TEXAS
The University of Texas at Austin

Submodular Observation Selection and Information Gathering for Quadratic Models

Abolfazl Hashemi*, Mahsa Ghasemi, Haris Vikalo, and Ufuk Topcu

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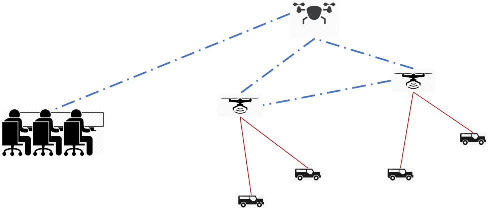
Observation Selection and Information Gathering

- Resource-constrained inference problems
  - Target tracking, experimental design, sensor networks
- Access to expensive / limited observations
  - Communication cost, power consumption, computational burden



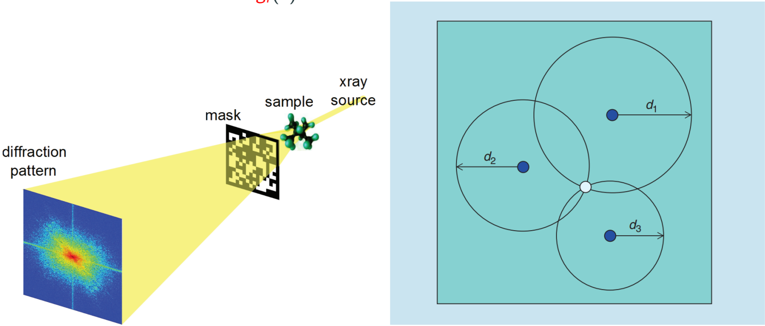
Goal

Cost-effectively identify the most useful subset of information

Observation Selection for Quadratic Models

- Quadratic relation between observations and unknown parameters

$$\mathbf{y}_i = \underbrace{\frac{1}{2}\mathbf{x}^\top \mathbf{Z}_i \mathbf{x} + \mathbf{h}_i^\top \mathbf{x}}_{g_i(\mathbf{x})} + \mathbf{v}_i\ , \quad i \in \{1, 2, \ldots, n\}$$



(a) Phase retrieval: $y_i = \frac{1}{2}\mathbf{x}^*(\mathbf{z}_i\mathbf{z}_i^*)\mathbf{x} + v_i$ (b) Localization: $\mathbf{y}_i = \frac{1}{2}\|\mathbf{h}_i - \mathbf{x}\|_2^2 + \mathbf{v}_i$

(Figures from [Candes'15] and [Gezici'05])

Locally-Optimal Observation Selection

- Challenge: Unknown optimal estimator and error covariance matrix
- Locally-optimal observation selection [Flaherty'06, Krause'08]: Linearize around a guess $\mathbf{x}_0$

$$\hat{y}_i := y_i - g_i(\mathbf{x}_0) \approx \nabla g_i(\mathbf{x}_0)^\top \mathbf{x} + v_i,$$

and find an approximate covariance matrix:

$$\hat{\mathbf{P}}_{\mathcal{S}} = \left(\boldsymbol{\Sigma}_x^{-1} + \sum_{i \in \mathcal{S}} \frac{1}{\sigma_i^2} \nabla g_i(\mathbf{x}_0) \nabla g_i(\mathbf{x}_0)^\top \right)^{-1}$$

- Observation selection task

$$\begin{aligned} \underset{\mathcal{S}}{\text{minimize}} \quad & \operatorname{Tr}\left(\hat{\mathbf{P}}_{\mathcal{S}}\right) \\ \text{s.t.} \quad & \mathcal{S} \subset [n], \;\; |\mathcal{S}| = K \end{aligned}$$

Proposed Approach: VTB for Quadratic Models

Main Idea

Exploiting Van Trees' bound (VTB) on error covariance of weakly biased estimators

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- A closed-form expression for VTB of quadratic models

Theorem 1

For any weakly biased estimator $\hat{\mathbf{x}}_{\mathcal{S}}$ with error covariance $\mathbf{P}_{\mathcal{S}}$ it holds that

$$\mathbf{P}_{\mathcal{S}} \succeq \left(\sum_{i \in \mathcal{S}} \frac{1}{\sigma_i^2} \left(\mathbf{Z}_i \Sigma_x \mathbf{Z}_i^\top + \mathbf{h}_i \mathbf{h}_i^\top \right) + \mathbf{I}_x \right)^{-1} = \mathbf{B}_{\mathcal{S}}$$

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- Proposed method: Find $\mathcal{S}$ by greedily maximizing Tr(.) scalarization of $\mathbf{B}_{\mathcal{S}}$: $f^A(\mathcal{S}) := \text{Tr}(\mathbf{I}_x^{-1} - \mathbf{B}_{\mathcal{S}})$

Characterizing $f^A(\mathcal{S})$

- Submodularity: $f_j(\mathcal{S}) \geq f_j(\mathcal{T})$ for all $\mathcal{S} \subseteq \mathcal{T} \subset \mathcal{X}$ and $j \in \mathcal{X} \backslash \mathcal{T}$
- α_f-Weak Submodularity [Zhang'16, Chamon17]: $\alpha_f \times f_j(\mathcal{S}) \geq f_j(\mathcal{T})$ where $\alpha_f > 1$ for all $\mathcal{S} \subseteq \mathcal{T} \subset \mathcal{X}$ and $j \in \mathcal{X} \backslash \mathcal{T}$

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- Greedy maximization performance:

$$f(\mathcal{S}) \geq (1 - e^{-\frac{1}{\alpha_f}}) f(\mathcal{O})$$

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$f^A(\mathcal{S})$ is a normalized, monotone set function with bounded α_{f^A}.

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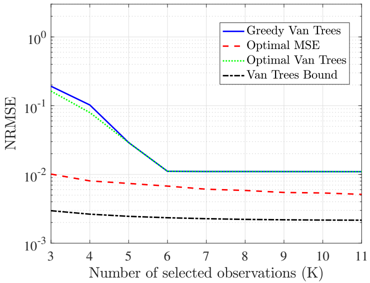
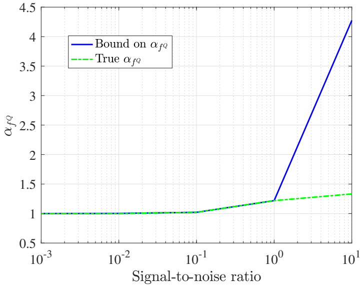
- Interpretation of bound on α_{f^A} as SNR condition

- Phase retrieval problem with $n = 12$ observations



(c) Tightness of VTB



(d) Bound on α_{fA}

- Asymptotic tightness of VTB
- Tightness of weak submodularity bound in low SNR regime

Thank you!

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Poster #167

Wed Jun 12th 06:30 PM – 09:00 PM @ Pacific Ballroom

Correspondance: Abolfazl Hashemi (email: abolfazl@utexas.edu)
Mahsa Ghasemi (email: mahsa.ghasemi@utexas.edu)