Improving Lossless Compression Rates via Monte Carlo Bits-Back Coding

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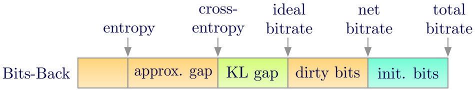
ICML 2021 (Long Talk)

Overview

Bits-back coding [8]...

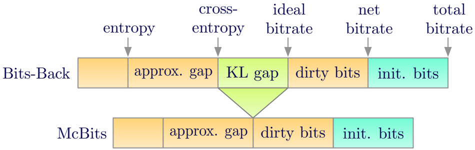
- ☺ successfully applies latent variable models to lossless compression
- ☺ achieves a bitrate equal to the negative ELBO
- ☹ suffers from a KL gap in the bitrate to the cross-entropy



Overview

We derive better bits-back schemes from tighter variational bounds..

- ✔ remove the KL gap with better bitrates
- ✔ introduce little additional cost
- ✔ better for out-of-distribution data compression



Background

Lossless Compression

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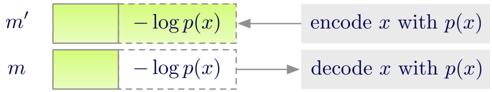
Key to understand: entropy coder is a store of randomness



Lossless Compression with Latent Variable Models

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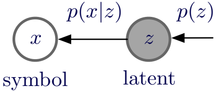
- Introduce an unobserved latent variable $z \in \mathbb{S}'$



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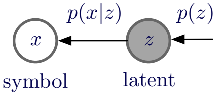

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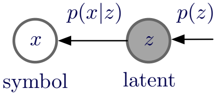

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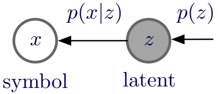

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☹ Communicating z is redundant!

Bits-Back Compression with ANS

Bits-Back with Asymmetric Numeral Systems (BB-ANS) [11] achieves a better bitrate!

- Adopt a *stack*-like entropy coder ANS
- Compress a sequence of symbols in a chain
- Decode latents z with an *approximate posterior* $q(z \mid x)$ from the intermediate message state instead of picking z

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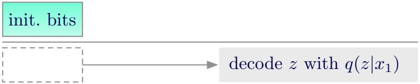

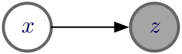

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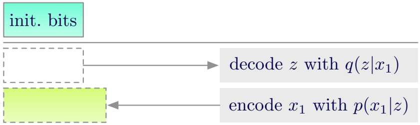

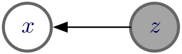

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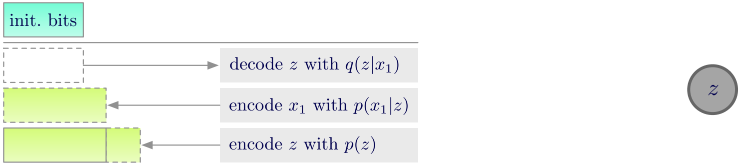

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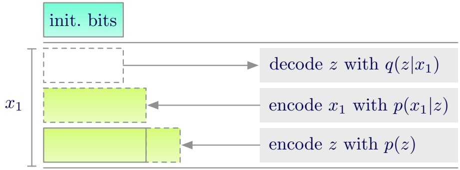

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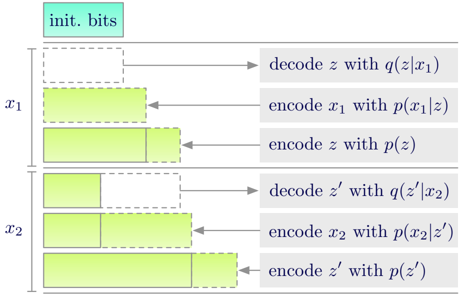

