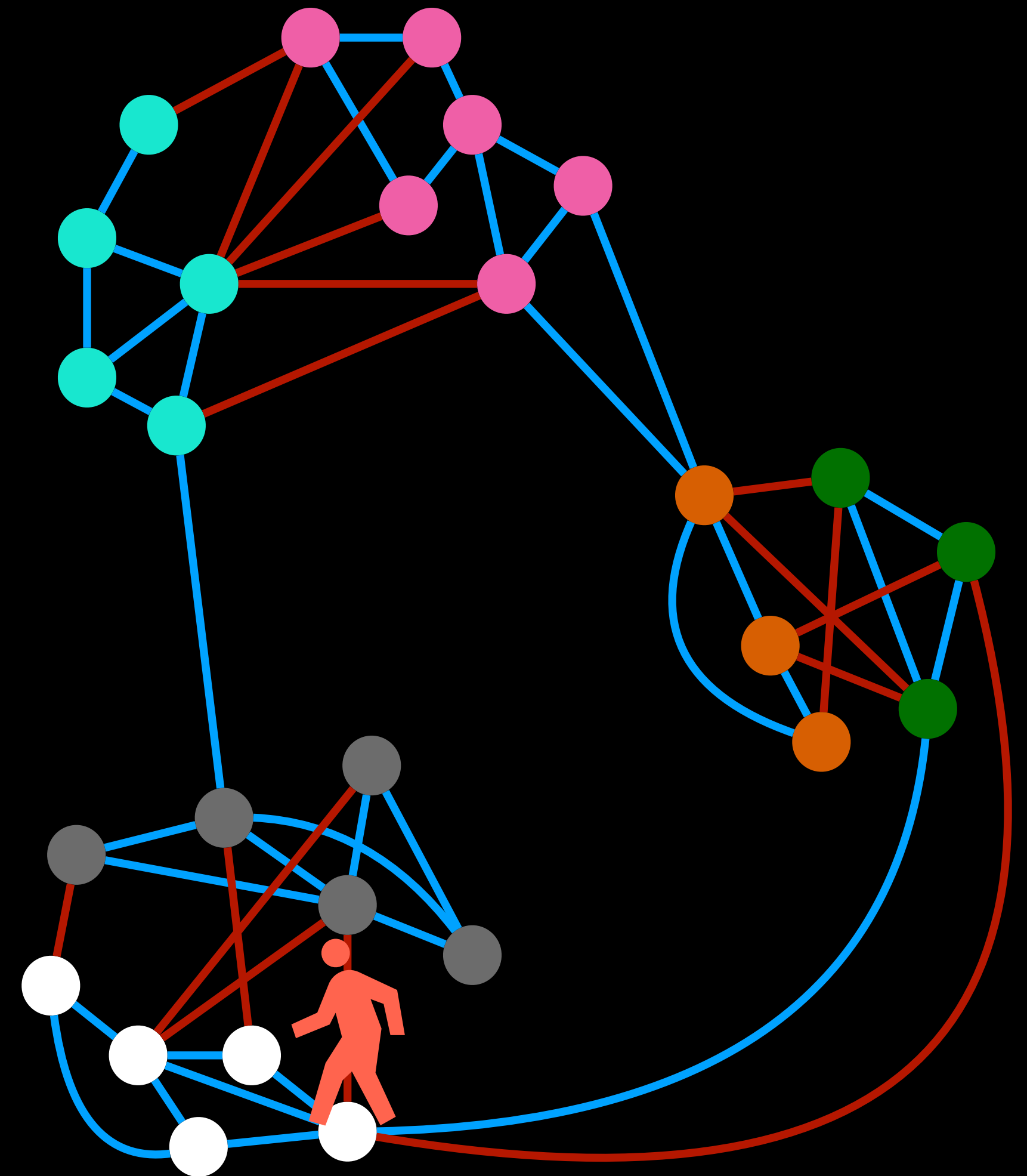
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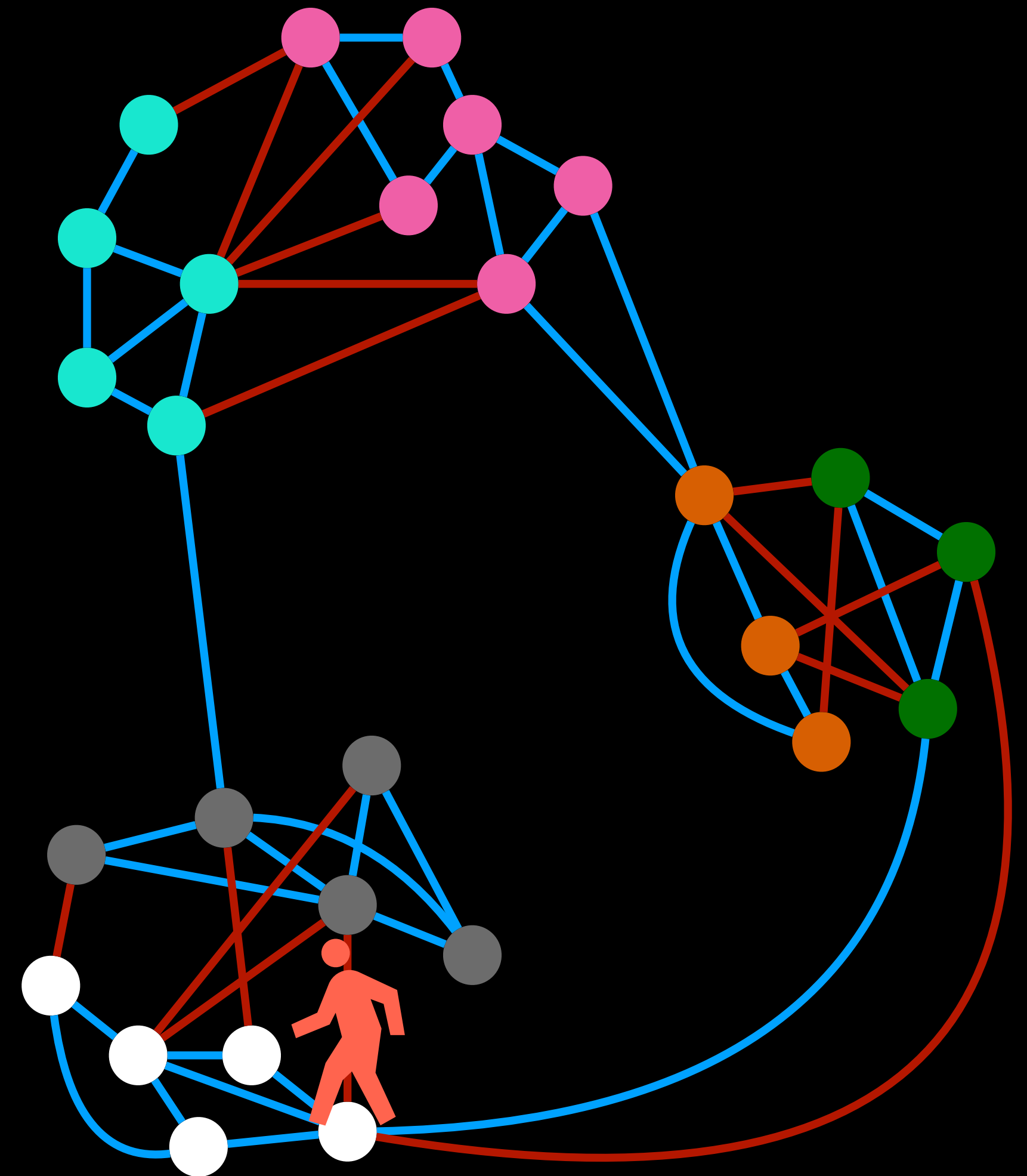
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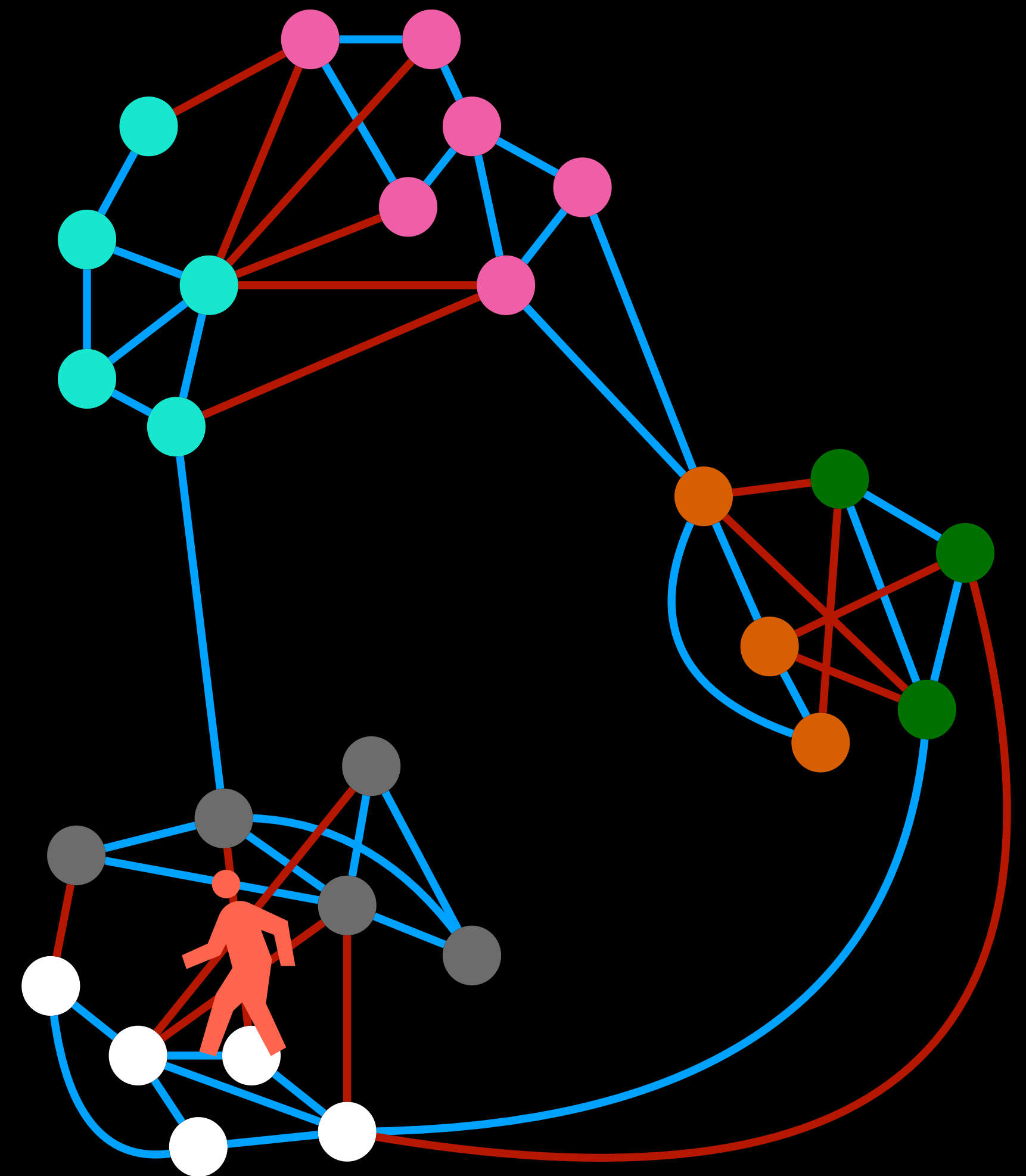
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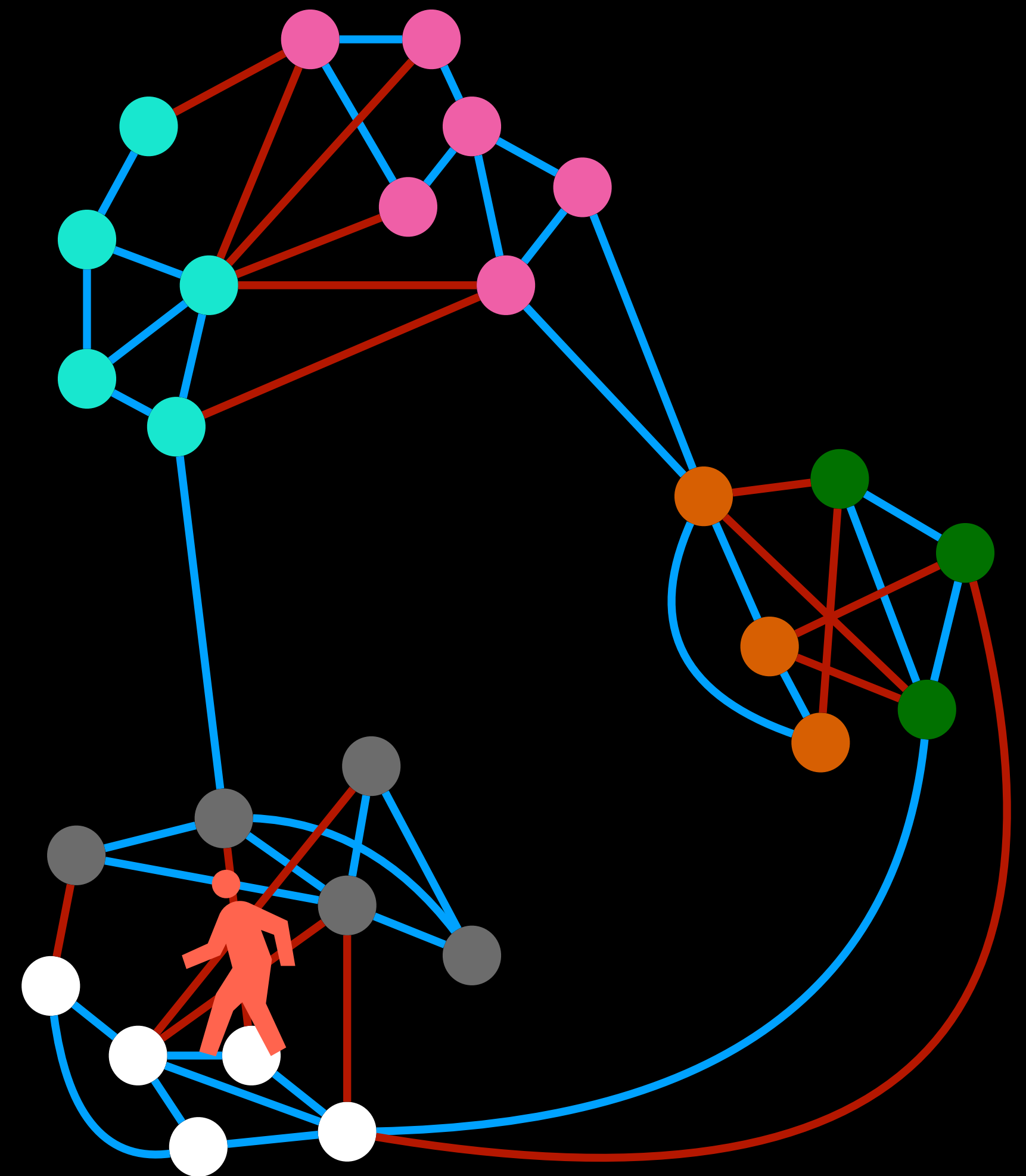
