# Cautious Regret Minimization: Online Optimization with Long-Term Budget Constraints

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Depuis 80 ans, nos connaissances bâtissent de nouveaux mondes

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# Motivating Example: Online Advertisement

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- Select over *d* websites to place bids for advertisements $\mathbf{x_t} = \{x_t^1, ..., x_t^d\}$, over $\mathrm{T}$ slots up to a budget of $b_T$
- You select your bid and then pay the a priori unknown $\mathbf{p_t^\intercal x_t}$
    - Depends on: competitor fraud, website popularity change, customer behavior
- Long term budget constraint $\sum_{t=1}^{T} \mathbf{p_t^\intercal x_t} \leq b_t$
- Return profit is revealed after the decision $\mathbf{r_t^\intercal x_t}$

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***Target Online Optimization***

$$\begin{aligned} \text{maximize} \quad & \sum_{t=1}^{T} \mathbf{r_t^\intercal x_t} \\ \text{subject to} \quad & \sum_{t=1}^{T} \mathbf{p_t^\intercal x_t} \leq b_t \\ & \mathbf{x_t} \in \mathcal{X}, \ \forall t \in \{1, ..., T\} \end{aligned}$$

# Prior Work

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***Online Convex Optimization* with**
***Long Term Adversarial Constraints***

$$\begin{aligned} \text{maximize} \quad & \sum_{t=1}^{T} f_t(\mathbf{x_t}) \\ \text{subject to} \quad & \sum_{t=1}^{T} g_t(\mathbf{x_t}) \\ & \mathbf{x_t} \in \mathcal{X}, \ \forall t \in \{1, ..., T\} \end{aligned}$$

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| Paper | Type | Bench. | Regret | Residual |
|---|---|---|---|---|
| Yuan et al. 2018 | Fix. | $T$ | $\mathcal{O}(\sqrt{T} + \frac{T}{V})$ | $\mathcal{O}(\sqrt{VT})$ |
| Yu et al. 2017 | Stoc. | $T$ | $\mathcal{O}(\sqrt{T})^\star$ | $\mathcal{O}(\sqrt{T})^\star$ |
| Neely & Yu 2017 | Adv. | 1 | $\mathcal{O}(\sqrt{T})^\dagger$ | $\mathcal{O}(\sqrt{T})^\dagger$ |
| Sun et al. 2017 | Adv. | 1 | $\mathcal{O}(\sqrt{T})$ | $\mathcal{O}(T^{3/4})$ |
| Mannor et al. 2009 | Adv. | $T$ | $\Omega(T)$ | $o(T)$ |
| **Us** | **Adv.** | $\boldsymbol{K}$ | $\boldsymbol{\mathcal{O}(\sqrt{T} + \frac{KT}{V})}$ | $\boldsymbol{\mathcal{O}(\sqrt{VT})}$ |

"No Regret" against the weakest benchmark

Impossibility against the strongest benchmark

# Cautious Online Lagrangian Descent

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**K-Benchmark**

$$x_*^K \in \operatorname*{argmin}_{x \in \mathcal{X}_K} \sum_{t=1}^{T} f_t(x)$$

$$\mathcal{X}_K \triangleq \left\{ x \in \mathcal{X} \;\middle|\; \sum_{\tau=t}^{t+K-1} g_\tau(x) \leq 0, \;\; \forall t \in \{1, \ldots, T-K+1\} \right\}$$

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**COLD**

$$\mathbf{x_t} = \Pi_{\mathcal{X}}\left[\mathbf{x_{t-1}} - \frac{V\mathbf{f}'_{\mathbf{t-1}}(\mathbf{x_{t-1}}) + Q(t)\mathbf{g}'_{\mathbf{t-1}}(\mathbf{x_{t-1}})}{2\alpha}\right]$$

