PAC Identification of Many Good Arms in Stochastic Multi-Armed Bandits

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What Is It All About?



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Get around:

- Identifying 1 from the best ρ -fraction is possible.
- Redefine the problem to identify 1 from the best *m* arms.
- Defining $\rho = \frac{m}{n}$, generalise the problem.
- What if we *n* is relatively small?



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• k = m = 1: Best arm identification.

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Contributions:

- LUCB-k-m (Fully sequential + Adaptive).
- Worst case upper and lower bound.

- **k** = 1: Any 1 arm out of the best *subset* of size *m*.
- **k** = **m**: Best *subset* identification.
- k = m = 1: Best arm identification.

 (\mathbf{k}, ρ) : To identify **any** distinct **k** arms from the **best** ρ fraction of arms.

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ho)$ Not well-posed $P_{\mathcal{A}}$ $(\mathbf{k},
ho)$ Well-posed (\mathbf{k}, ρ) Which one is easier to solve (1, m, n) or (1, P)?



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