# Gaussian Process Uniform Error Bounds with Unknown Hyperparameters for Safety-Critical Applications

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### Motivation: Gaussian Processes for Safety-Critical Settings



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Posterior mean Posterior variance  
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Control, Bayesian optimization

e.g., Ostafew at al. (2016), Berkenkamp et al. (2017), Umlauft and Hirche (2019), Kirschner et al. (2019)...



• GP error bound **given hyperparameters**, (Srinivas et al., 2012), (Lederer et al., 2019)

$$P(|f(x) - \mu(x)| \le \beta \sigma(x) \quad \forall x \in \Omega) \ge 1 - \delta$$
  
Prior knowledge of hyperparameters required



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Log-likelihood min. fundamentally wrong for classical bounds













#### Same excellent regression performance as likelihood-optimal.







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Easy to compute and scalable.

Can easily replace bounds in standard GP-based algorithms.





# **Robust Performance Certificates for Real-World Applications**

Validation with several benchmark data sets with different level of sparsity



#### Novel hyperparameter training superior for safety-critical applications





#### Also in the paper...

- Full theoretical results
- Theoretical guarantees for control problem
- Comparisons across different benchmark data sets, also with full Bayes
- Numerical control example

and much more...

DATA SET	BSTN <sub>50</sub>	BSTN <sub>450</sub>	$ML_{50}$	ML300	WINE <sub>200</sub>	WINE <sub>1000</sub>
DIMENSION	d = 13	d = 13	d = 1	d = 1	d = 11	d = 11
OUR APPROACH	0.19	0.35	0.00	0.00	0.01	0.01
VANILLA GP	0.41	0.48	0.11	0.01	0.04	0.04
FULL BAYES	0.36	0.44	0.00	0.00	0.04	0.04





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