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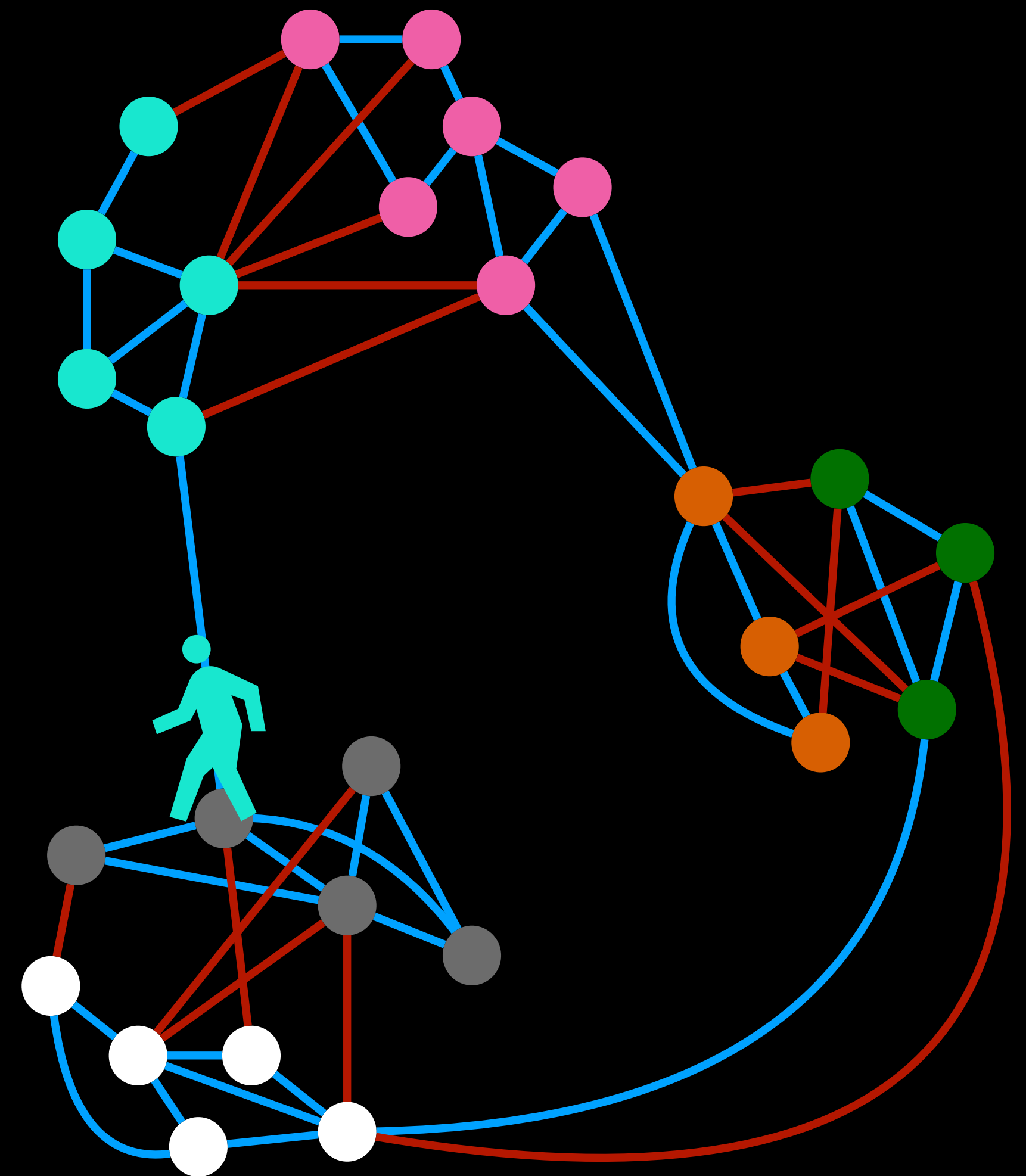
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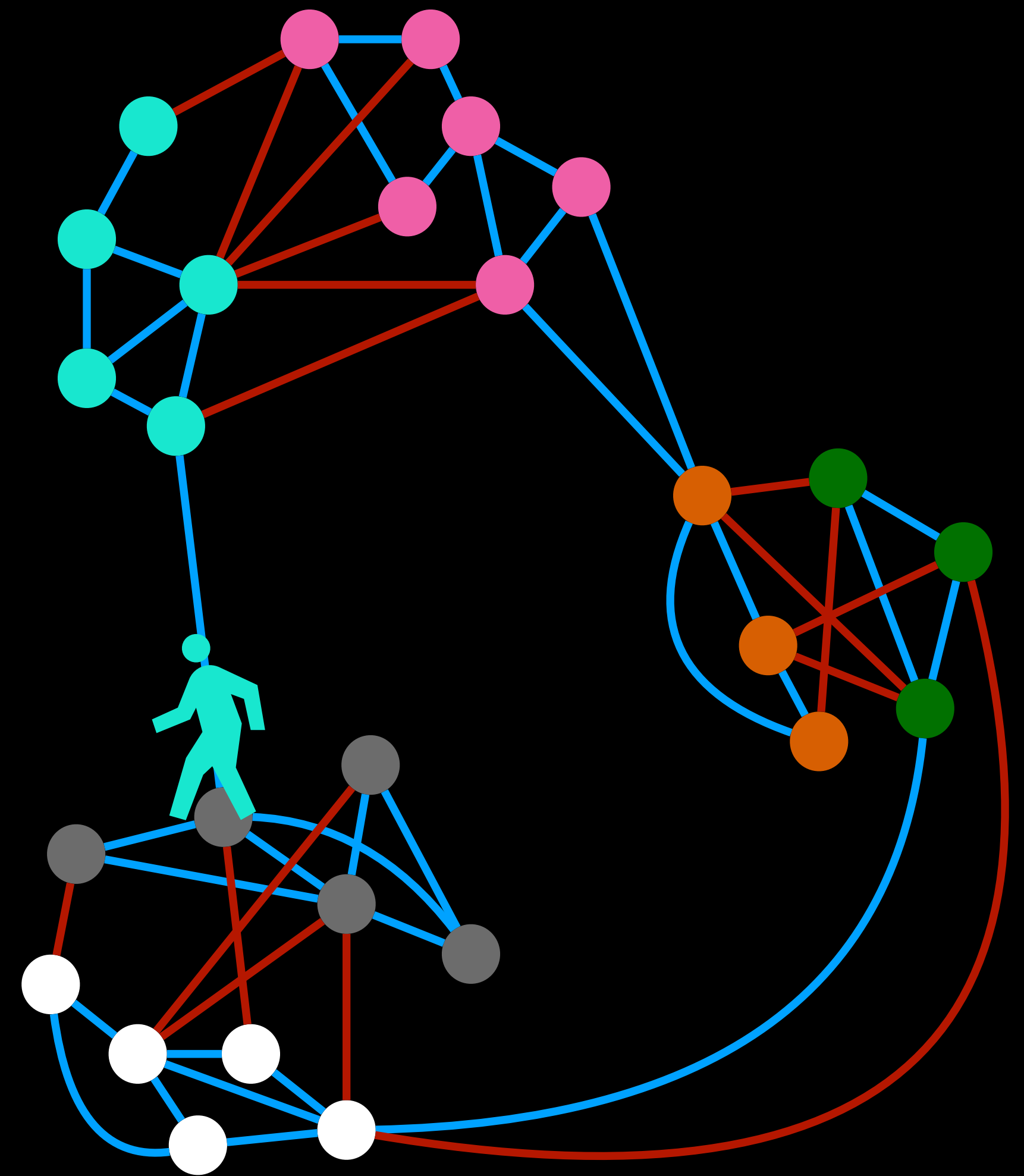
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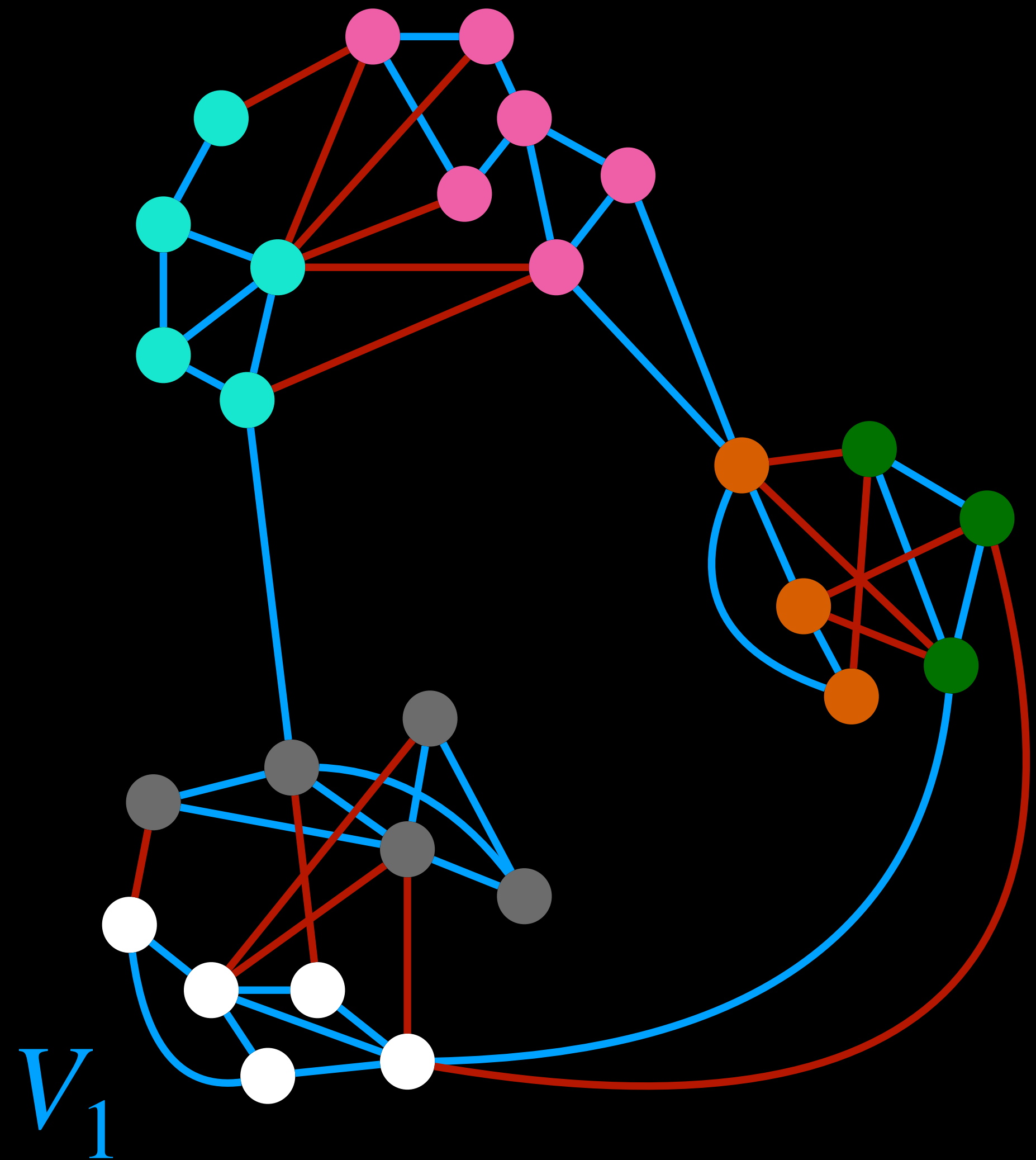
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- We do not make any changes to the randomness of the walk



