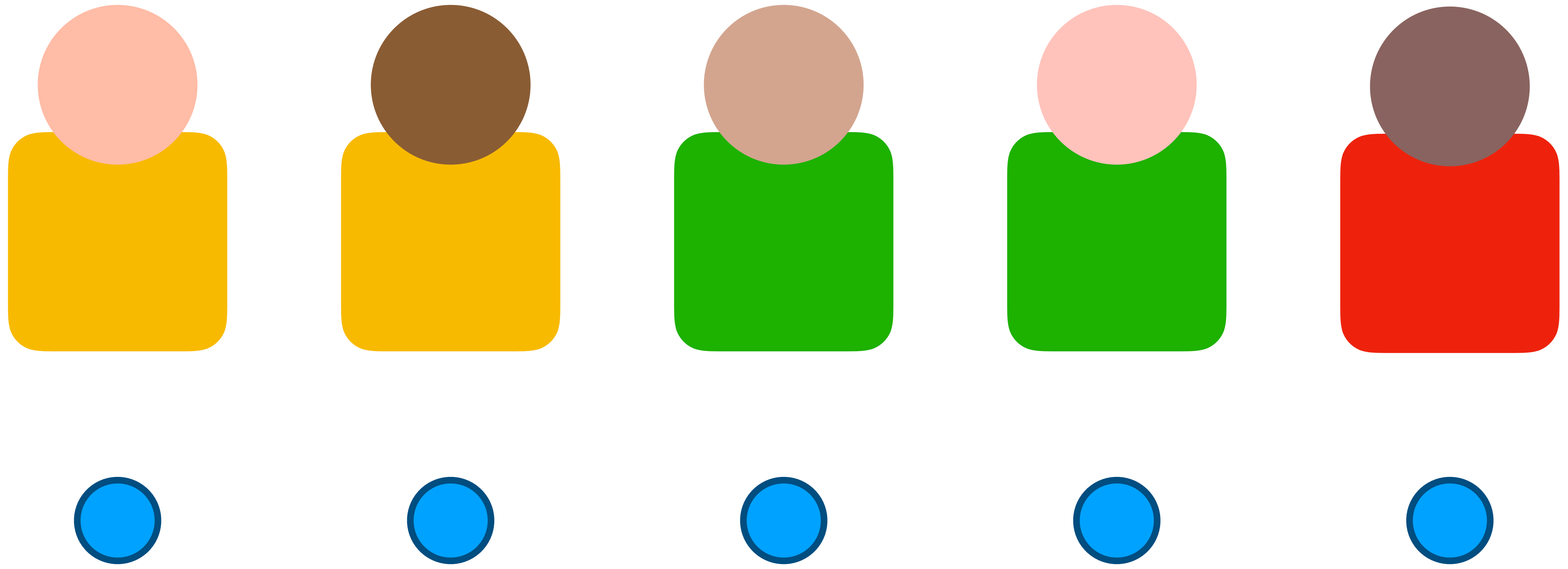
Bounding User Contributions: A Bias-Variance Trade-off in Differential Privacy

