

# Automated Model Selection Using Bayesian Qaudarture

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# Posterior Probability of a Model

The posterior probability of model  $i$  (PPM) is defined as

$$z_i = Z_i / \sum_j Z_j$$

where

$$Z_i = \int f(\mathcal{D}|\theta_i)\pi(\theta_i)d\theta_i$$

is the model evidence for model  $i$  and  $\theta_i$  are the parameters of model  $M_i$ ,  $f(\mathcal{D}|\theta_i)$  is the likelihood and  $\pi(\theta_i)$  is the prior.

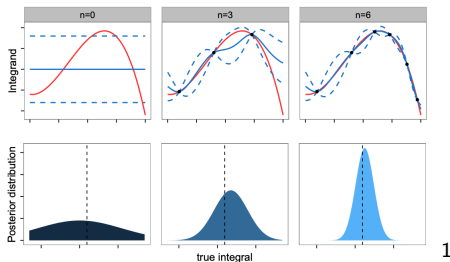
# Motivation

Comparing the model evidences usually relies on Monte Carlo (MC) methods, which converge slowly and are unreliable for expensive models.

# Bayesian Quadrature

Given some intractable integral

$$Z = \int f(\theta)\pi(\theta)d\theta$$



The goal is it to choose the points where we evaluate the GP efficiently. For example it does not make sense to take 2 points that are really "close".

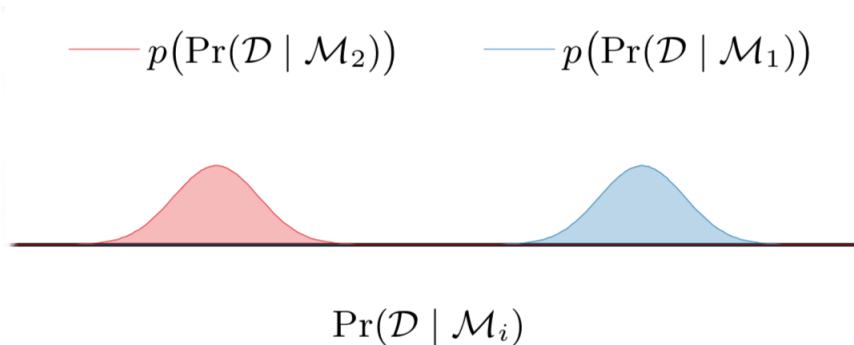
<sup>1</sup><https://warwick.ac.uk/girolami>

Hence we can apply Bayesian Quadrature to estimated the model evidences

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However, this may waste computation by producing an overly-accurate estimate for the evidence of a clearly poor model

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NOTE that our task is to decide where to evaluate the likelihood  $f$

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Hence we utilize the following Mutual Information

$$MI(z_i, f(\mathcal{D}|\theta_i))$$

yielding efficient acquisition of samples across disparate model spaces when likelihood observations are limited.

# Synthetic Data

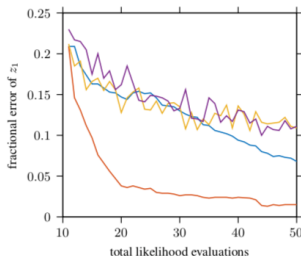
## Synthetic dataset

Model selection task between two zero-mean GP models

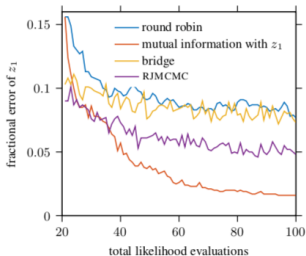
One with RBF kernel

Other with Matern5/2

The observed dataset  $\mathcal{D}$  consists of  $5d$  observations from a  $d$ -dimensional, zero-mean GP with a RBF kernel



(a)  $d = 1$



(b)  $d = 2$

End

Thanks you for your attention  
If interested visit poster 06:30 – 09:00 PM @ Pacific Ballroom

#219