Some Intuition

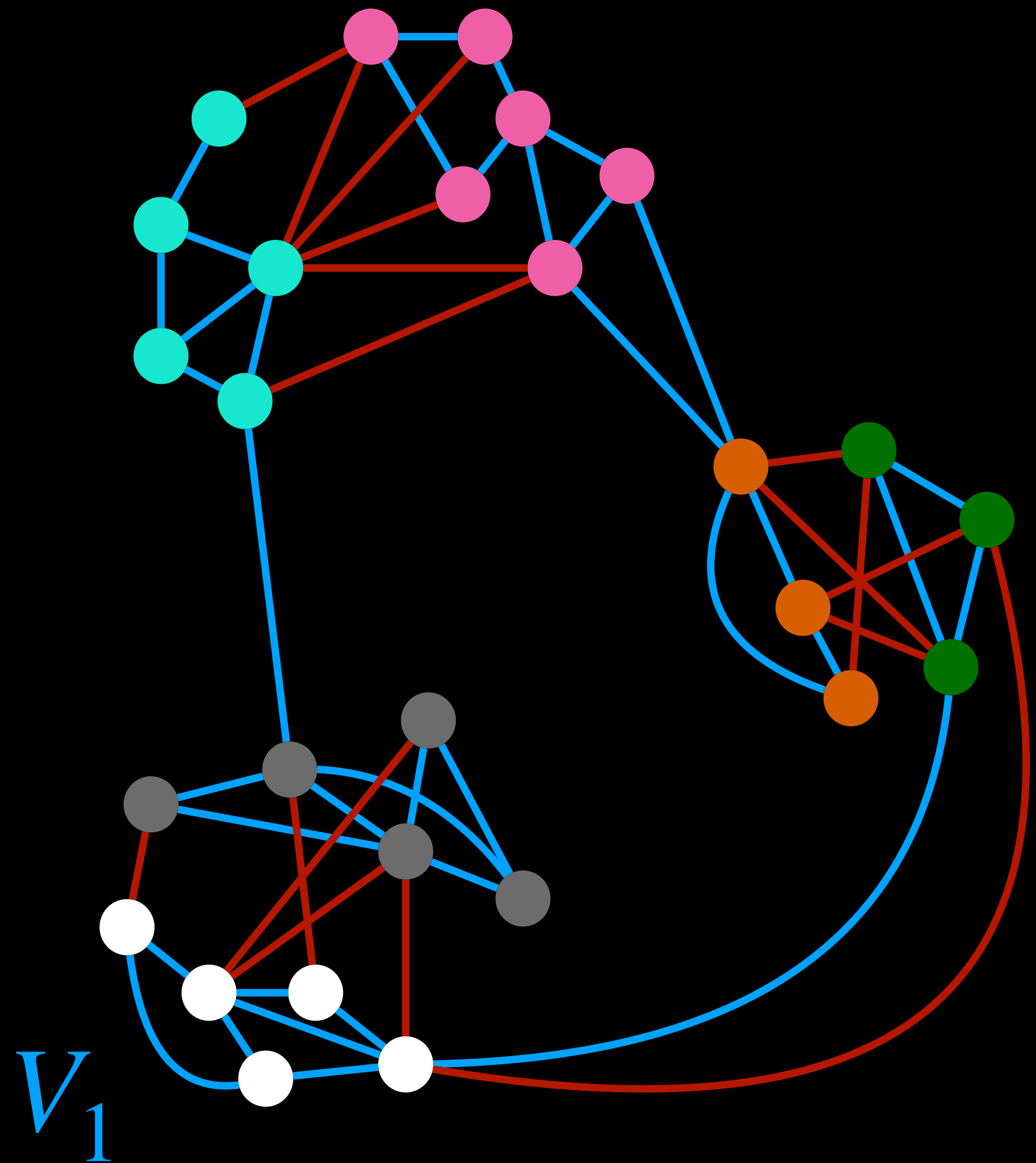
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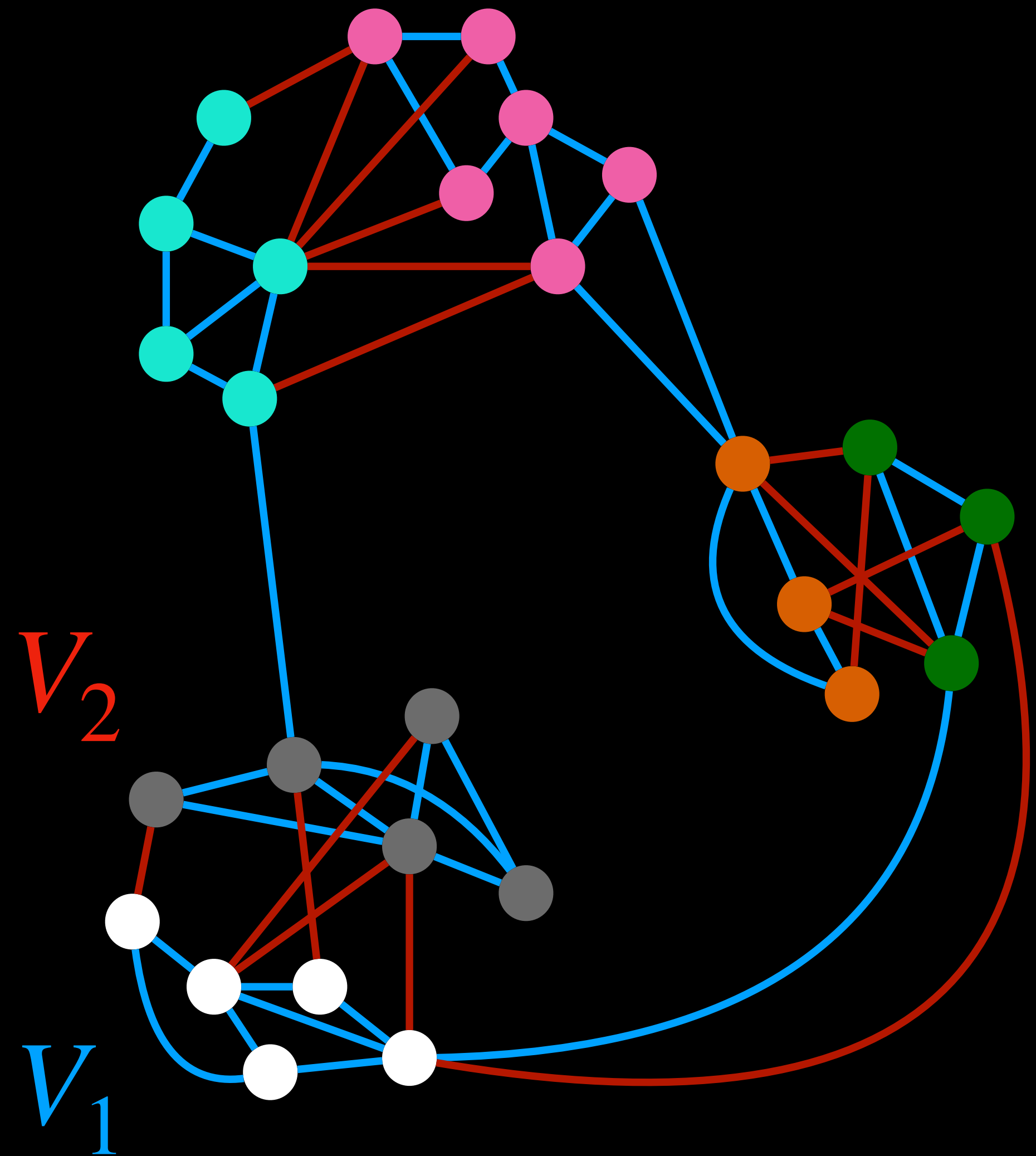
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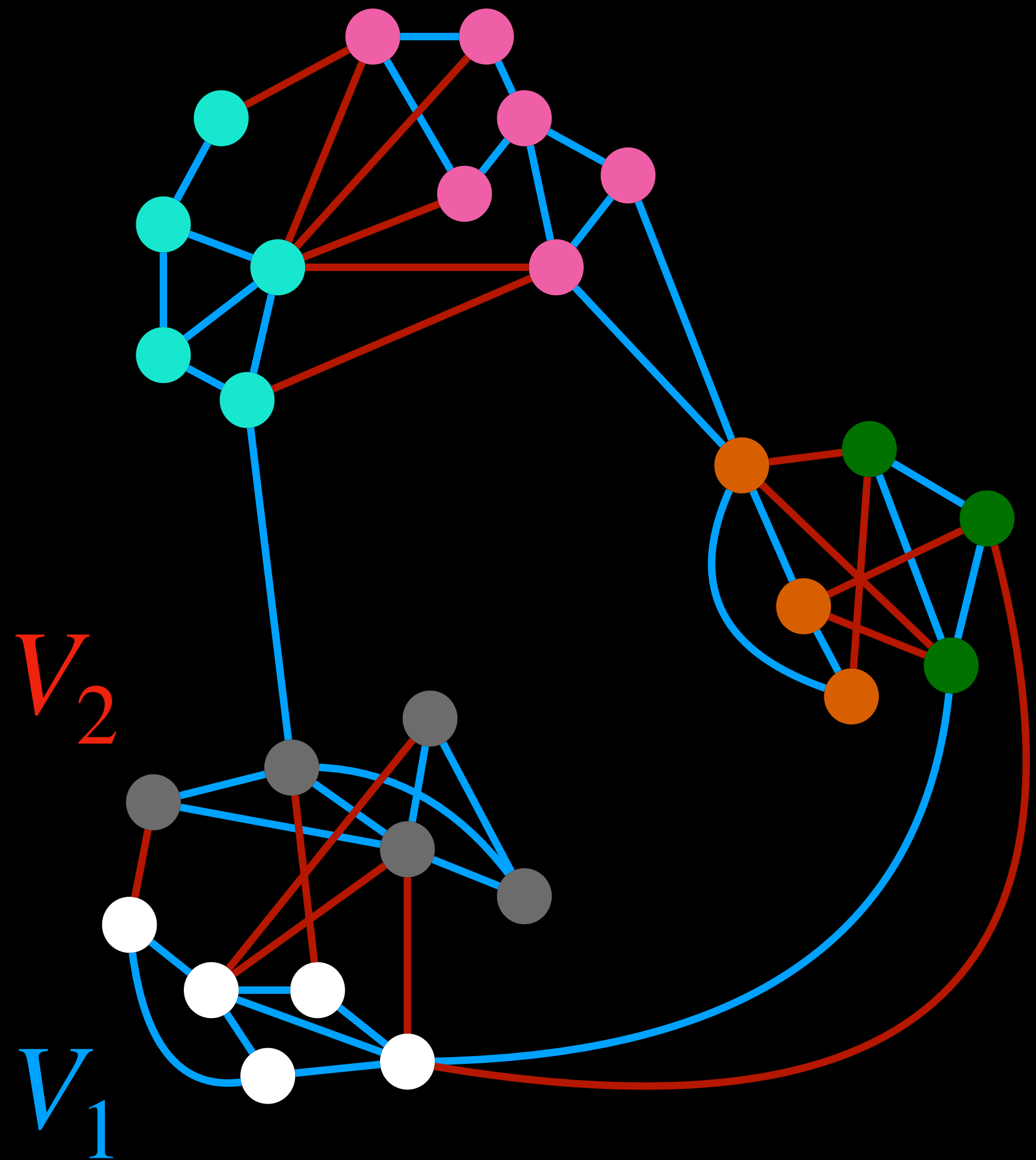
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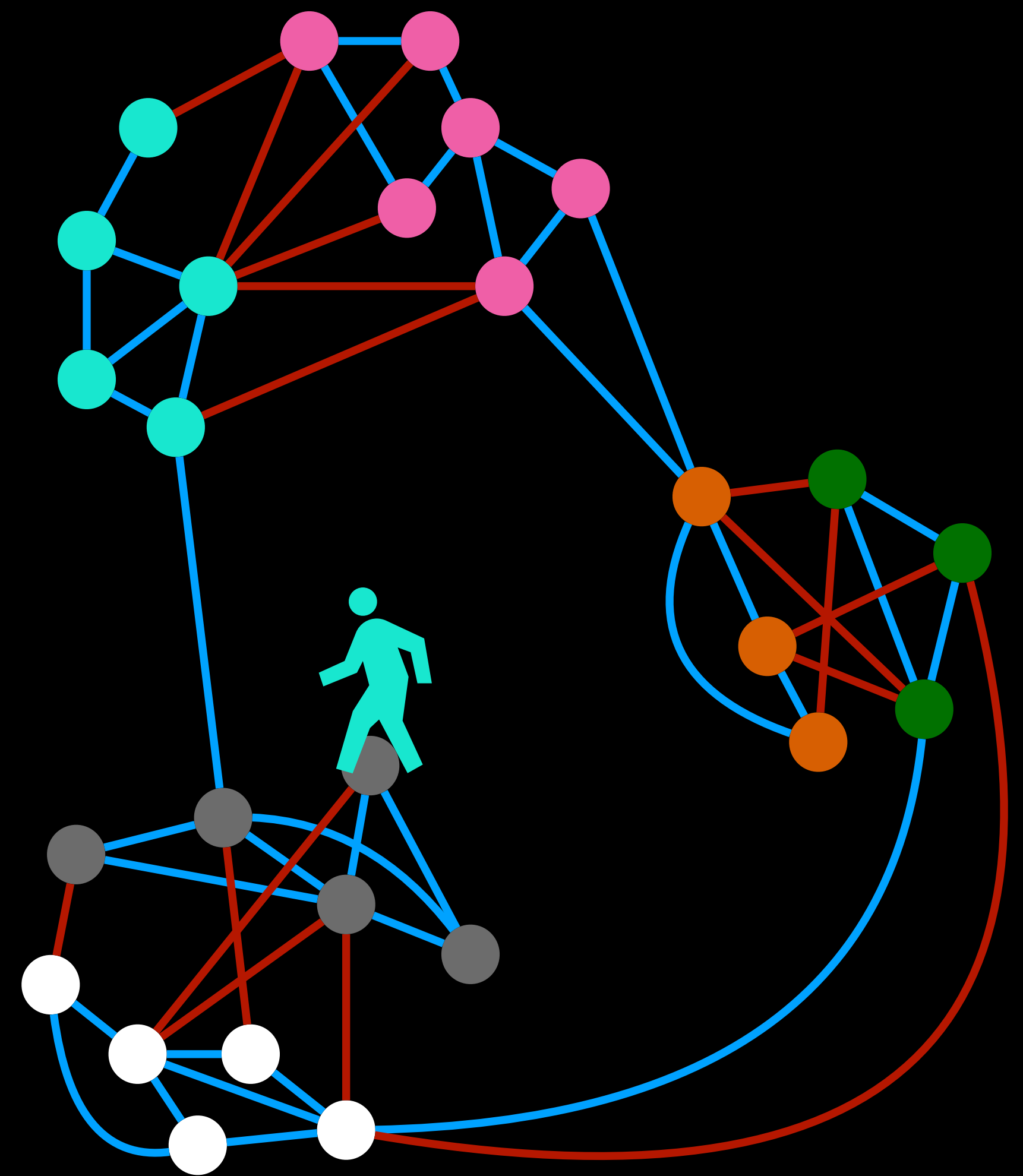
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- If we start in V_1 and end in V_1 , sign should be $+$
- If we start in V_1 and end in V_2 , sign should be $-$
- If we start in V_1 then the probability to end in $V \setminus (V_1 \cup V_2)$ should be very small