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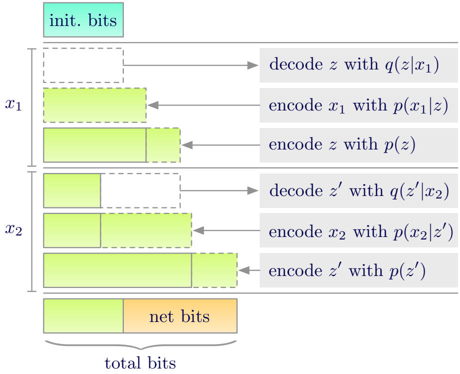

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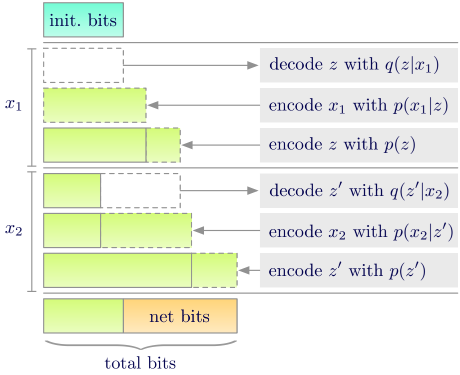

Quantities of interest

- Initial bits
- Net bitrate
- Total bitrate

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In an ideal scenario, if we assume $z \sim q(z \mid x)$, the *net* bitrate of BB-ANS achieves the (negative) 'evidence lower bound' (ELBO):

$$\begin{aligned}&\mathbb{E}_{z \sim q(z \mid x)} [-\log p(x, z) + \log q(z \mid x)] \\ &= -\log p(x) + D_{\mathrm{KL}}(q(z \mid x) \,\|\, p(z \mid x))\end{aligned}$$

We refer to vanilla BB-ANS as BB-ELBO

Motivation

Tighter variational bound is better!

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Can we derive bits-back coders from those tighter bounds and approach the cross-entropy?

Our Method: McBits
Monte Carlo Bits-Back Coding

General Framework

McBits coders are built from elaborate Monte Carlo estimators

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- Given a positive unbiased MC estimator of the marginal likelihood $\hat{p}_N(x)$ that can be simulated with $\mathcal{O}(N)$ random variables

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- A variational bound on $\log p(x)$ can be derived from $\hat{p}_N(x)$ by Jensen's inequality

$$\mathbb{E}[\log \hat{p}_N(x)] \leq \log p(x)$$

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Key step: identity the extended latent space representation of $\hat{p}_N(x)$

- Extended latent variables $\mathcal{Z} \sim Q(\mathcal{Z} \mid x)$
- Proposal distribution $Q(\mathcal{Z} \mid x)$
- Target distribution $P(x, \mathcal{Z})$

$$\hat{p}_N(x) = \frac{P(x, \mathcal{Z})}{Q(\mathcal{Z} \mid x)}$$

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☺ The variational bound can be viewed as an ELBO!

$$\mathcal{Z} \leftrightarrow z$$

$$Q(\mathcal{Z} \mid x) \leftrightarrow q(z \mid x)$$

$$P(x, \mathcal{Z}) \leftrightarrow p(x, z)$$

General Framework

Derive McBits coders in a similar way to BB-ELBO

Algorithm: General Procedures of McBits Coders

Procedure *Encode(sym x, msg m)*
- decode $\mathcal{Z}$ with $Q(\mathcal{Z} \mid x)$
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☺ It achieves an ideal net bitrate of $-\mathbb{E}[\log \hat{p}_N(x)]$!

☺ If $-\mathbb{E}[\log \hat{p}_N(x)] \to -\log p(x)$, it approaches the cross-entropy!