Kareem Amin, Alex Kulesza, Andrés Muñoz Medina, Sergei Vassilvitskii

Google Research NY

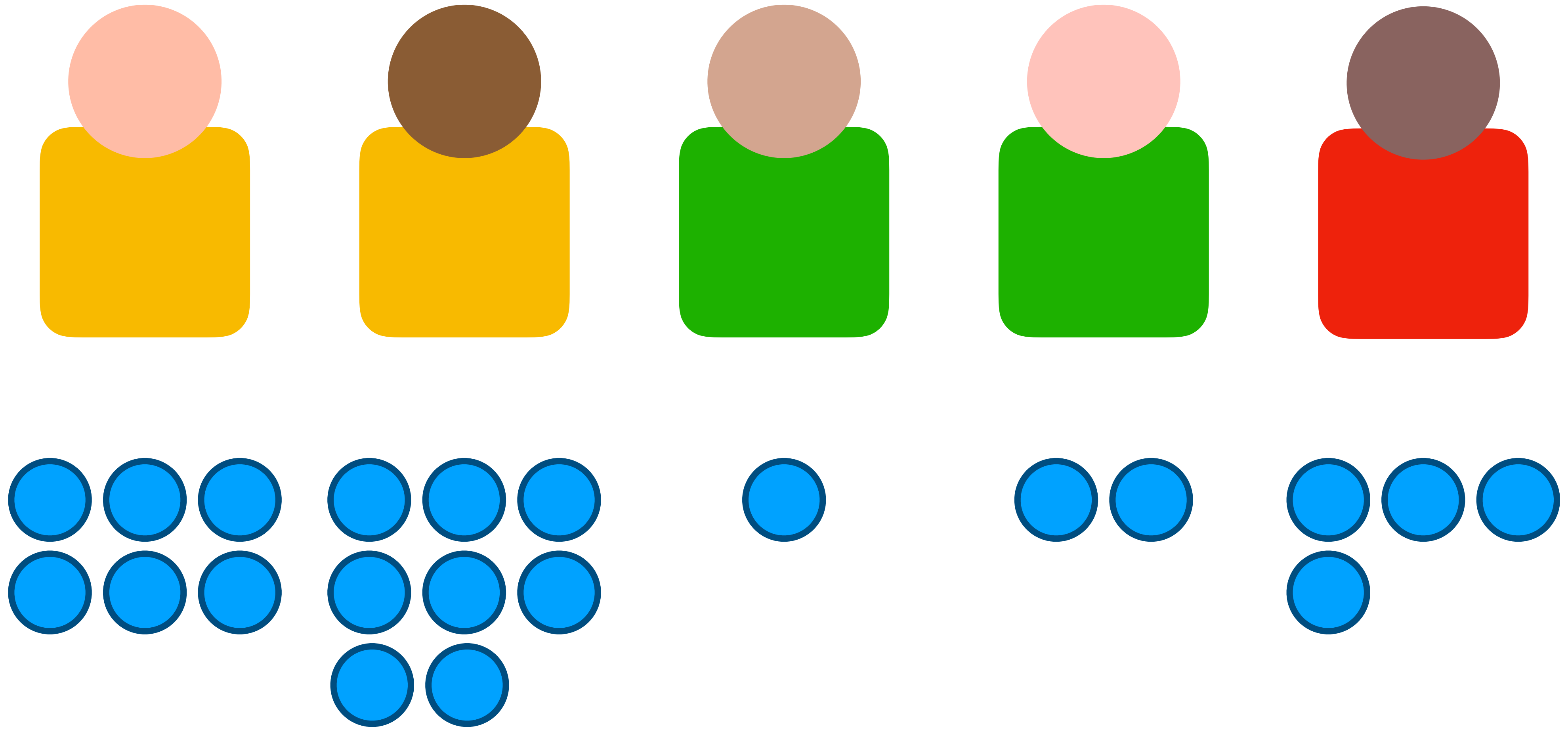
Typical DP assumption:

One user = one example

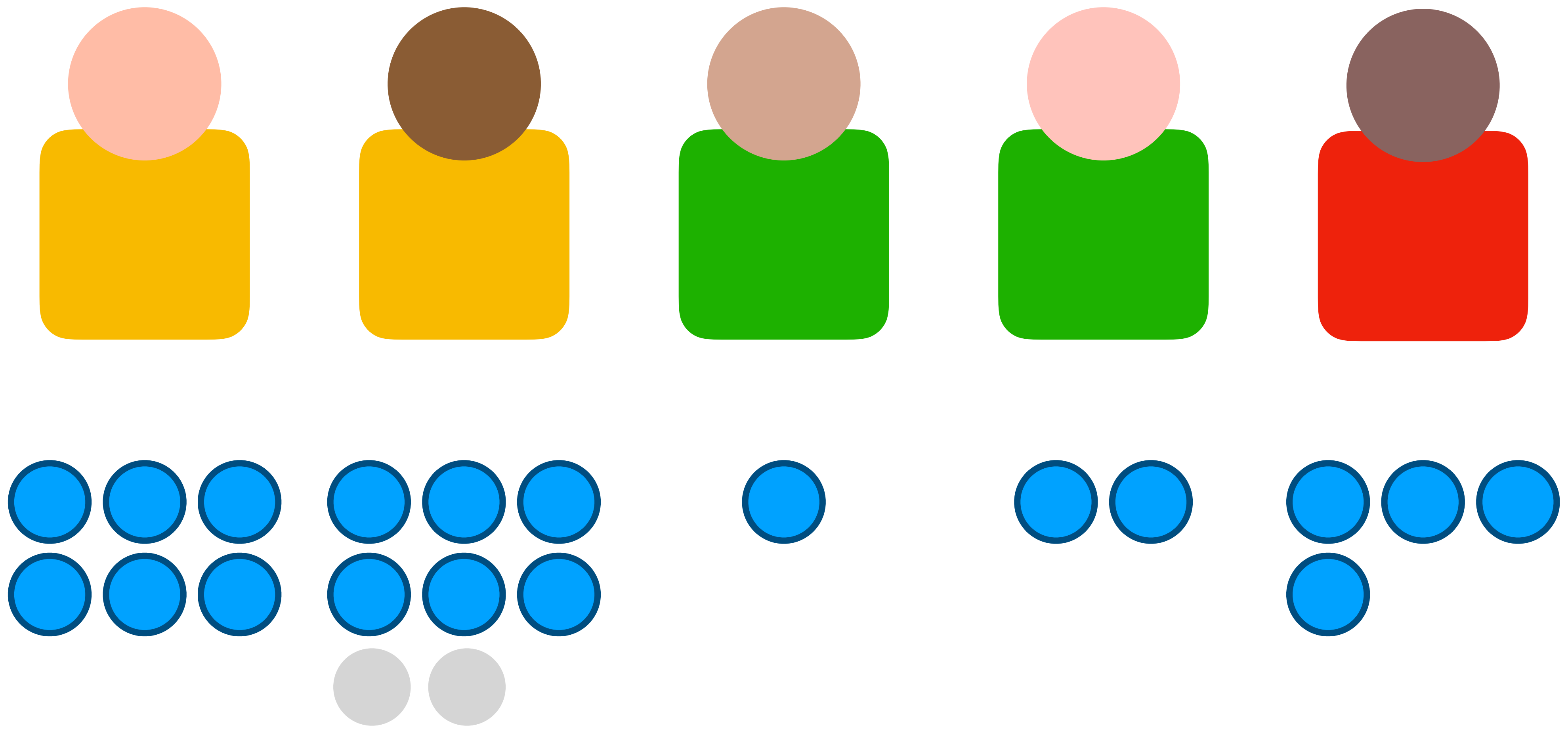


Reality:

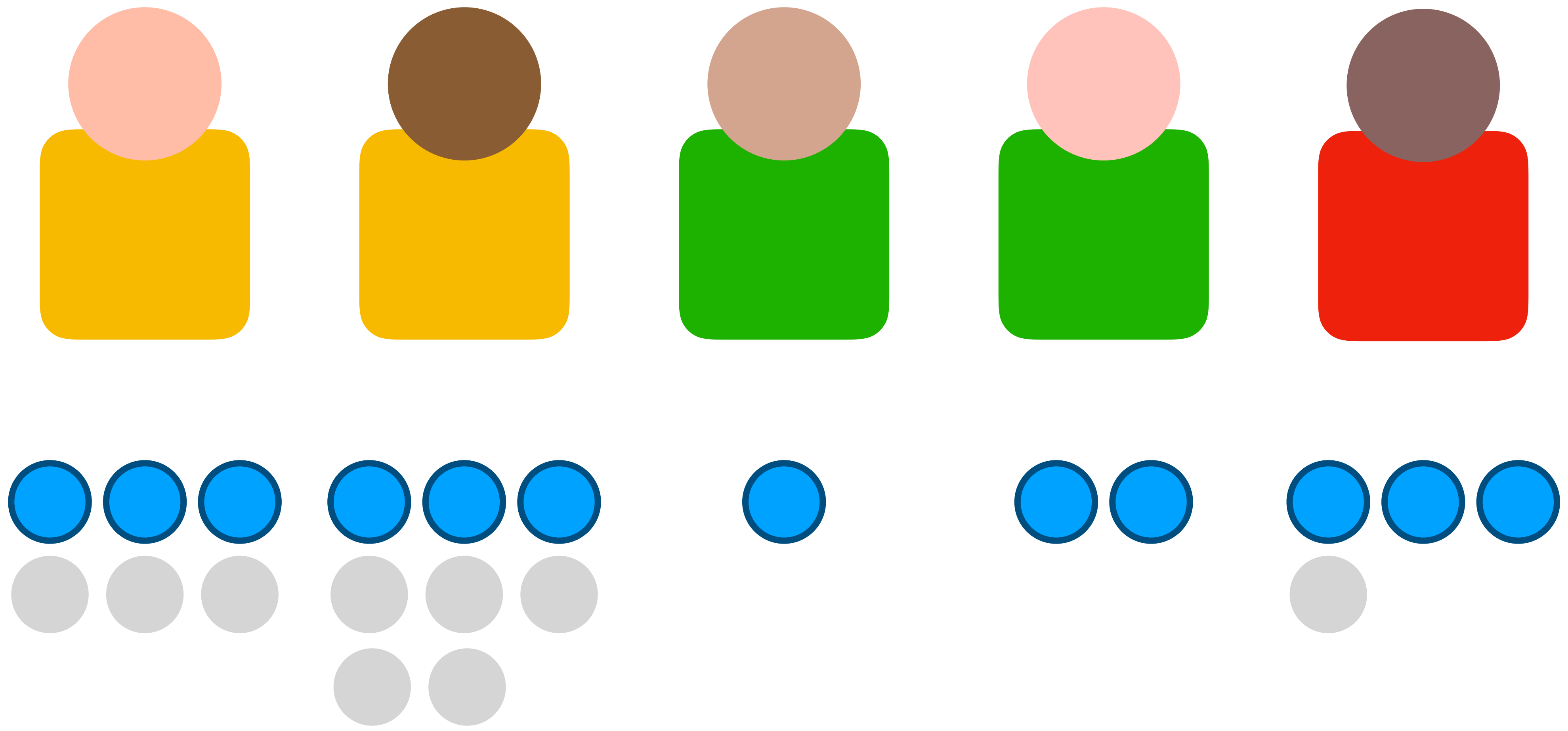
Users contribute many times



High cap = excessive noise



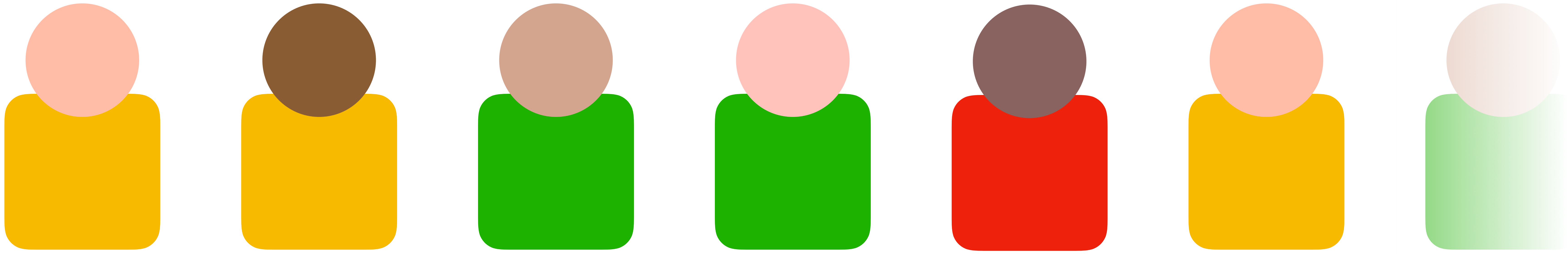
Low cap = biased data



We investigate this bias-variance trade-off using tools from learning theory

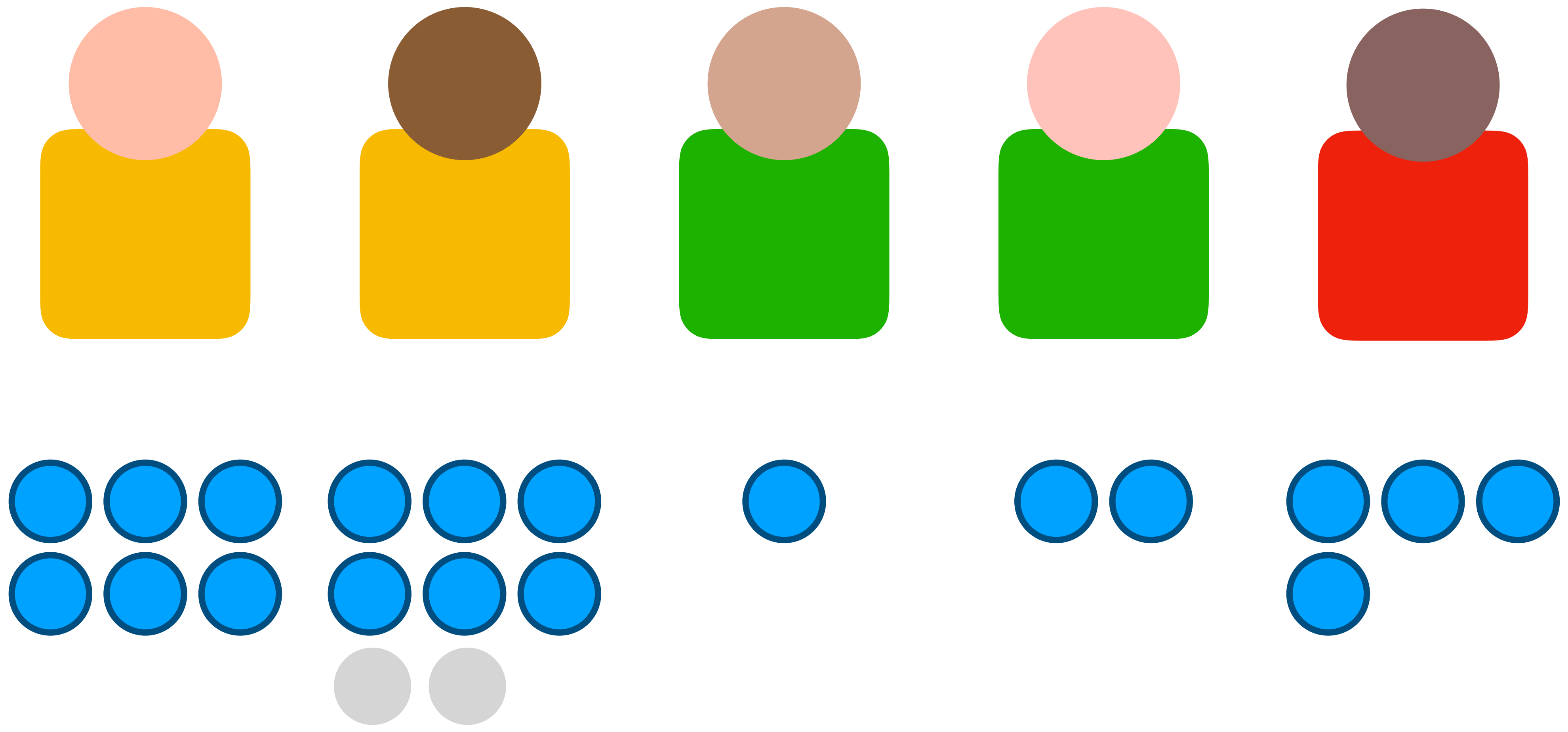
Setting

Infinite collection of users



- Distribution ***P*** over users
- Each user has a unique distribution over examples
- I.i.d. data: first sample a user from ***P***, then sample the user's distribution

Learning



- Cap each user at a τ_0 fraction of the dataset
- Run a standard differentially private ERM algorithm

Result

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + \text{Bias due to capping} + \text{Finite sample variance} + \text{Privacy noise variance}$$

Result

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \quad \text{Finite sample variance} \quad + \quad \text{Privacy noise variance}$$

Bias due to capping

Result

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \tilde{O}\left(\sqrt{\frac{1}{\tau_0 n}}\right) + \text{Privacy noise variance}$$

Bias due to capping

Finite sample variance

Result

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \tilde{O}\left(\sqrt{\frac{1}{\tau_0 n}}\right) + O\left(\frac{1}{K^2(\tau_0)}\right)$$