$$Q(t+1) = [Q(t) + \hat{g}_t(x_t)]^+$$

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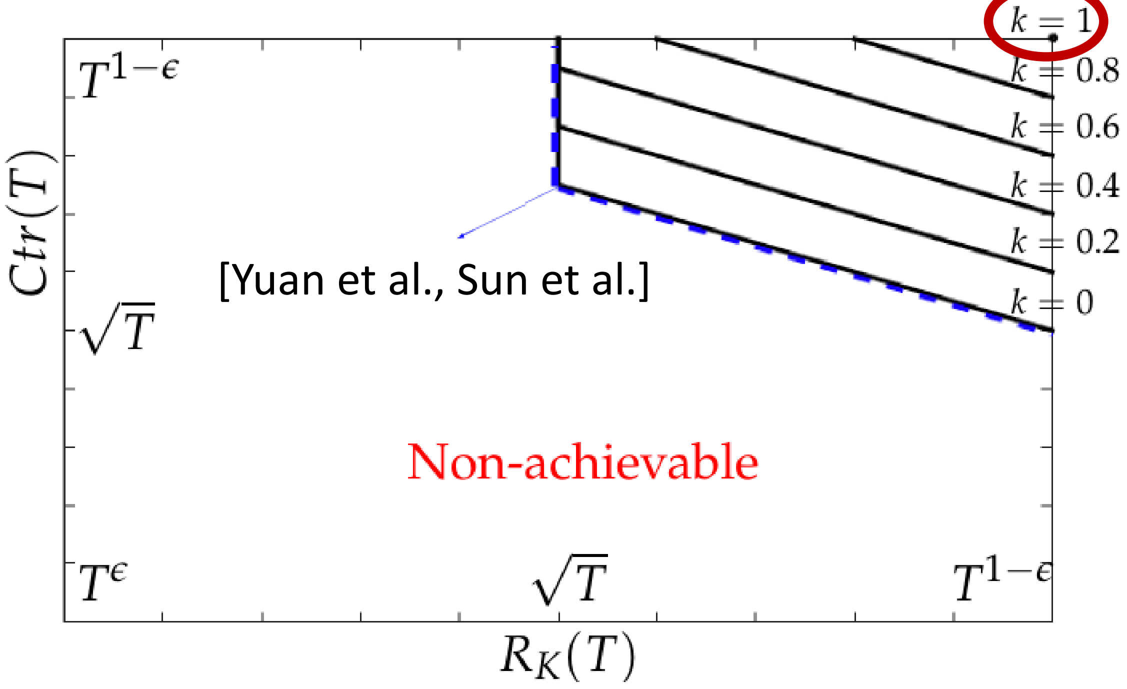
**Achievable Bounds**

“No regret” against $\mathbf{K = \mathcal{O}(T^{1-\epsilon})}$

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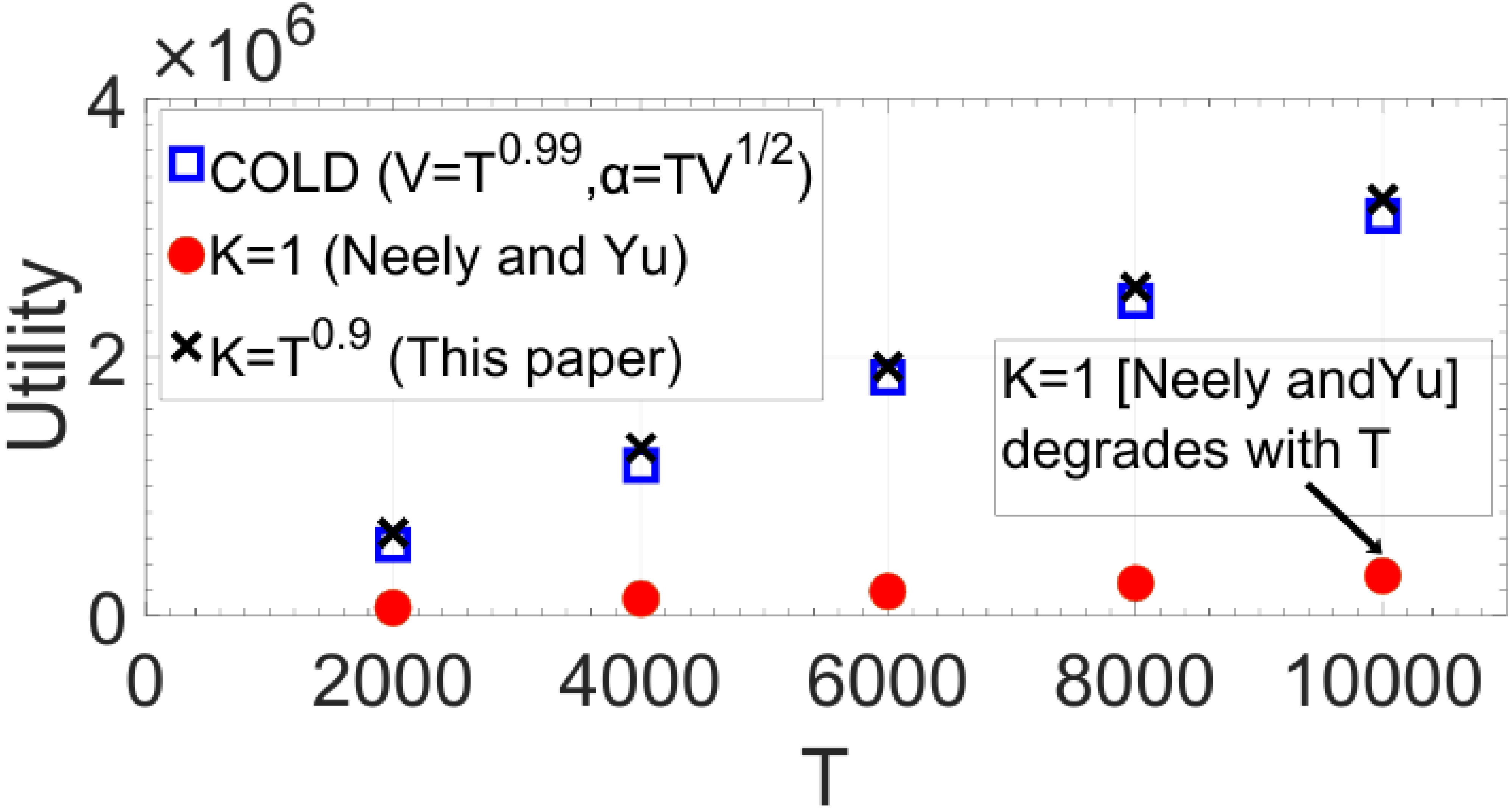
# Impact of Guarantee