Bits-Back Importance Sampling

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- IS samples N particles $z_i \sim q(z_i \mid x)$ i.i.d. and uses the average importance weights to estimate $p(x)$

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- The corresponding variational bound (IWAE) [3]:

$$\mathbb{E}_{\{z_i\}_{i=1}^N} \left[\log \left(\sum_{i=1}^{N} \frac{1}{N} \frac{p(x, z_i)}{q(z_i \mid x)} \right) \right] \leq \log p(x)$$

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- Under mild conditions, the IWAE bound converges monotonically to $\log p(x)$ as $N \to \infty$

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- The extended space variables $\mathcal{Z}$ include the configurations of the N particles $\{z_i\}_{i=1}^N$ and an index $j \in \{1 \,..\, N\}$
- IWAE is the ELBO between a different pair of distributions $P(x, \mathcal{Z})$ and $Q(\mathcal{Z} \mid x)$ over the extended space

$$\frac{P(x, \mathcal{Z})}{Q(\mathcal{Z} \mid x)} = \sum_{i=1}^{N} \frac{1}{N} \frac{p(x, z_i)}{q(z_i \mid x)}$$

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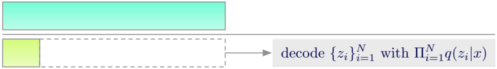

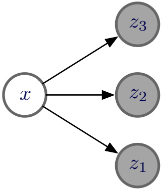

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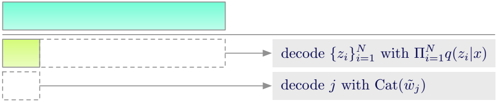

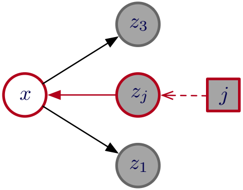

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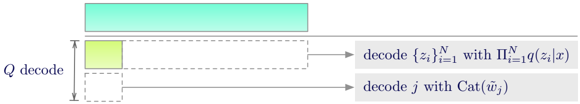

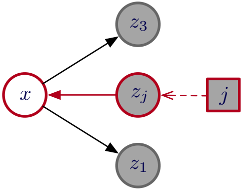

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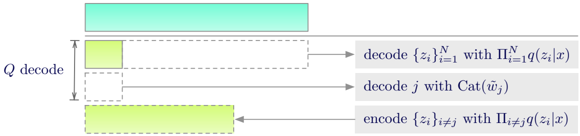

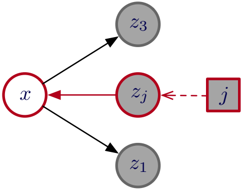

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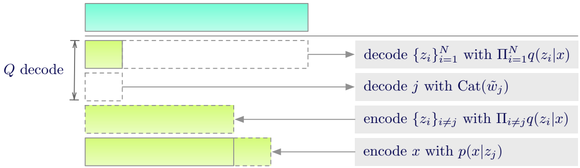

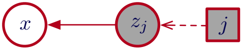

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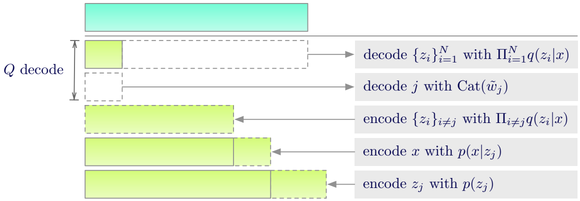

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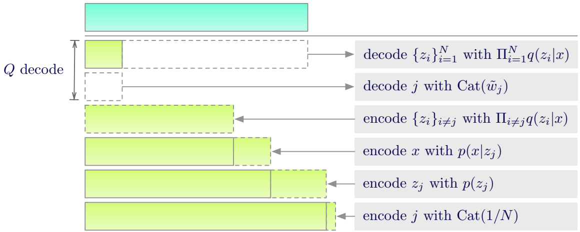

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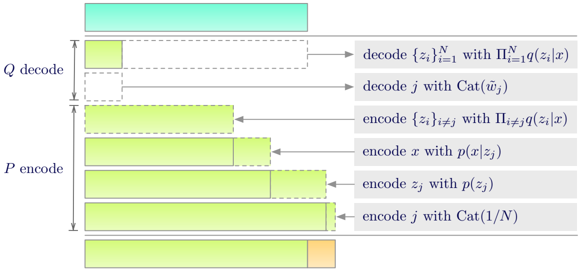

Bits-Back Importance Sampling

BB-IS...

