An Information-Geometric Distance on the Space of Tasks

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Motivation. We would like to theoretically characterize the distance between two learning tasks.

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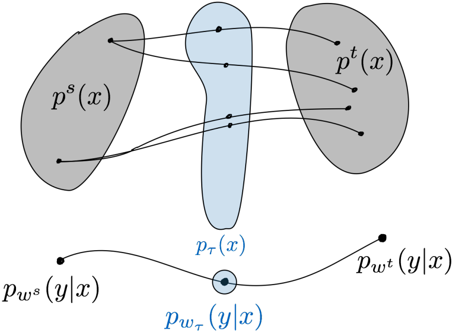
4. Task distance should be comparable across different network architectures.

Coupled transfer distance: Modifying the task and classifier synchronously during transfer

$$p^s(x;y) = p^s{}_{w_s}(yjx)\; p^s(x) \quad ! \quad p^t(x;y) = p^t{}_{w_t}(yjx)\; p^t(x): \qquad (1)$$



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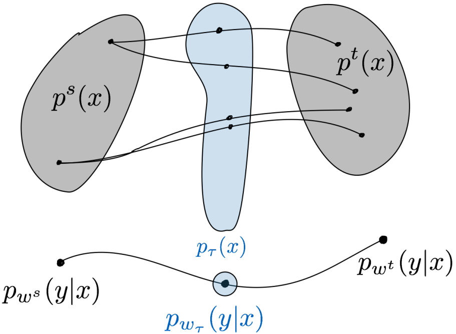

Roughly speaking, the length of shortest trajectory connecting $p^s(x;y)$ and $p^t(x;y)$ on statistical manifold parametrized by $w\ 2\ W$ is our transfer distance.

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$$\mathrm{d}s^2 = 2\mathrm{KL}[p_w, p_{w+\mathrm{d}w}] = \sum_{i,j=1}^{p} g_{ij}\, \mathrm{d}w_i \mathrm{d}w_j, \qquad (2)$$

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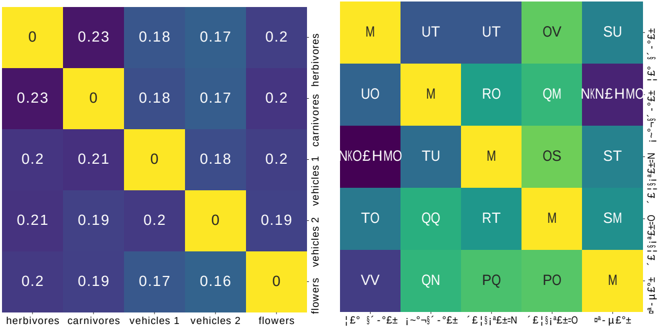
3. **Coupled transfer distance** between learning tasks is the solution of the following optimization problem.

$$\min_{\in} \int_0^1 \mathop{\mathbb{E}}_{(x,y) \sim p\ (x,y)} \sqrt{2\mathrm{KL}\, p_{w(\ )}(\cdot|x),\ p_{w(\ +\mathrm{d}\ )}(\cdot|x)}$$

$$\text{subject to } \frac{\mathrm{d}w(\ )}{\mathrm{d}} = \nabla_w \mathop{\mathbb{E}}_{(x,y) \sim p} \log p_{w(\ )}(y|x) \qquad (3)$$

$$\text{and } p\ (x) = \sum_{i=1}^{N_s} \sum_{j=1}^{N_t} {}_{ij}\ (1\ \ )x_i + x'_j(x)$$

Experiments: transferring across super-classes of CIFAR-100



(a) Coupled Transfer Distance ($r = 0{:}14; p = 0{:}05$ with fine-tuning)

(b) Fine-Tuning ($r = 0{:}36; p = 0{:}03$ with itself)

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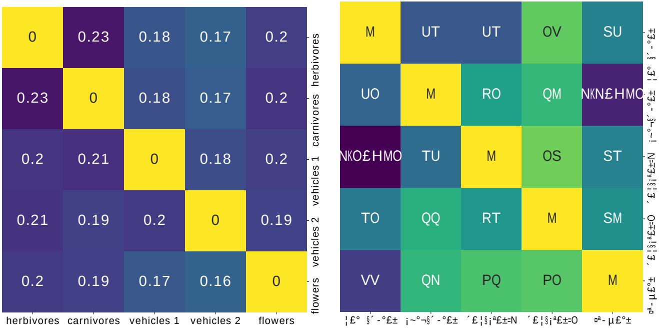

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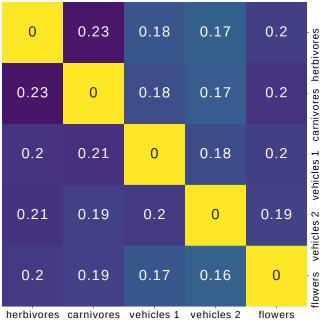

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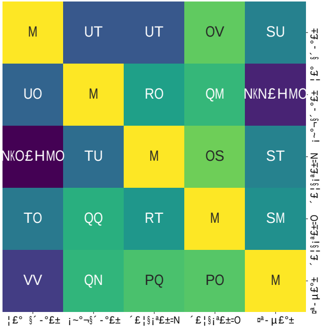
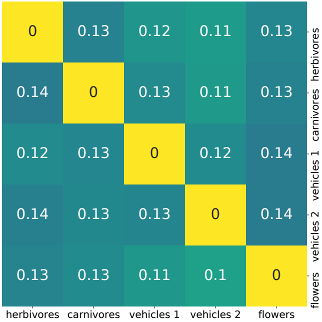


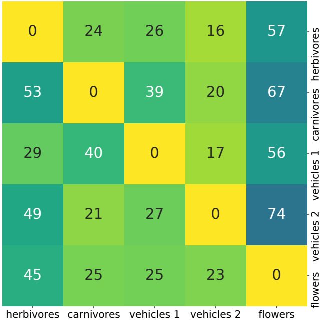
(b) Fine-Tuning (r = 0:36; p = 0:03 with itself)

1. We use the Mantel test to accept/reject the null hypothesis that variations in two distance matrices are correlated. Large r with small p indicates better correlation.
2. Task2Vec(Achille et al., 2019) does NOT correlate with the difficulty of fine-tuning well.

Larger model capacity results in smaller task distance



(a) Couple Transfer Distance(WideRes), (r = 0:15; p = 0:01 with fine-tuning)



(b) Fine-Tuning(WideRes), (r = 0:39; p = 0:01 with itself)

The larger WRN-16-4 model has a smaller task distance for all pairs compared to the smaller convolutional network on the previous slide.

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2. Coupled transfer distance accurately reflects the difficulty of transfer/fine-tuning.
3. Future work: Both task and weights are modified synchronously here, we would like to use the tools developed here for practical applications, e.g., to design methods that can select the best source task or the best architecture to transfer.