Differentially Private Learning of Geometric Concepts

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joint work with
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Privately Learning Union of Polygons

**Given:** $n$ points in $\mathbb{R}^2$ with binary labels: $\{(x_i, y_i)\}_{i=1}^n$

**Assume:** $\exists$ collection of polygons $\{P_1, ..., P_t\}$ with a total of at most $k$ edges s.t. $\forall i \in [n]: x_i \in \bigcup_j P_j \iff y_i = 1$

**Find:** Hypothesis $h: \mathbb{R}^2 \rightarrow \{0, 1\}$ with small error
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Find: Hypothesis $h: \mathbb{R}^2 \to \{0, 1\}$ with small error, while providing differential privacy for the training data:
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Find: Hypothesis \( h: \mathbb{R}^2 \rightarrow \{0, 1\} \) with small error, while providing differential privacy for the training data:

- Every labeled example represents the (private) information of one individual
- **Goal:** the output hypothesis does not reveal information that is specific to any single individual
Given: $n$ points in $\mathbb{R}^2$ with binary labels: $\{(x_i, y_i)\}_{i=1}^n$

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- Requirement: the output distribution is insensitive to any arbitrarily change of a single input example (an algorithm satisfying this requirement is differentially private)
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**Why is that a good privacy definition?**

Even if an observer knows all other data point but mine, and now she sees the outcome of the computation, then she still cannot learn “anything” on my data point
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Motivation: Analyzing Users’ Location Reports

- Analyzing GPS navigation data
- Learning the shape of a flood or a fire based on reports
- Identifying regions with poor cellular reception based on reports
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- Impossibility results for differential privacy show that this problem (and even much simpler problems) cannot be solved over infinite domains
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- We assume that input points come from $[d]^2 = \{1, 2, ..., d\} \times \{1, 2, ..., d\}$ for a discretization parameter $d$
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- Furthermore, the sample complexity must grow with the size of the discretization
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**Previous Result**

Private learner with sample complexity

\( O(k \cdot \log d) \) and runtime \( \approx d^k \)

(using a generic tool of MT’07)
Private learner with sample complexity $O(k \cdot \log d)$ and runtime $\approx d^k$ (using a generic tool of MT’07)

New Result
Private learner with sample complexity $\tilde{O}(k \cdot \log d)$ and runtime $\text{poly}(k, \log d)$
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Summary
✓ New algorithm for privately learning union of polygons
✓ Efficient runtime and sample complexity
✓ Applications to privately analyzing users’ location data