Learning node representations using stationary flow prediction on large payment and cash transaction networks

Ciwan Ceylan[1,2] Salla Franzén[2] Florian T. Pokorny[1]

[1]RPL, EECS, KTH Royal Institute of Technology [2]SEB Group

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Transaction data

- Vast number of payments and cash transactions executed daily
- This data is useful for business intelligence and financial crime prevention

Row	block_timestamp	from_address	to_address	amount
1	2018-06-01 07:06:17 UTC	0x8e5d083f	0x30727e88	150000.0
2	2018-06-01 07:12:35 UTC	0x30727e88	0x876eabf4	149999.0
3	2018-06-01 07:16:15 UTC	0x876eabf4	0x742d35cc	135130.0
4	2018-06-06 06:53:46 UTC	0x8e5d083f	0x052af704	259457.0
5	2018-06-08 06:33:24 UTC	0x53a46c10	0x6f0881f9	188087.0
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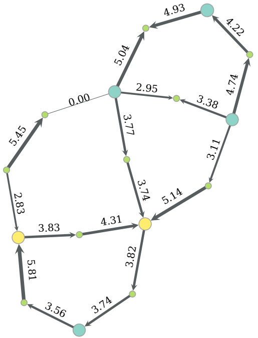
Aggregate → over time



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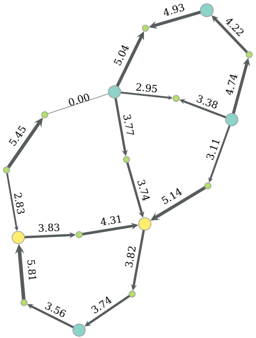

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- Dataset used: 1.5 years of publicly available ethereum transactions

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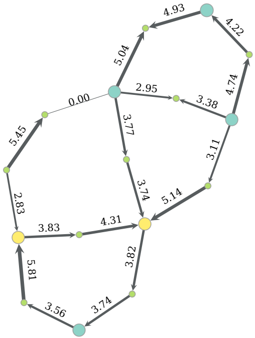

Gradient Flow Model[1]

- Hodge Decomposition of flow $\mathbf{y} \in \mathbb{R}^{|E|}$ on a graph $G = (V, E)$
  - $\mathbf{y} = \mathbf{y}_{\text{grad}} + \mathbf{y}_{\text{div}}$ s.t. $\mathbf{y}_{\text{grad}} \perp \mathbf{y}_{\text{div}}$
  - $y^{(ij)}_{\text{grad}} = z^{(j)} - z^{(i)}$ with $z^{(i)} \in \mathbb{R}$ and $(i, j) \in E$

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Two issues:

1. Gradient model cannot learn $\mathbf{y}_{\text{div}}$.
   For ethereum dataset:
   $\| \mathbf{y}_{\text{div}} \|_2 = 42\text{M}$, $\| \mathbf{y}_{\text{grad}} \|_2 = 6\text{M}$

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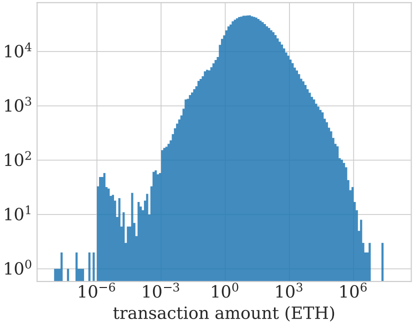
2. Squared error unsuitable for multi-scale and heavy-tail transactions



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Addressing expressivity

- Gradient model is limited by scalar potentials
  - Use multiple potentials for each node, $\mathbf{z}^{(i)} \in \mathbb{R}^K$
  - Use a gate function $\bar{\sigma} : \mathbb{R}^{K \times K} \mapsto [0, 1]^K$
- Gated gradient flow model: $f^{(ij)} = \bar{\sigma}(\mathbf{u}^{(i)}, \mathbf{u}^{(j)})^T \left(\mathbf{z}^{(j)} - \mathbf{z}^{(i)}\right)$

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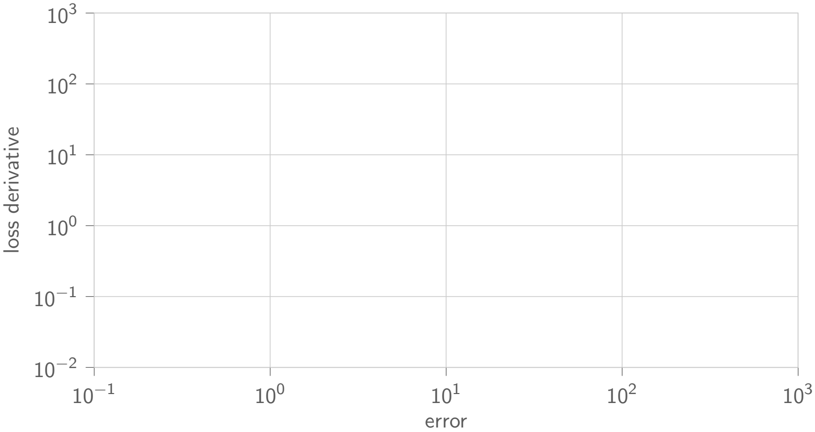
- Theorem: This model can express any flow on a graph, given sufficiently expressive $\bar{\sigma}(\cdot,\cdot)$ and $K = 2\Delta(G)$
- We use a simple form for $\bar{\sigma}$ with learnable $\mathbf{u}^{(i)}$:

$$[\bar{\sigma}(\mathbf{u}^{(i)}, \mathbf{u}^{(j)})]_k = \frac{1}{1 + e^{-(u_k^{(i)} + u_k^{(j)})}}$$