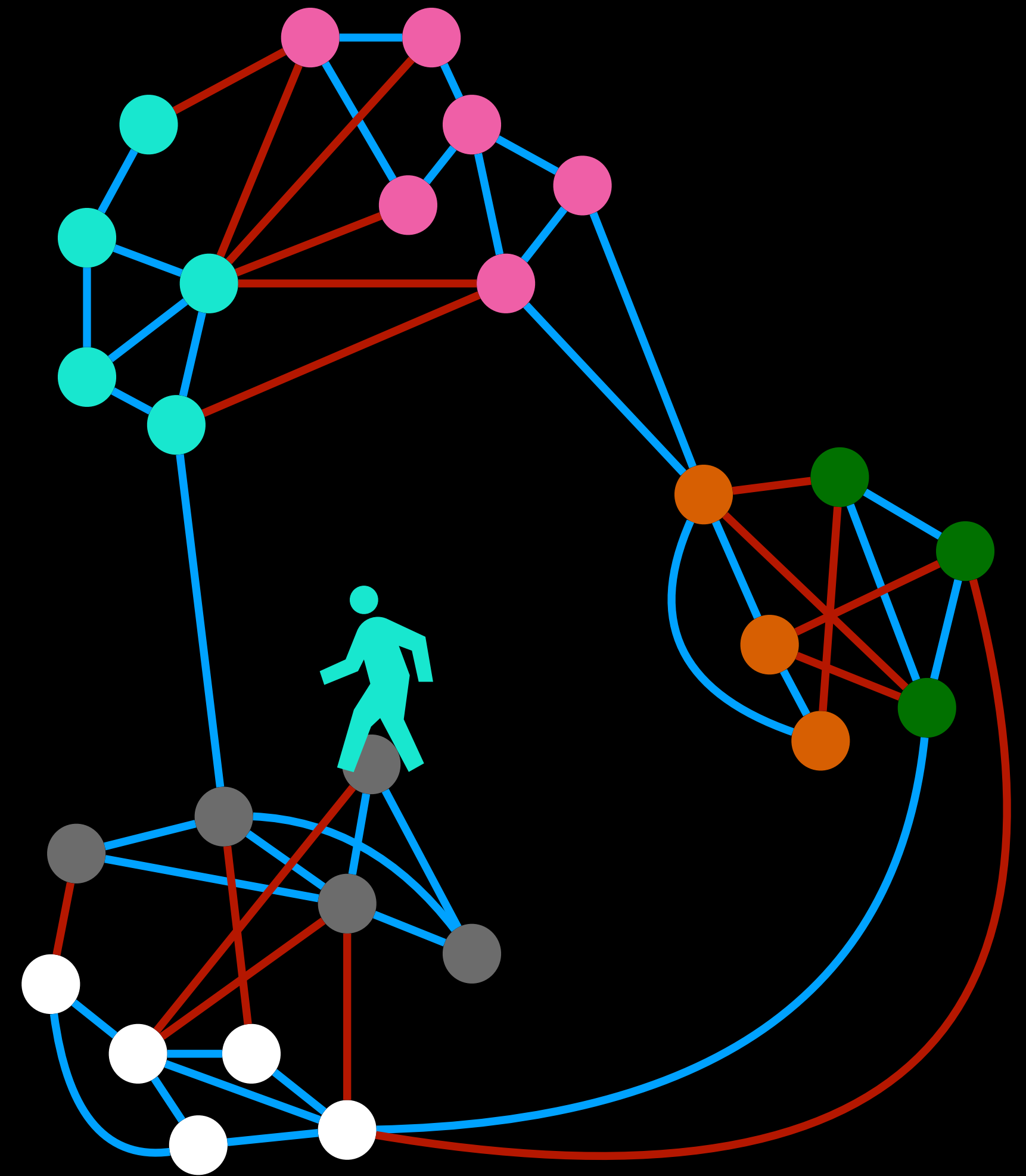


Community Vectors



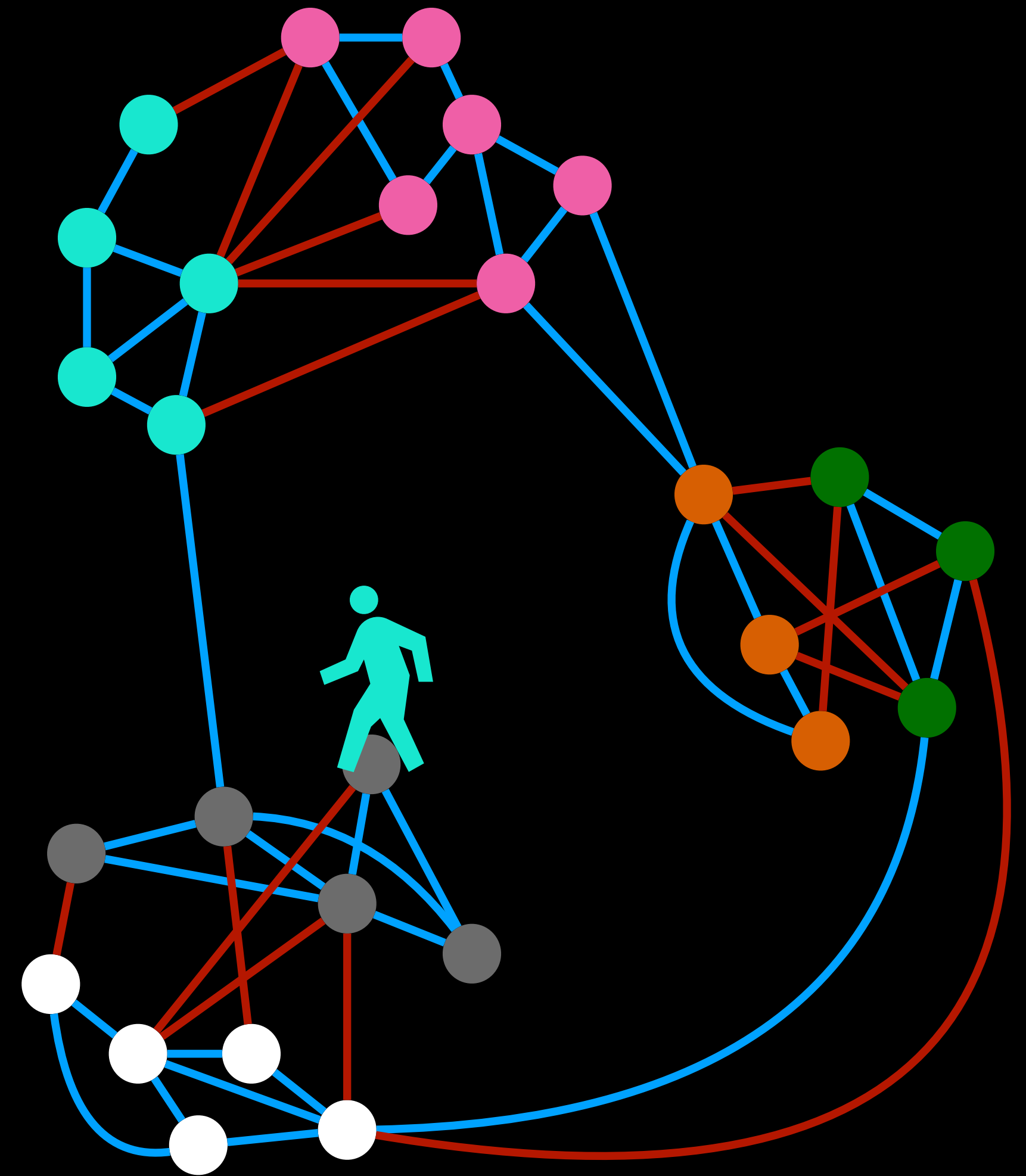
Community Vectors

- SignedCommunityVector(u):



