Learning Optimal Linear Regularizers

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Google

Setup

- Want to produce a model θ
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- Ultimately, we care about test loss: $L_{test}(\theta)$

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- Will minimize training loss + regularizer: $L_{train}(\theta) + R(\theta)$
- Ultimately, we care about test loss: $L_{test}(\theta)$
- An optimal regularizer: $R(\theta) = L_{test}(\theta) L_{train}(\theta)$

• suggests that a good regularizer should upper bound the generalization gap

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Approximate by maximizing over small set of models (estimating test loss using validation set)

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- **b** Under certain assumptions, can "jump" to optimal λ given data from just 1 + $|\lambda|$ models
- **TuneReg:** uses LearnReg iteratively to do hyperparameter tuning