Addressing training loss function



Stationary flow prediction

minimize $\sum_{ij \in E} \ell(y^{(ij)}, f^{(ij)})$

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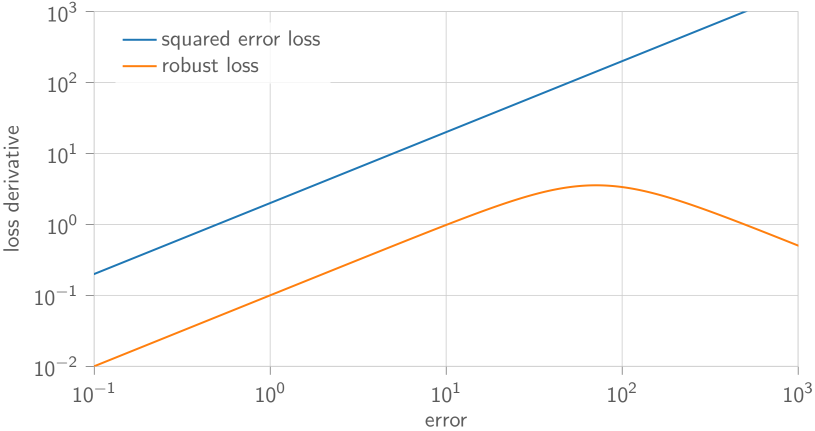

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minimize $\sum_{ij \in E} \ell(y^{(ij)}, f^{(ij)})$

Robust loss

$\ell(y^{(ij)}, f^{(ij)}) = \log\left(1 + \frac{(y^{(ij)} - f^{(ij)})^2}{\nu |y^{(ij)}|_\tau}\right)$

Baseline models

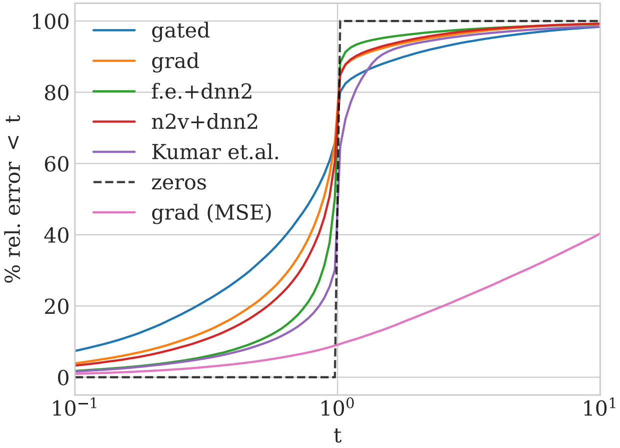
- Gradient model (Robust loss)
- Gradient model, MSE (Squared loss)
- 6 engineered node features + 2 layer MLP (Robust loss)
- 128 node2vec[2] node representations + 2 layer MLP (Robust loss)
- Weight prediction model by Kumar et. al.[3]

[2] Aditya Grover and Jure Leskovec. "node2vec: Scalable feature learning for networks". In: *Proceedings of the 22nd ACM SIGKDD international conference on Knowledge discovery and data mining*. 2016, pp. 855–864.

[3] S. Kumar et al. "Edge Weight Prediction in Weighted Signed Networks". In: *2016 IEEE 16th International Conference on Data Mining (ICDM)*. 2016, pp. 221–230.

Quantitative results

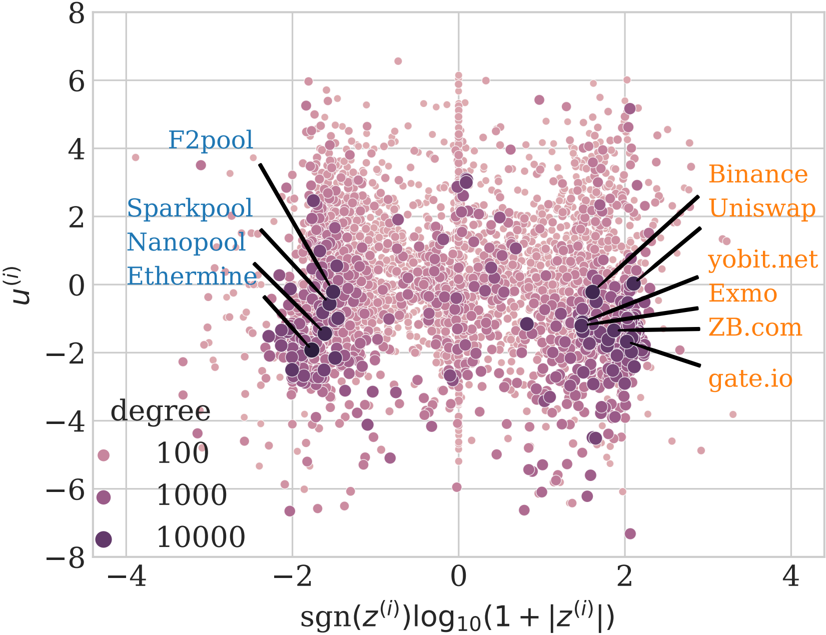
- Rel. error: $|y^{(ij)} - f^{(ij)}|/|y^{(ij)}|$
- Trivial baseline: $f^{(ij)} = 0$



Median abs. error

GATED	GRAD	F.E.+DNN2	N2V+DNN2	KUMAR ET.AL.	ZEROS	GRAD (MSE)
9.34	10.39	13.10	11.25	14.19	14.61	396.63

Qualitative results



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Thank you for your attention!

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