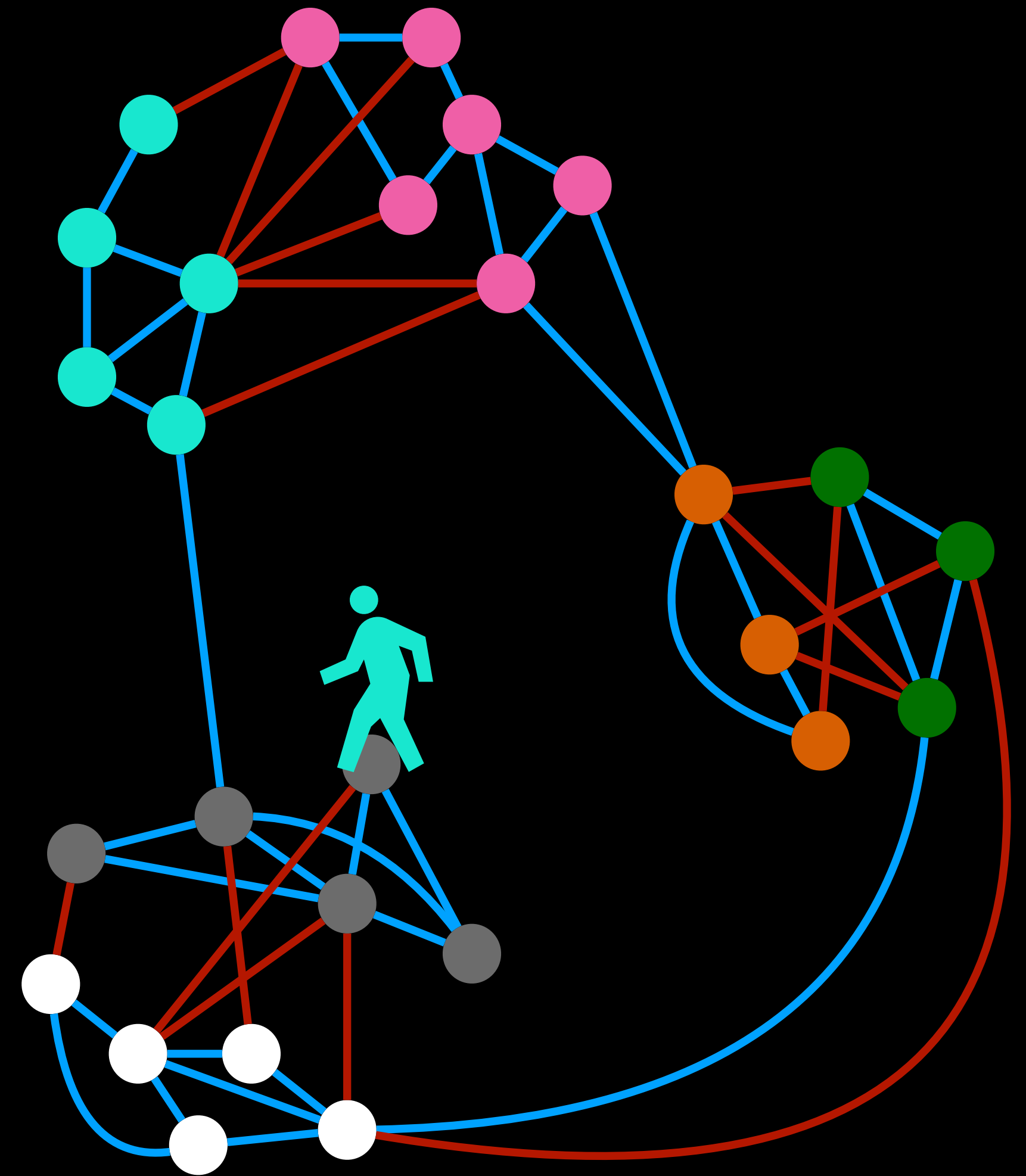
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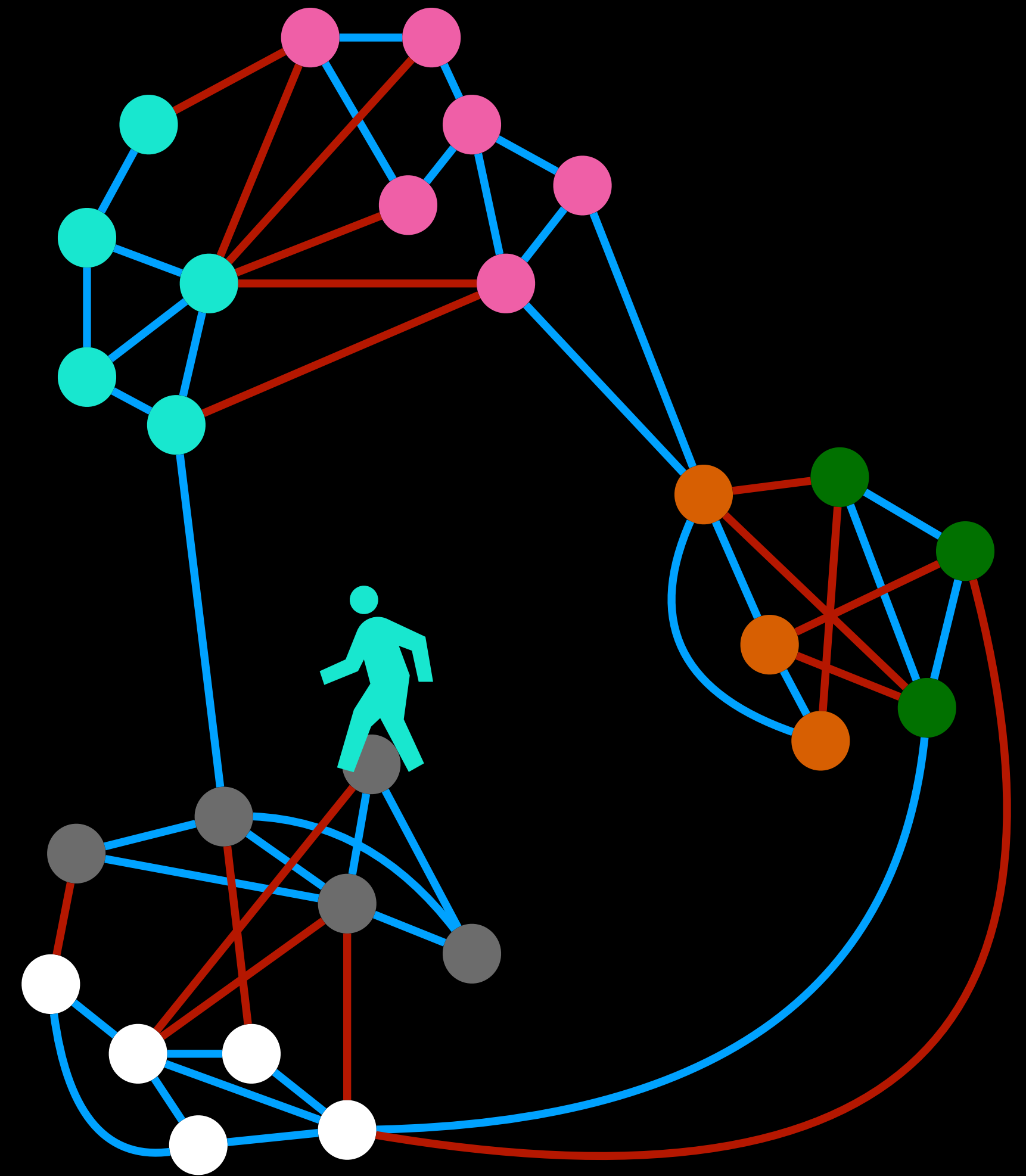
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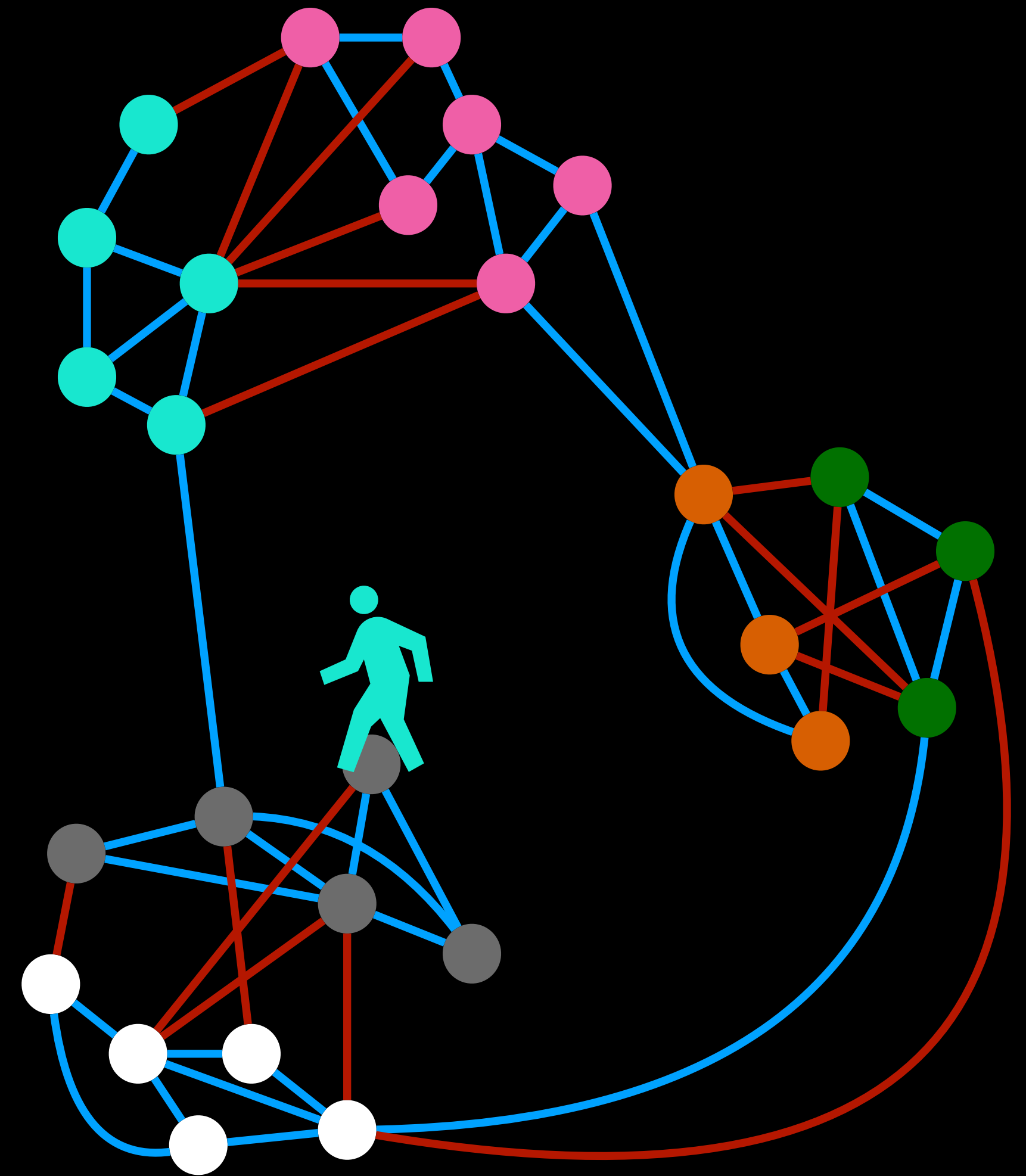
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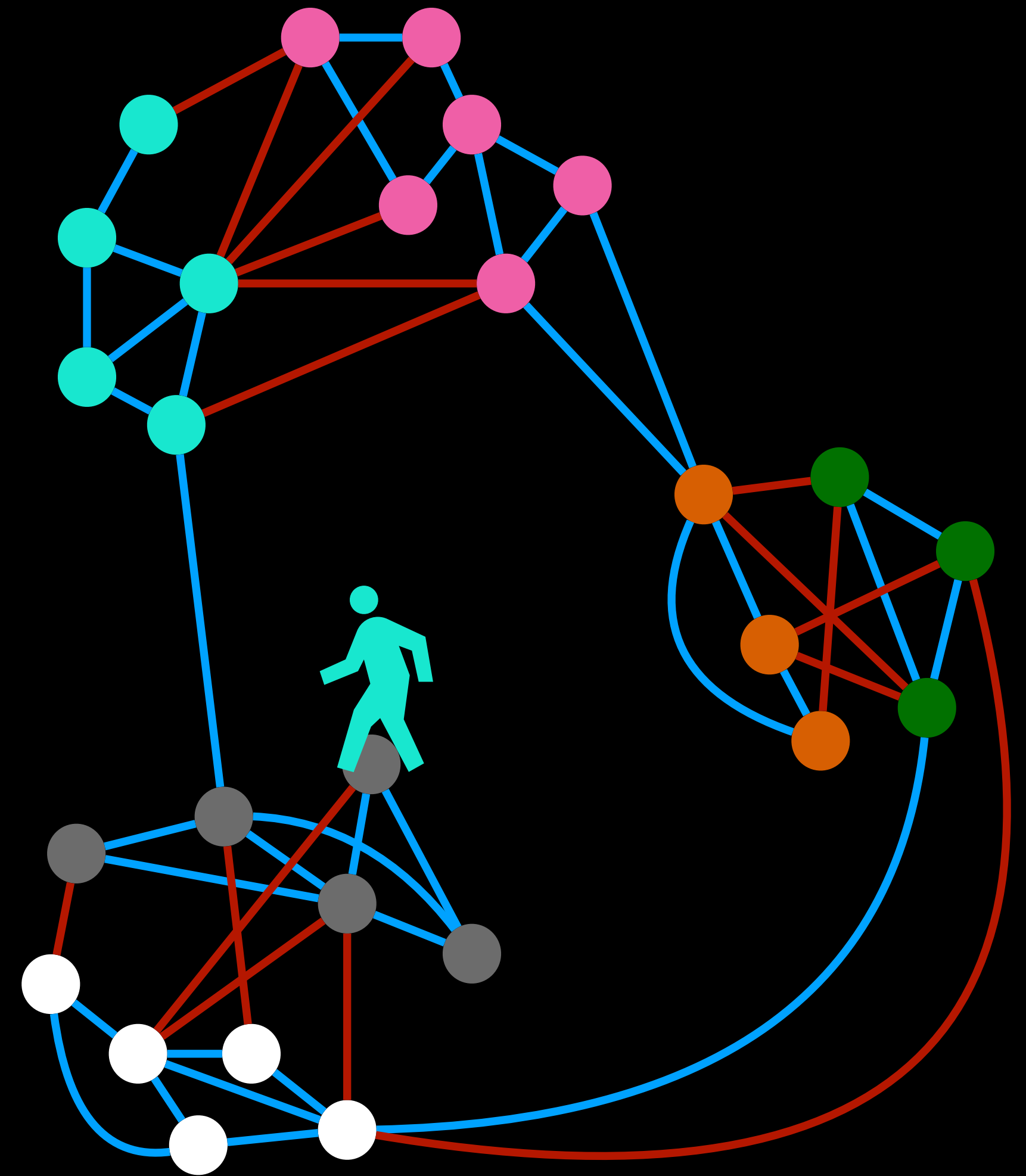
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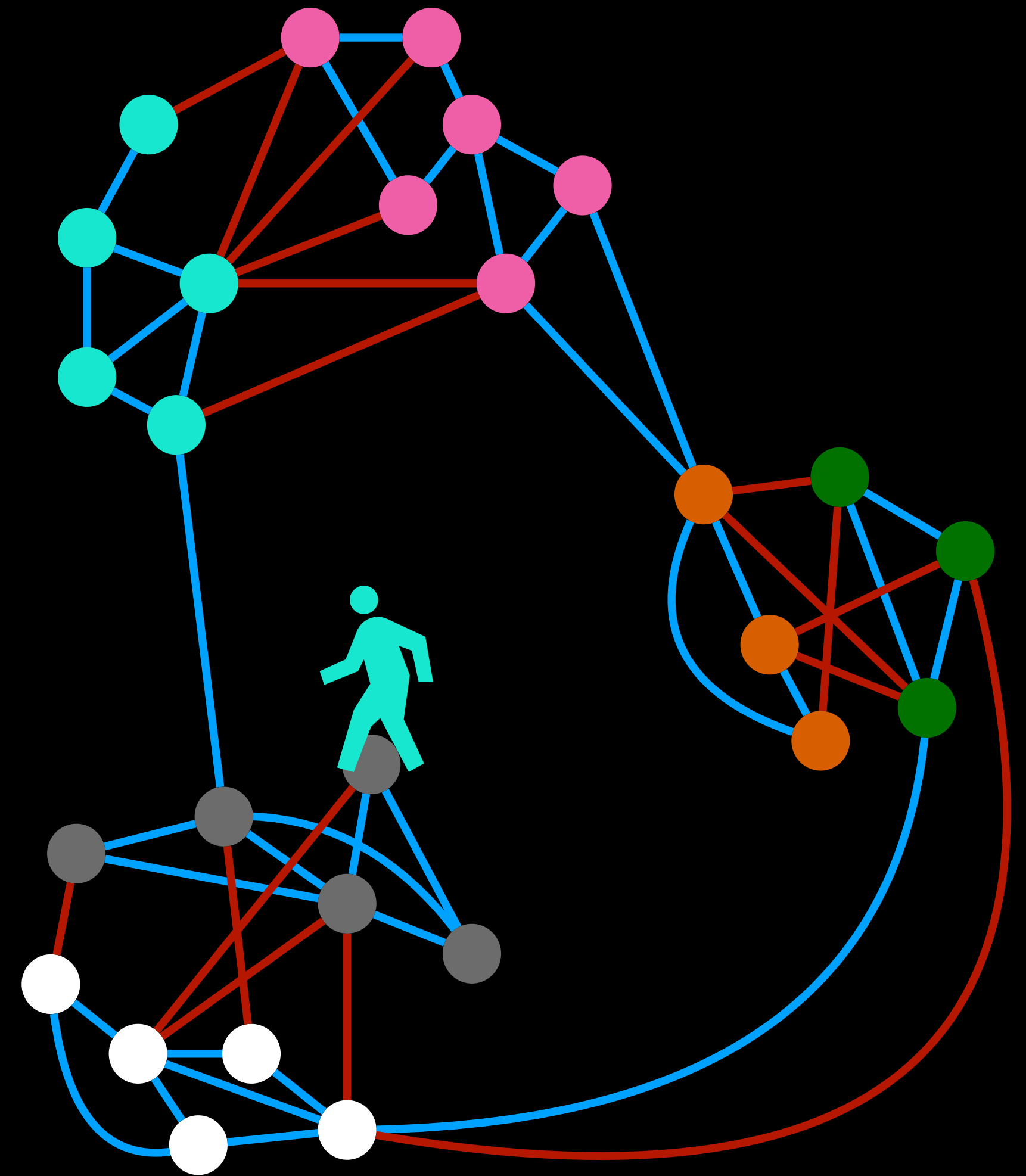