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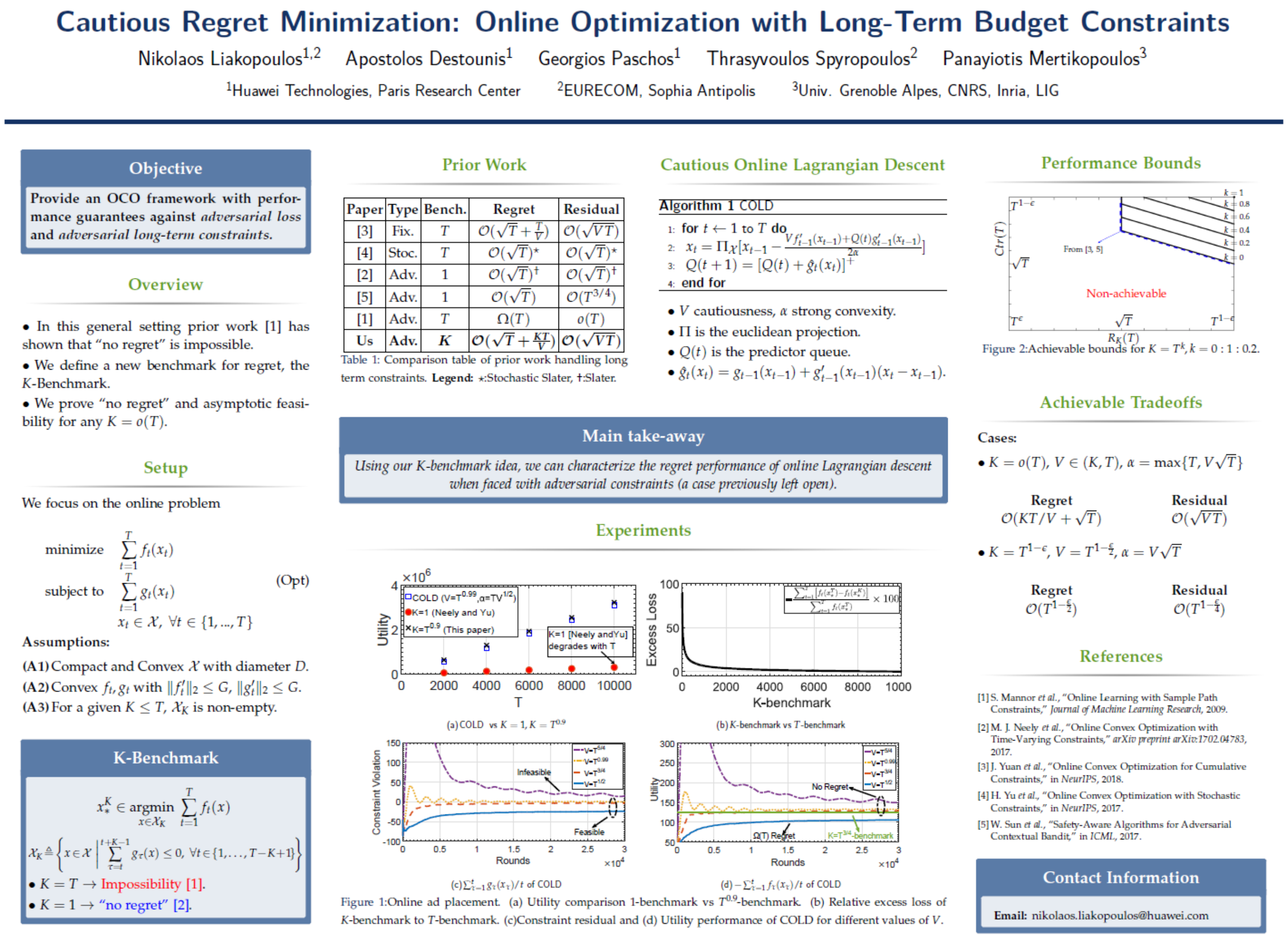
# Thanks!

## See you at our poster!

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