Test of Time Award **Online Dictionary Learning for Sparse Coding**

Julien Mairal, Francis Bach, Jean Ponce, Guillermo Sapiro

International Conference on Machine Learning, 2019





Test of Time Award

Online Learning for Matrix Factorization and Sparse Coding

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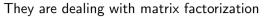


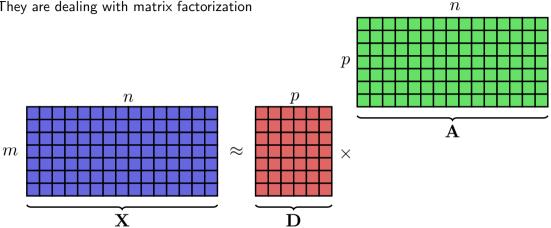


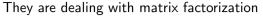
Francis

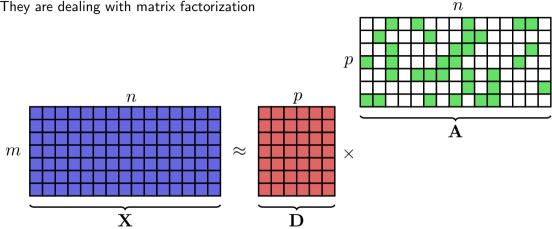
Jean

Guillermo

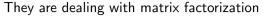


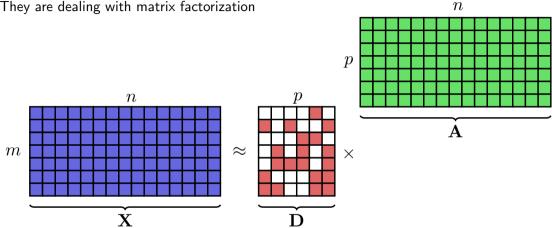




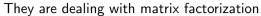


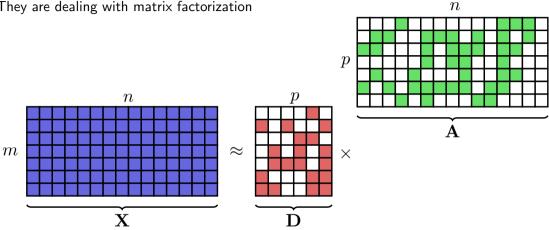
when a factor is **sparse**.



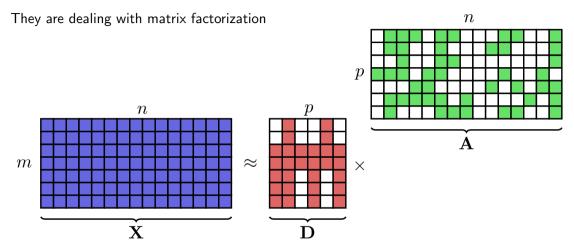


or the other one.

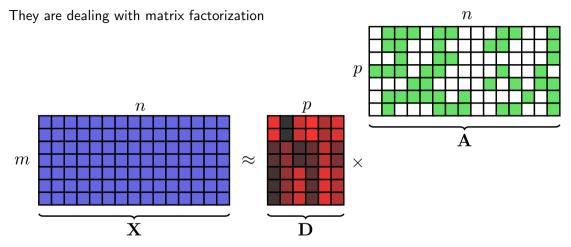




or both.

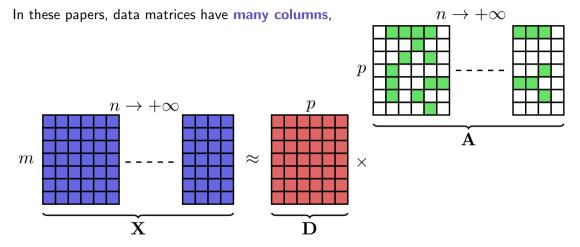


or not only one factor is sparse, but it admits a particular structure.



or one factor admits a particular structure (e.g., piecewise constant), but it is not sparse.

What these papers are about?



or an infinite number of columns, or columns are streamed online.

- $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]$ is a data matrix.
- We may call $\mathbf{D} = [\mathbf{d}_1, \dots, \mathbf{d}_p]$ a dictionary.
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Interpretation as signal/data decomposition

$$\mathbf{X}pprox \mathbf{D}\mathbf{A} \qquad \Longleftrightarrow \qquad orall \, i, \, \, \mathbf{x}_ipprox \mathbf{D}oldsymbol{lpha}_i = \sum_{j=1}^p oldsymbol{lpha}_i[j] \mathbf{d}_j.$$

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Generic formulation

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Generic formulation / stochastic case

$$\min_{\mathbf{D}\in\mathcal{D}} \mathbb{E}_{\mathbf{x}} \left[L(\mathbf{x},\mathbf{D}) \right] \quad \text{ with } \quad L(\mathbf{x},\mathbf{D}) \stackrel{\scriptscriptstyle \Delta}{=} \min_{\boldsymbol{\alpha}\in\mathcal{A}} \frac{1}{2} \|\mathbf{x}-\mathbf{D}\boldsymbol{\alpha}\|^2 + \lambda \psi(\boldsymbol{\alpha}).$$

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Which formulations does it cover?