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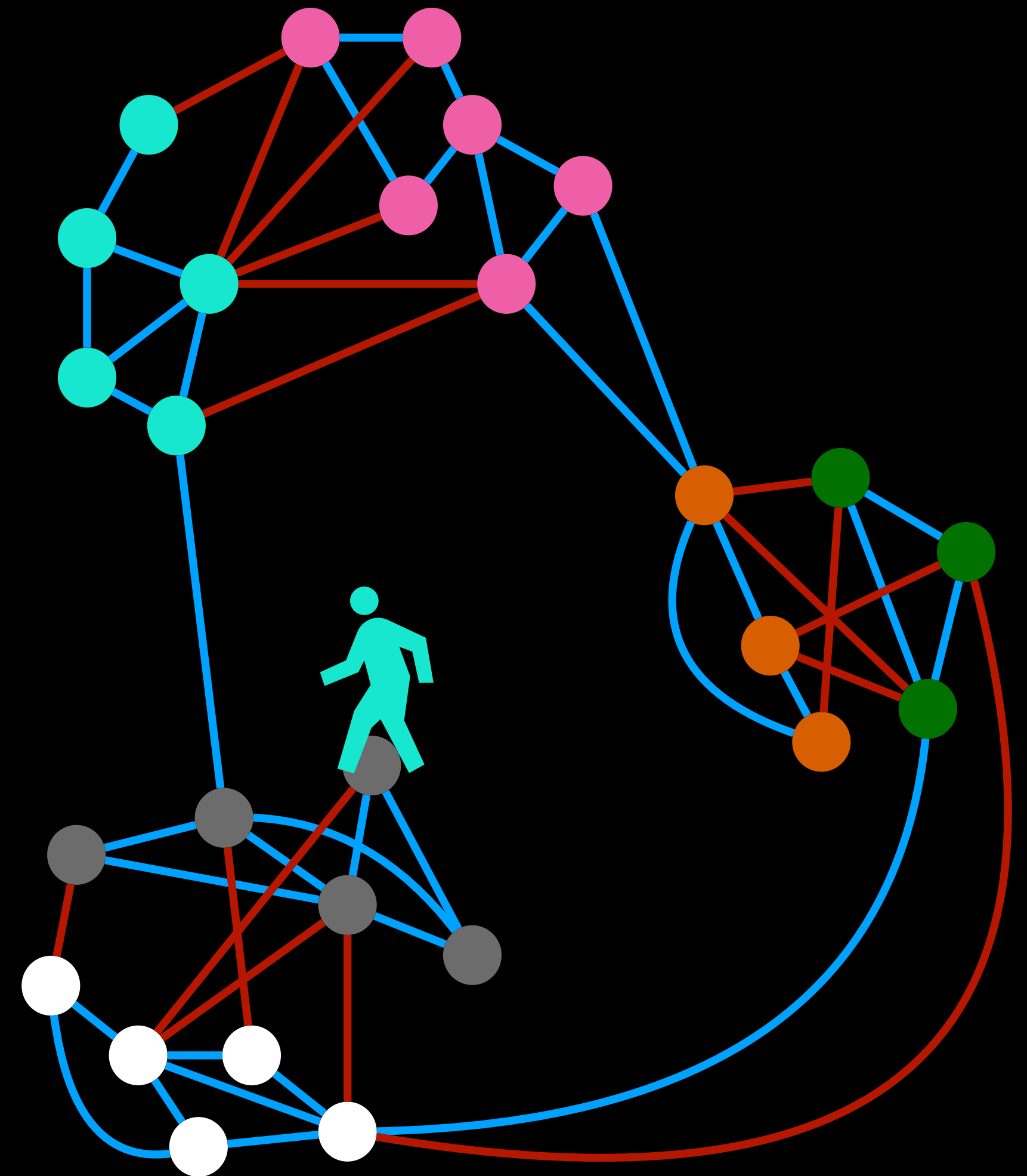
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 - $m(w) \leftarrow m_+(w) - m_-(w)$
- ➔ we expect that if w is on the same side as u then $m(w) \gg 0$,
if w is on the other side of u then $m(w) \ll 0$, and
if w is from a different cluster then $m(w) \approx 0$