- ☺ ideally achieves a net bitrate equal to the negative IWAE and asymptotically reaches the cross-entropy
- ☹ requires $\mathcal{O}(N)$ *initial bits* $\Leftarrow \mathcal{O}(N)$ decoded latent variables

Bits-Back Coupled Importance Sampling

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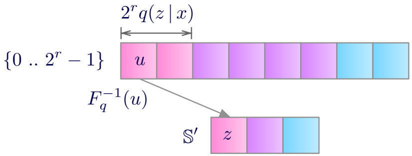
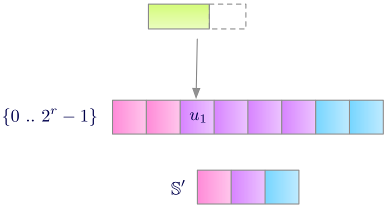
Discrete analog: suppose q is approximated to an integer precision r such that $2^r q(z \mid x)$ is an integer for all $z \in \mathbb{S}'$. The discrete analog of the inverse CDF function F_q^{-1} maps the uniform samples on $\{0 \,..\, 2^r - 1\}$ into samples from q



Bits-Back Coupled Importance Sampling

Reparamerization: reparamerize the particles $\{z_i\}_{i=1}^N$ by a *single* uniform random variable u_1



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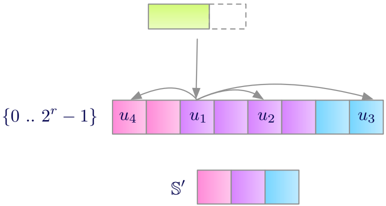

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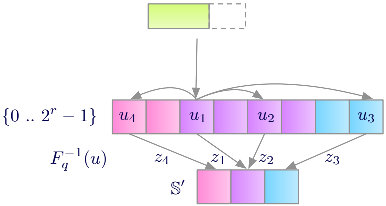

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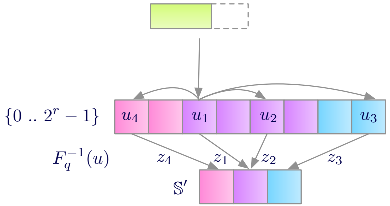

☺ Decoding a single uniform is all you need!

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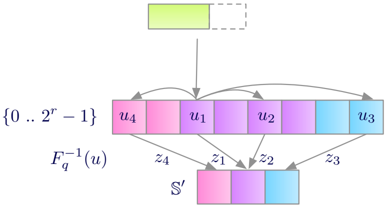

- ☺ Decoding a single uniform is all you need!
- ☺ The initial bit cost is reduced to

$$r - \log \tilde{w}_j \in \mathcal{O}(1) + \mathcal{O}(\log N) = \mathcal{O}(\log N)$$

In practice, the $\mathcal{O}(1)$ term dominates

Bits-Back Coupled Importance Sampling

BB-CIS...

☺ achieves a net bitrate comparable to BB-IS

$$-\mathbb{E}_{u_1}\left[\log\left(\sum_{i=1}^{N}\frac{1}{N}\frac{p(x, z_i)}{q(z_i \mid x)}\right)\right]$$

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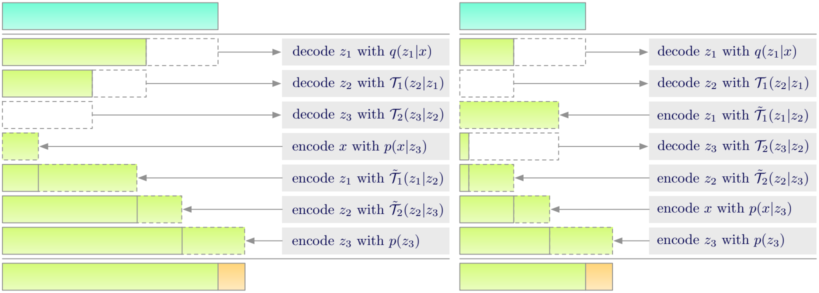
- ☺ significantly reduces the initial bit cost of BB-IS
  $\Rightarrow$ may motivate other coupling schemes that reduce initial bits