Bias due to capping

Finite sample variance

Privacy noise variance

The Cost of Privacy

As n→∞ …

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \tilde{O}\left(\sqrt{\frac{1}{\tau_0 n}}\right) + O\left(\frac{1}{K^2(\tau_0)}\right)$$

The Cost of Privacy

For privacy noise to vanish, $\tau_0 \to 0$

As $n \to \infty$ …

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \tilde{O}\left(\sqrt{\frac{1}{\tau_0 n}}\right) + O\left(\frac{1}{K^2(\tau_0)}\right)$$

The Cost of Privacy

As $n \to \infty$ …

But then bias grows without bound

For privacy noise to vanish, $\tau_0 \to 0$

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \tilde{O}\left(\sqrt{\frac{1}{\tau_0 n}}\right) + O\left(\frac{1}{K^2(\tau_0)}\right)$$

The Cost of Privacy

As $n \to \infty$ …

But then bias grows without bound

For privacy noise to vanish, $\tau_0 \to 0$

$$\mathcal{L}(h_{\text{priv}}) \leq \inf_{h \in H} \mathcal{L}(h) + O\left(\sqrt{\frac{\text{Var}(H)}{\tau_0}}\right) + \tilde{O}\left(\sqrt{\frac{1}{\tau_0 n}}\right) + O\left(\frac{1}{K^2(\tau_0)}\right)$$

Privacy incurs a fixed cost: we cannot recover optimal error even when $n \to \infty$