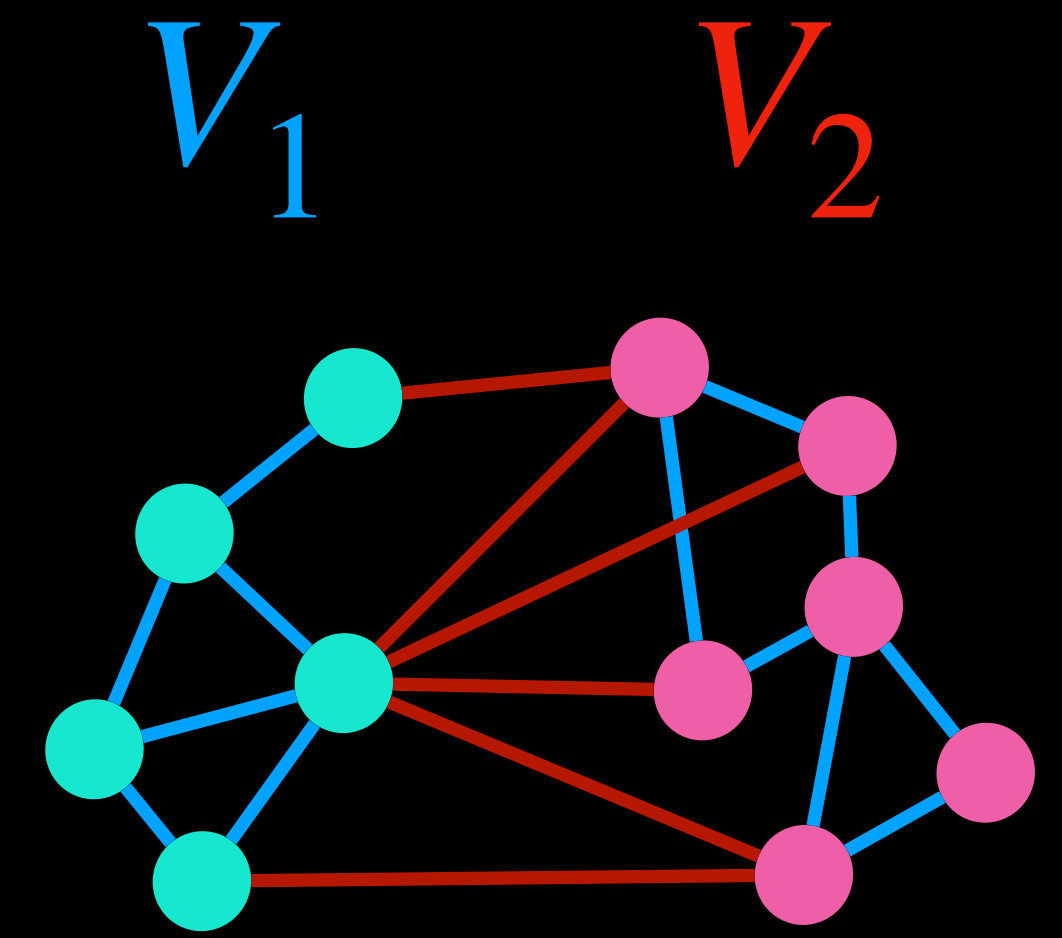


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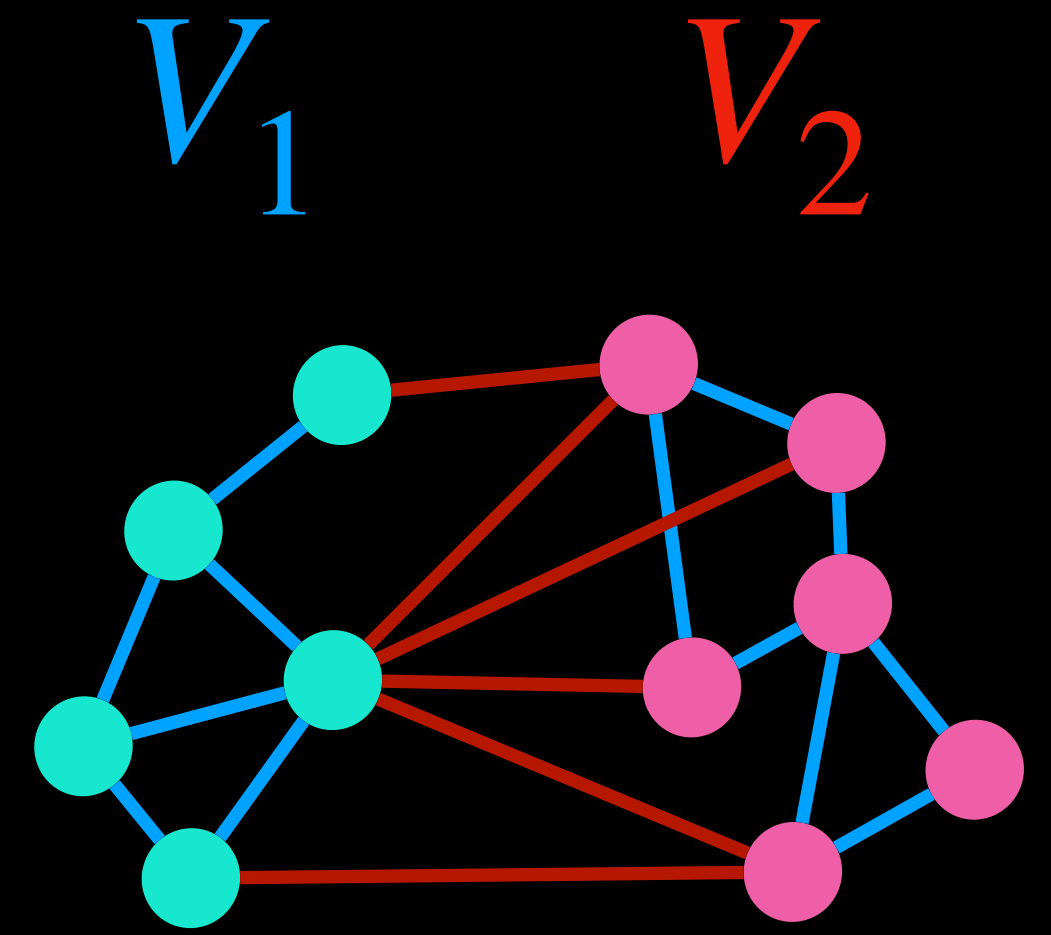
- Suppose that for each cluster C_1, \dots, C_k we know a seed-vertex $v_i \in C_i$
 - **WhichCluster(u):**
 - $m_u \leftarrow$ SignedCommunityVector(u)
 - $m_{v_i} \leftarrow$ SignedCommunityVector(v_i) for all i
 - $i^* = \arg \min_i \min\{\|m_u - m_{v_i}\|_2, \|m_u + m_{v_i}\|_2\}$
 - Return that u is from cluster i^*
- ➔ **Intuition:** if u is from cluster i^* , then the community vectors of m_u and $m_{v_{i^*}}$ should be similar

Main Technical Result



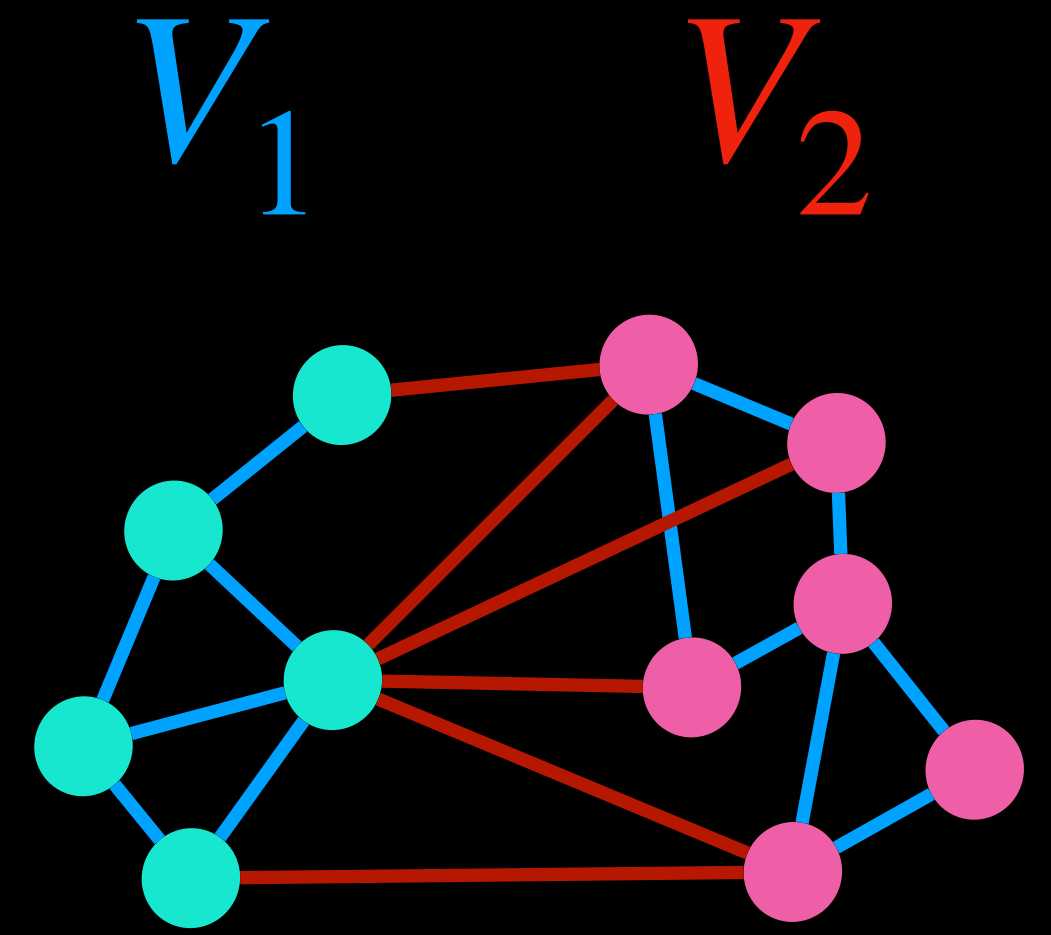
Main Technical Result

- Let $\mathcal{L} = \mathbf{I} - \mathbf{D}^{-1/2} \mathbf{A} \mathbf{D}^{-1/2}$ be the *signed normalized Laplacian*