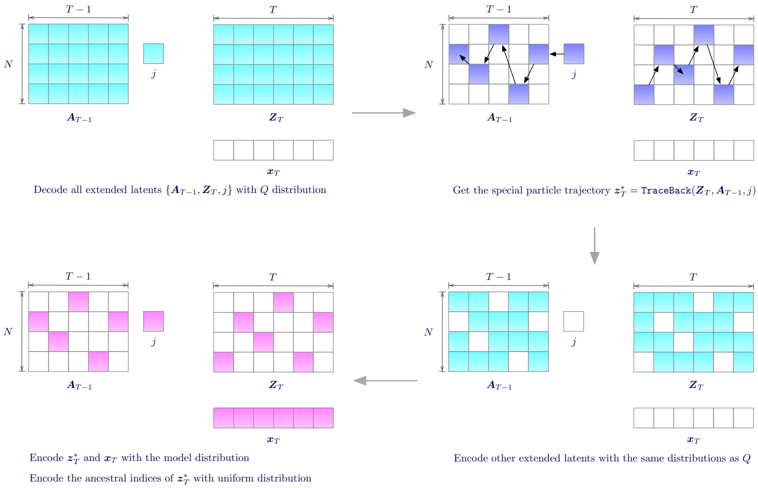
Bits-Back Annealed Importance Sampling

Bits-Back Annealed Importance Sampling (BB-AIS) and its Bit-Swap [9] variant (BB-AIS-BitSwap) for reducing initial bit cost



(a) BB-AIS **(b)** BB-AIS-BitSwap

Bits-Back Sequential Monte Carlo (BB-SMC) and its coupled variant (BB-CSMC) for reducing initial bit cost



Experiments

Computational Cost

- McBits coders scale linearly with the number of particles N

[1] Available at *https://github.com/j-towns/crayjax*

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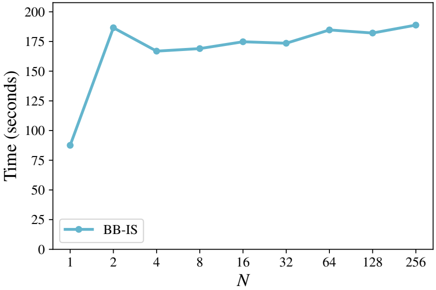
Computational Cost

- McBits coders scale linearly with the number of particles N
- However, particles are amenable to parallelization
- We implemented[1] vectorized rANS on the GPU, which allows McBits coders to scale sublinearly with particles

Total encode + decode times for the binarized MNIST test set



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Toy Mixture Model: Net Bitrate $\rightarrow$ Entropy

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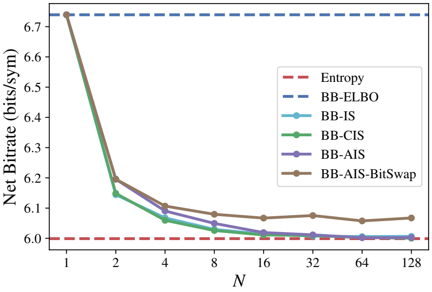
- We performed experiments on a toy mixture model, where the data generating distribution is randomly initialized and known
- A uniform approximate posterior was used to ensure a large mismatch with the true posterior
- As $N \rightarrow \infty$, the net bitrate converges to the entropy for most coders, as expected



- Observation and latent alphabet sizes were 64 and 256, respectively
- Compression was performed on 5000 symbols

