



## Causal Discovery and Forecasting in Nonstationary Environments with State-Space Models

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Two tasks:

- 1. Identify time-varying causal relations
- 2. Forecast the values of variables of interest

- Forecasting benefits from causal knowledge
  - Each causal module changes independently
  - Causal knowledge makes the forecasts more interpretable

Time-varying causal model:

$$x_{i,t} = \sum_{x_j \in \mathbb{P}\mathbb{A}_i} b_{ij,t} x_{j,t} + e_{i,t}$$

with 
$$e_{i,t} \perp \{b_{ij,t}, x_{j,t}\}_{x_j \in \mathbb{P}\mathbb{A}_i}$$
.

• 
$$b_{ij,t}$$
 and  $\sigma_{i,t}^2$  change over time

$$\begin{split} b_{ij,t} &= \alpha_{ij,0} + \sum_{p=1}^{p_l} \alpha_{ij,p} b_{ij,t-p} + \epsilon_{ij,t}, \\ h_{i,t} &= \beta_{i,0} + \sum_{q=1}^{q_l} \beta_{i,q} h_{i,t-q} + \eta_{i,t}, \quad \begin{array}{c} e_{i,t}, \epsilon_{ij,t}, \eta_{i,t} \\ &\text{where } h_{i,t} = \log \sigma_{i,t}^2. \\ \end{split}$$

## Causal Model, Identifiability, and Estimation

- Goal: Find time-varying causal relations & make prediction
- Causal relations change over time
  - **Model**: causal coefficients modeled by *autoregressive models*
- **Identifiability**: The causal model identifiable if the underlying causal structure is acyclic
- Model Estimation: A specific nonlinear state-space model
  - Estimated by Stochastic approximation EM with Conditional particle filter

Forecasting with time-varying causal model

 Treat forecasting as a Bayesian inference problem in the causal model



• Metropolis-Hastings to forecast  $Y_{T+1}$ 

$$p(Y_{T+1}|\mathcal{M}_{Y,T+1}) \propto p(Y_{T+1}|\mathcal{P}_{Y,T+1}) \prod_{\tilde{X}_{C_i} \in \mathcal{C}_Y} p(\tilde{X}_{C_i,T+1}|\mathcal{P}_{C_i,T+1})$$

Causal discovery:

0.9

Ours: highest F1 score!

Forecasting:

**Ours: lowest RMSE!** 



0.90

⊖Ours

♦IB

**₽**MC

\*LiNGAM

★CD-NOD

## Macroeconomics data

(quarterly data, 1965-2017, USA)





Methods	RMSE	Methods	RMSE
Ours	0.32	Lasso	0.38
Kalman filtering	0.42	Window Lasso	0.37
SSM (CPF)	0.43	GP	0.37

RMSE of the forecasts on inflation (2007 - 2017).



## Conclusion

- A unified framework for causal discovery and forecasting
- Establish the identifiability results, even when data is conditional Gaussian
- Future work
  - Improve the scalability
  - Nonlinear causal relationships, partially observable processes, and causal models with instantaneous cycles...