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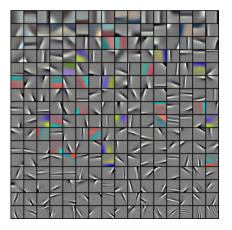
The sparse coding context

was introduced by Olshausen and Field, '96. It was the first time (together with ICA, see [Bell and Sejnowski, '97]) that a simple unsupervised learning principle would lead to

various sorts of "Gabor-like" filters, when trained on natural image patches.

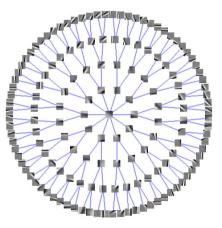


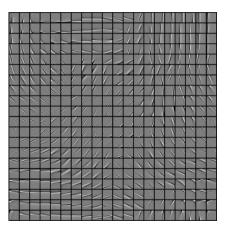




The sparse coding context

Remember that we can play with various structured sparsity-inducing penalties:





[Jenatton et al. 2010], [Kavukcuoglu et al., 2009], [Mairal et al. 2011], [Hyvärinen and Hoyer, 2001].



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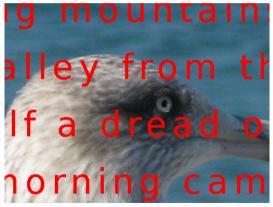
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- 2004: compressed sensing (Candes, Romberg and Tao).
- 2006: Elad and Aharon's image denoising method.

Many successful stories of dictionary learning in image processing

• image denoising, inpainting, demosaicing, super-resolution

[Elad and Aharon., 2006], [Mairal et al., 2008], [Yang et al., 2008] . . .

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- dictionary learning on top of SIFT wins the PASCAL VOC'09 challenge.
- another variant wins the ImageNet 2010 challenge.

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Matrix factorization becomes a key technique for unsupervised data modeling

- recommender systems (Netflix prize) and social networks.
- document clustering.
- genomic pattern discovery.

Θ...

[Koren et al., 2009b], [Ma et al. 2008], [Xu et al. 2003], [Brunet et al., 2004]...

Classical approach for matrix factorization: alternate minimization

$$\min_{\mathbf{D}\in\mathcal{D},\mathbf{A}\in\mathcal{A}}\frac{1}{2}\|\mathbf{X}-\mathbf{D}\mathbf{A}\|_{\mathsf{F}}^{2}+\lambda\psi(\mathbf{A}).$$

which requires loading all data at every iteration (batch optimization).

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see Léon's tutorial at NIPS'07, or NeurIPS'18 test of time award [Bottou and Bousquet, 2008].

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which makes the risk minimization point of view relevant:

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Idea 1: If we knew optimal codes α^{*}_i for all x_i's in advance, then the problem becomes

$$\min_{\mathbf{D}\in\mathcal{D}}\left\{\frac{1}{2}\mathsf{trace}\left(\mathbf{D}^{\top}\mathbf{D}\mathbf{B}\right)-\mathsf{trace}\left(\mathbf{D}^{\top}\mathbf{C}\right)\right\} \text{ with } \mathbf{B}=\frac{1}{n}\sum_{i=1}^{n}\boldsymbol{\alpha}_{i}^{\star}\boldsymbol{\alpha}_{i}^{\star\top} \text{ and } \mathbf{C}=\frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_{i}\boldsymbol{\alpha}_{i}^{\star\top},$$

which yields parameter-free block coordinate descent rules for updating D.

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What about theory?

We could provide guarantees of convergence to stationary points, even though the problem is **non-convex**, **stochastic**, **constrained**, and **non-smooth**.

[Neal and Hinton, '98], [Mairal, 2013], [Mensch, 2018].

A timely context (\approx luck)

Datasets were becoming larger and larger, and there was **suddenly a need for more scalable** matrix factorization methods.

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A combination of mathematics and engineering?

- an efficient software package: the SPAMS toolbox.
- robustness to hyper-parameters: default setting that works (many times) in practice.
- (try it with pip install spams in Python, or download R/Matlab packages).

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Flexibility in the constraints/penalty design

• allowing the method to be used in unexpected contexts.