Toy Mixture Model: Initial Bit Cost

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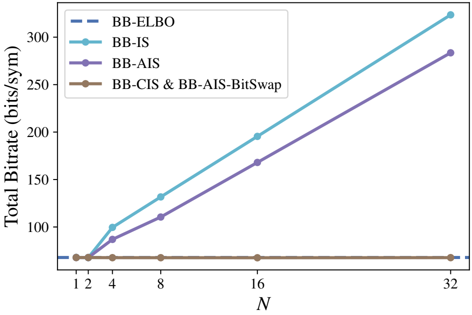
Toy Mixture Model: Initial Bit Cost

- We quantified the initial bit cost by computing the total bitrate (the net bitrate plus initial bits per symbol) after the first symbol
- The initial bit cost of naive coders scales linearly with particles, but coupled and BitSwap [9] variants significantly reduce it



Toy Hidden Markov Model: Net Bitrate $\rightarrow$ Entropy

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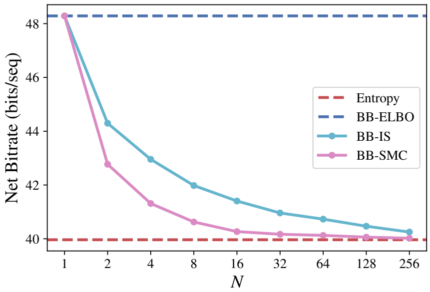
Toy Hidden Markov Model: Net Bitrate $\rightarrow$ Entropy

- We performed experiments on data from a toy Hidden Markov Model where prior, emission and transition probabilities were known, and a uniform approximate posterior was used
- As $N \rightarrow \infty$, the net bitrates of BB-IS and BB-SMC converge to the entropy, but BB-SMC converges much faster



- Observation and latent alphabet sizes were 16 and 32, respectively
- A uniform approximate posterior was used, and other distributions were randomly initialized
- Compression was performed on 5000 sequences with 10 time-steps each
- The entropy was estimated empirically using the forward algorithm

EMNIST: Transfer Learning Setting

- We trained a VAE, with Gaussian latents and Bernoulli observations, on the binarized EMNIST-Letters and EMNIST-MNIST datasets [5]

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- We trained a VAE, with Gaussian latents and Bernoulli observations, on the binarized EMNIST-Letters and EMNIST-MNIST datasets [5]
- Compression performance was evaluated on both tests sets
- BB-IS achieves greater rate savings than BB-ELBO in the out-of-distribution setting

Trained on	MNIST		Letters	
Compressing	MNIST	Letters	MNIST	Letters
BB-ELBO	0.236	0.310	0.257	0.250
BB-IS ($N = 5$)	0.231	0.289	0.249	0.243
BB-IS ($N = 50$)	0.228	0.280	0.244	0.239
Savings	3.4%	9.7%	5.1%	4.4%

Polyphonic Music Datasets: BB-SMC for Sequential Data

- We used a chunked version of 4 polyphonic music datasets from [2] to evaluate the compression performance of BB-SMC on sequential datasets

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- We used a chunked version of 4 polyphonic music datasets from [2] to evaluate the compression performance of BB-SMC on sequential datasets
- Models were based on the variational RNN [4], with Gaussian latents and Bernoulli observations, and trained following [10]
- BB-SMC achieves the best net bitrates (bits/timestep) on all piano roll test sets.

	Musedata	Nott.	JSB	Piano.
BB-ELBO	10.66	5.87	12.53	11.43
BB-IS ($N = 4$)	10.66	4.86	12.03	11.38
BB-SMC ($N = 4$)	**9.58**	**4.76**	**10.92**	**11.20**
Savings	10.1%	18.9%	12.8%	2.0%

Conclusion

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Conclusion

- McBits are bits-back coders that exploit the extended space representations of tighter variational bounds for better bitrates
- The initial bit cost of McBits coders scales linearly with particles, but can be significantly reduced by coupling the extended latents
- When parallelizing computation over particles, McBits coders can achieve better bitrates than BB-ANS with little overhead
- Experiments indicate that BB-IS enjoys better bitrate savings in out-of-distribution compression settings

Thank you!

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