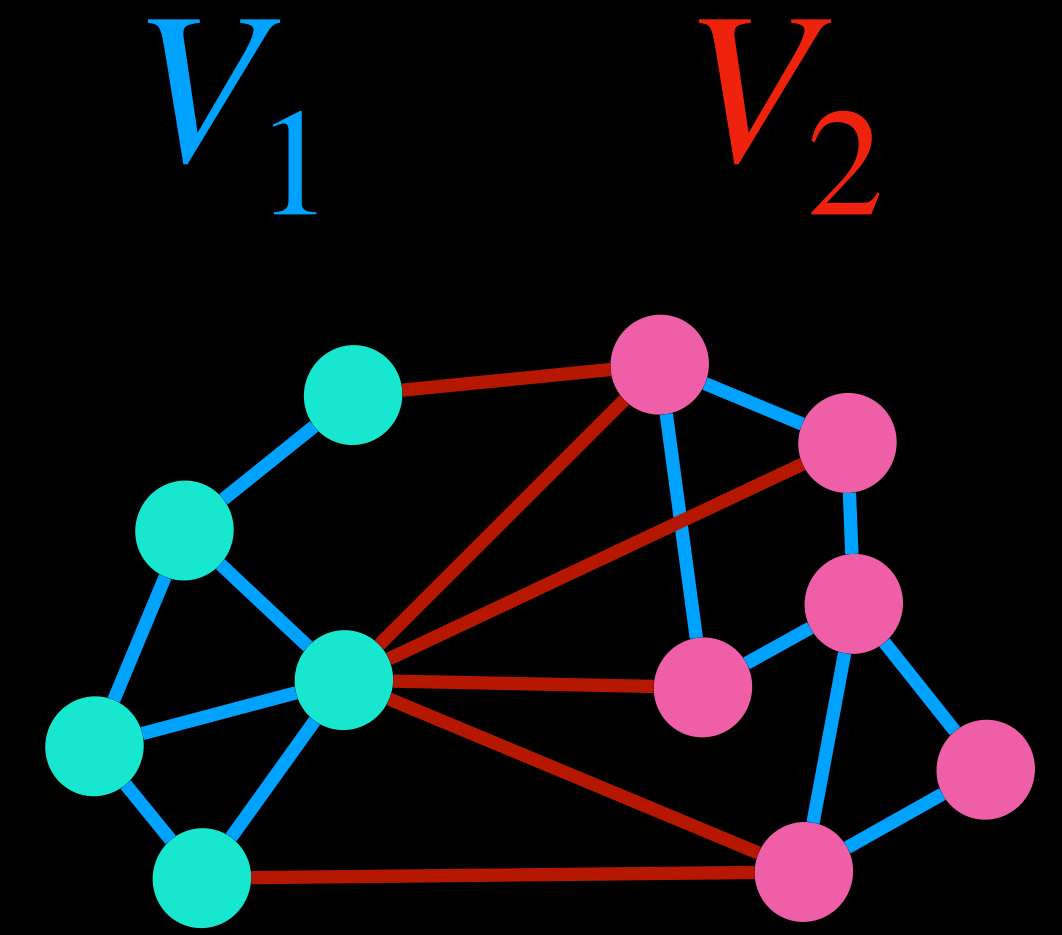
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- Let $\mathbf{v}_1, \dots, \mathbf{v}_n$ be the orthonormal row eigenvectors of \mathcal{L} , i.e., $\lambda_i \mathbf{v}_i = \mathbf{v}_i \mathcal{L}$, and assume that $0 \leq \lambda_1 \leq \dots \leq \lambda_n$

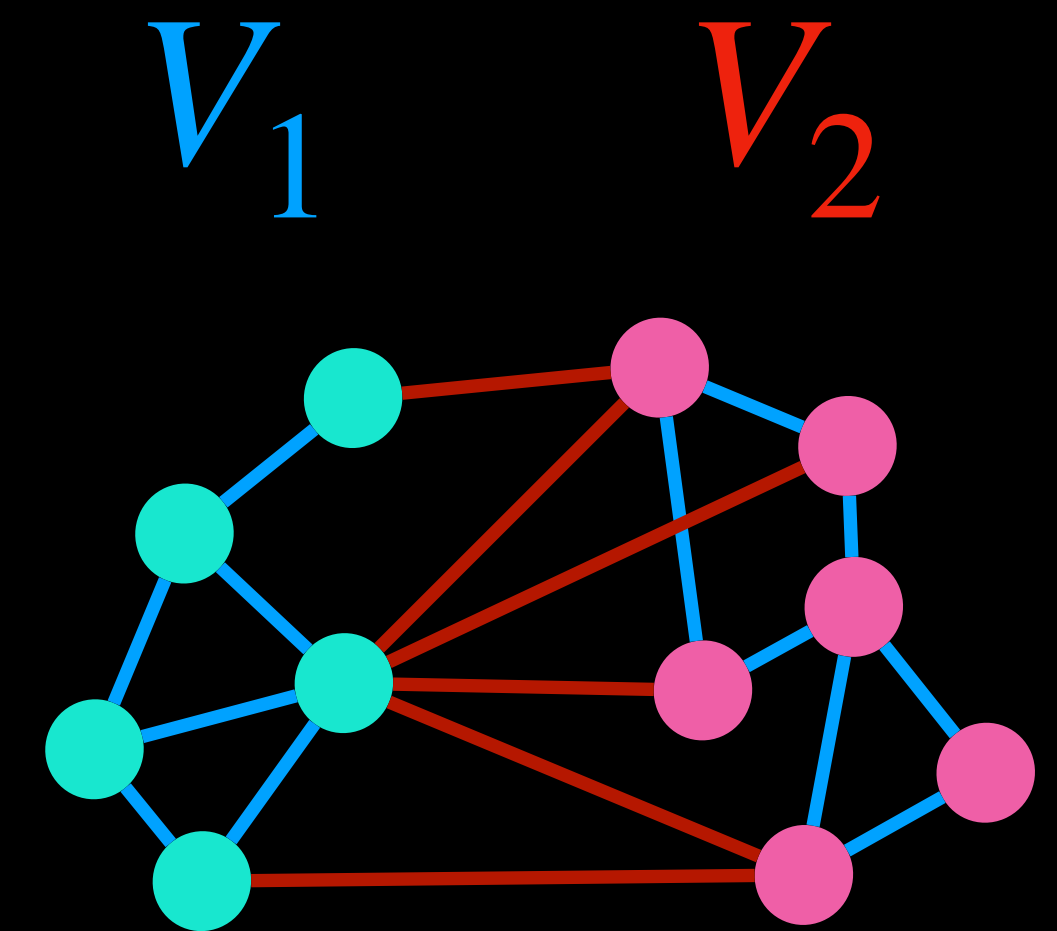


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➔ *The vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$ essentially reveal the polarized communities*

- More formally, consider a polarized cluster $U = (V_1, V_2)$.

Then (if G is degree-bounded, clusterable, and still simplifying a lot) for all $i \leq 1, \dots, k$,

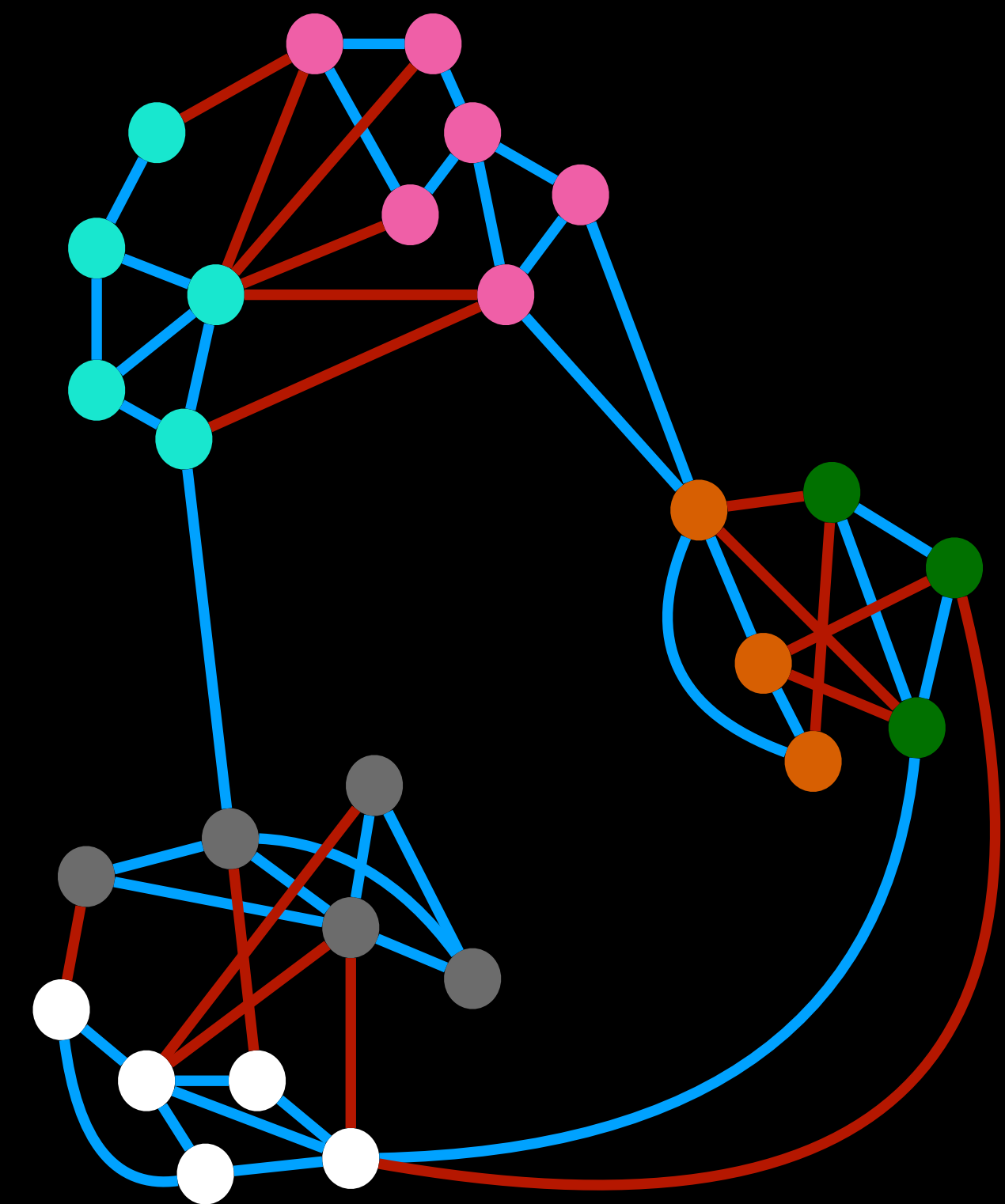
$$\mathbf{v}_i(u) \approx +\frac{1}{|U|} \text{ if } u \in V_1 \quad \text{and} \quad \mathbf{v}_i(u) \approx -\frac{1}{|U|} \text{ if } u \in V_2$$

Sublinear-Time Clustering Oracle for Signed Graphs

Stefan Neumann (KTH) and Pan Peng (USTC)

@StefanResearch

- We provide a **sublinear-time clustering oracle** that returns the communities of vertices in signed graphs
- We prove that it works and give new insights into **spectral graph theory for signed random walks**
- **Highly scalable** and works well in practice for large clusters
- We provide a **new signed graph dataset** with large ground-truth communities
- **Open questions:**
 - Give guarantees for our heuristic
 - Use our theoretical insights for better practical algorithms
 - What else can we do with signed random walks?



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Thank you!