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SANG

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Computer vision cracks the leaf code

Peter Wilf^{a,1}, Shengping Zhang^{b,c,1}, Sharat Chikkerur^d, Stefan A. Little^{a,e}, Scott L. Wing^f, and Thomas Serre^{b,1}

^aDepartment of Geosciences, Pennsylvania State University, University Park, PA 16802; ^bDepartment of Cognitive, Linguistic and Psychological Sciences, Brown Institute for Brain Science, Brown University, Providence, RI 02912; ^cSchool of Computer Science and Technology, Harbin Institute of Technology, Weihai 264209, Shandong, People's Republic of China; ^dAzure Machine Learning, Microsoft, Cambridge, MA 02142; ^cLaboratoire Ecologie, Systématique et Evolution, Université Paris-Sud, 91405 Orsay Cedex, France; and ⁱDepartment of Paleobiology, National Museum of Natural History, Smithsonian Institution, Washington, DC 20013

Edited by Andrew H. Knoll, Harvard University, Cambridge, MA, and approved February 1, 2016 (received for review December 14, 2015)

Understanding the extremely variable, complex shape and venation characters of angiosperm leaves is one of the most challenging species (15–19), and there is community interest in approaching this problem through crowd-sourcing of images and machine-identifi-

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NAS

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Stability-driven nonnegative matrix factorization to interpret spatial gene expression and build local gene networks

Siqi Wu^{a,b}, Antony Joseph^{a,b,c}, Ann S. Hammonds^b, Susan E. Celniker^b, Bin Yu^{a,d,1}, and Erwin Frise^{b,1}

^aDepartment of Statistics, University of California, Berkeley, CA 94720; ^bDivision of Environmental Genomics and Systems Biology, Lawrence Berkeley National Laboratory, Berkeley, CA 94720; ⁶Walmart Labs, San Bruno, CA 94066; and ^dDepartment of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720

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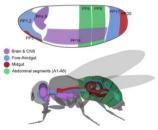
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Flexibility in the constraints/penalty design

Mapping a Cell's Destiny

New Berkeley Lab Tool Speeds Discovery of Spatial Patterns in Gene Networks

Science Shorts Sarah Yang (510) 486-4575 • MAY 4, 2016



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Datasets were becoming larger and larger, and there was suddenly a need for more scalable matrix factorization methods

A combination of mathematics and engineering?

an e Correcting gene expression data when neither the o robi unwanted variation nor the factor of interest are observed • (try LAURENT JACOB* Laboratoire de Biométrie et Biologie Évolutive, Université de Lyon, Université Lyon 1, Flexibilit CNRS. UMR. 5558 Lvon. France allow laurent.jacob@univ-lvon1.fr JOHANN A. GAGNON-BARTSCH Department of Statistics, University of California, Berkelev, CA 974720, USA TERENCE P. SPEED Department of Statistics, University of California, Berkelev, CA 974720, USA and Division of Online Dictionary Learning for Sparse Coding

Julien Mairal

practice.

A cheap way to obtain a sparse code β from x and D is

$$\boldsymbol{\beta} = \mathsf{relu}(\mathbf{D}^{\top}\mathbf{x} - \lambda),$$

versus

$$oldsymbol{lpha} \in \operatorname*{arg\,min}_{oldsymbol{lpha} \in \mathcal{A}} rac{1}{2} \| \mathbf{x} - \mathbf{D}oldsymbol{lpha} \|^2 + \lambda \| oldsymbol{lpha} \|_1.$$

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Then, not surprisingly, for dictionary learning,

- end-to-end feature learning is feasible.
- one can design convolutional and multilayer models.

[Mairal et al., 2012], [Zeiler and Fergus, 2010]

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$$\boldsymbol{\beta} = \mathsf{relu}(\mathbf{D}^{\top}\mathbf{x} - \lambda),$$

versus

$$oldsymbol{lpha} \in \operatorname*{arg\,min}_{oldsymbol{lpha} \in \mathcal{A}} rac{1}{2} \| \mathbf{x} - \mathbf{D} oldsymbol{lpha} \|^2 + \lambda \| oldsymbol{lpha} \|_1.$$

Then, not surprisingly, for dictionary learning,

- end-to-end feature learning is feasible.
- one can design convolutional and multilayer models.
- sparse decomposition algorithms perform neural network-like operations (LISTA).

[Mairal et al., 2012], [Zeiler and Fergus, 2010], [Gregor and LeCun, 2010].

Thoughts



Is Wrinch and Jeffrey's simplicity principle still relevant?

- Simplicity is a key to interpretability and to model/hypothesis selection.
- Next form will probably be different than ℓ_1 . Which one?
- Simplicity is not enough. Various forms and robustness and stability are also needed.

[Yu and Kumbier, 2019].